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Strategic Interactions among Firms, Interest Groups and Policy Makers

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Introduction

The term “Political Economy” has a very long story that dates back at least to Adam Smith, when it was used to define the discipline, derived from moral philosophy, that studied production and trade in relationship with national income and wealth and their distribution.

This is how Classical Economists used this expression at least until 1848, when John Stuart Mill published his *Principles of Political Economy*.

It is only after the publication, in 1890, of *Principles of Economics* by Alfred Marshall, that what used to be called Political Economy became “Economics”, and the use of the former became less and less frequent.

The word Political Economy has been revitalised in the second half of the XX century when economists became increasingly interested in how political and institutional constraints affect economic policies and the economy in general, and how it would be possible to study these interactions through the standard economic techniques. This led to a great expansion of what had historically been the field of interest of Economics. There are two seminal contributions in founding the modern Political Economy.

The first one was *The Calculus of Consent. Logical Foundations of Constitutional Democracy* by Buchanan and Tullock (1962), where they formally proposed methodological individualism to study the political process, thus refusing the concept of a public interest as something different than just the sum of private ones. The second one was *The Logic of Collective Action: Public Goods and the Theory of Groups* by Olson (1965) who analysed the formation of interest groups assuming that individuals are rational and egoist. His conclusion was that collective action will not take place for large groups unless they rely on selective incentives or on coercion.

In the last decades the economic literature studying problems related to policy decisions has grown at an incredible rate, and the notion of “Political Economy” has finally come to define the “research program of applying economic theory to the study of politics” (Wallerstein, 2004).\(^1\)

This interest of economists for the political sphere should not come as a surprise. After all, an economic analysis that does not take into account the social and political context fails to capture fundamental aspects of most of the problems that Economics itself should deal with. Overall, the risk is to propose economic policies that are not feasible from a social and political point of view, or to fail to understand why policy-makers makes decision that are different from what predicted by models that

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\(^1\) Actually some authors refer to this new approach as *New Political Economy* to distinguish from the words used by Classical Economy.
totally abstract from the social context.²

It is also not surprising that Game Theory is the most used tool in Political Economy. Game Theory was developed to study strategic interaction between rational decision-makers, and there are few human interactions more rational and strategic than politics. Politicians are well aware that each policy they choose or even each statement they make has an enormous range of potential consequences. Political success depends on the ability to anticipate such consequences in an extremely complex environment, and even more so in the era of social media. The same considerations also apply to all those individuals who gravitate around politicians: bureaucrats, senior civil servants and in particular lobbyists - who are often former politicians who have changed sides. It then follows that the concept of Nash equilibrium, intended as a situation in which every player’s best respond to the actions taken by all other players - although constrained by its well known limiting assumptions - has proven to be able to provide meaningful insights in identifying important channels and mechanisms in policy decision making.

The development of Political Economy has led on the one hand to the rigorous definition of the techniques that can be used in the theoretical treatment (the best example being Person & Tabellini, 2000) and, on the other hand, to the development of an empirical approach. More recently, Political Economy has also benefited from the introduction of new lines of research in Economics, with the development of “Behavioural Political Economy” (Schnellenbach & Schubert, 2015) and of “Experimental Political Economy” (Palfrey, 2016).

This thesis is the collection of three stand-alone theoretical contributions in which Game Theory is applied to analyse different issues connected to the interaction between the economic, political and institutional domains. These are issues which I believe are particularly relevant in the present context and have not been sufficiently addressed in the economic literature.

The first essay deals with the increasing role of bureaucrats in the political process, a phenomenon that in the last decades has been, in many ways, rather pervasive in Western countries. In particular, the focus is on the importance of bureaucrats in local governments where, following several reforms, often their role is not anymore only to help local politicians, but also to monitor their behaviour on behalf of the central government, thus performing a role similar to that of watchdogs.³ The idea behind these reforms was that bureaucrats, given that they do not run for offices, are less likely to extract income and pork barrel spending and, therefore, are in an ideal position to ensure better behaviour by politicians. However, this comes to the cost of a reduction in accountability, given that replacing a bureaucrat is much more difficult than removing a politician, and the absence of electoral concerns might not necessarily be sufficient to ensure a better behaviour from the bureaucrat.

More specifically, the paper discusses the effects on the quality of the decision of the introduction of a watchdog bureaucrat who provides policy recommendations

²For example, by studying the choice of make-or-buy service in the context of U.S. cities, Levin and Tadelis (2010) show that political economy assessments often play a more important role than strictly economic evaluations.

³Another example, that has attracted the attention of scholars interested in analysing the problem of delegation, is the proliferation of independent bureaucratic agencies which, following the model of Sweden, are responsible for the implementation of public policies.
that are costly to be ignored. Through a simple model, it is shown that, if both the politician and the bureaucrat are sensitive to the pressure of an interest group, the positive impact of bureaucrats is very small, even though not null. Moreover, by characterising the conditions that allow this effect, it is shown that if the bureaucrat’s recommendation is too binding, the decision power moves from the politician to the bureaucrat, producing a reduction in accountability without any improvement in the quality of the decision.

The second essay is part of the growing literature on partial decentralisation and its consequences on the efficiency of public expenditure in public good provision. Decentralisation has been another rather pervasive phenomenon in the last decades, both in developed and developing countries. In particular, in developed countries it has been presented as an additional way to further improve the quality of government: the closer voters to politicians, the better their ability to control them. This mechanism was supposed to be an increase in the overall quality of government. This idea has been so influential that even in the Maastricht Treaty there is an explicit mention of the “subsidiarity principle”, which was subsequently chosen as the pillar for building the European institutions and then also adopted by the national institutions of each member state.

However, this initial enthusiasm overlooked the negative aspects of decentralisation, and in fact the results of these reforms are, at best, mixed. Quoting Treisman (2007) “Empirical studies have found almost no solid, general result about the consequences of decentralisation. Decentralising government in a particular place and time is very much a leap in the dark”. Scholars have identified several reasons for these unsatisfactory results. One of the main ones is the difficulty of fully implementing these reforms, a situation that in the literature is referred to as partial decentralisation.

This essay belongs to a strand of literature pointing out that one of the possible side effects of partial decentralisation is a reduction in accountability because voters are imperfectly informed about each government’s contribution. In particular, the specific contribution is to take into account the possibility for politicians to manipulate voters’ imperfect information. The paper presents a simple model in which two levels of government are involved in the provision of a local public good with the local government that can decide to spend its budget either on the provision of the public good or to influence the information of the voters in its favour. A central result is that the conflict of interest that arises among the levels of government reduces public good expenditure at both levels, while it generates a wasteful spending to pander to voters.

Finally, the third essay belongs to the strand of literature studying environmental regulation. This topic has always attracted a great interest from public opinion and scholars. There has always been a great deal of interest in this topic, and in fact many scholars have focused on the analysis of this topic. In particular, public interest in the last few years has increased as a result of the so called Dieselgate. This scandal has once more shown how a very concentrated market, such as the automotive sector, is a favourable ground for the formation of cartels against environment-friendly innovations and how governments and public agencies of the producing countries are more than willing to support such cartels.
This work analyses the incentive for firms to externalise the environmental damage caused by the production process depending on the number of firms active into the market, provided that firms can form a lobby to influence the environmental regulation. The analysis is conducted through a Cournot competition model where, at the beginning of the game, firms have to decide whether to internalise the cost generated by their polluting production process or not. Firms that decide not to internalise this cost are subject to a linear taxation over production. The tax level is the result of a political game between the lobby of the firms that have decided not to internalise and a government characterised by different policy objectives that include the reduction of the externality level. The first result is that independently of the specific policy objective, the share of firms that decide to internalise the cost is increasing in the number of firms in the market. The second result is that, in general, the share of internalising firms is the highest if the government is interested only in minimising pollution. Finally, if the government is interested in consumers’ surplus, then either no firm chooses to internalise or all firms choose to internalise, depending on the number of firms.
Chapter 1
Contribution and bribe: lobbying in presence of a watchdog bureaucrat

1.1 Introduction

In the last decades, the interest in the increasing role of money in politics, and in particular its use by special interest groups as a mean to “buy” political favours has hugely increased with more and more request of reforms meant to reduce the importance of lobbies over the legislators. The economic literature gave its contribution with several theoretical works (see for example Bernheim & Whinston, 1986; Grossman & Helpman, 1994) highlighting how money could play an important role in influencing policy decisions.¹

One of the main consequences of this increased attention to the issue has been the idea that, since politicians looking for reelection are too inclined to accommodate the interests of lobbies in exchange for their support, an increasing role of bureaucrats, with no electoral concerns, into the decision making process would improve the goodness of the decision taken.

The first and most natural role that bureaucrats can play in the decision making process is the one of information providers since they often have technical skills on a specific subject that are superior to those of politicians who have to make a decision. In other words, they can act as advisers for incumbent politicians. Studies of information transmission from more informed experts to less informed decision makers have a long history in the economic literature dating back to the seminal paper of Crawford and Sobel (1982) and following developments of the so-called Cheap Talk literature (see for example Kartik, 2009; Pei, 2015). Even though in this literature the framework is usually very general, the main theoretical results can be easily applied to the specific focus of the role of bureaucrats.²

¹For a comprehensive discussion on the techniques and formal models as well as a review of the early literature see Grossman and Helpman (2001).
²It is worth to mention that the same framework has been applied to the literature of informational lobby, which studies the cases in which lobbies try to obtain their preferred policy by offering to the incumbent relevant information rather than money. Even though in this set up the interest
Many are the real cases where this second framework can be applied: one of the most notable examples is the Congressional Budget Office. This independent bureaucratic institution, formed by non partisan economists, was set up in 1974 and its role is to provide Congressmen with an independent costing of the policies proposed by the U.S. Government, in addition to the one provided by the Government itself. This design proved to work quite well, since it has actually improved the ability of Congressmen to make more informed choices over policies proposed, increasing their bargaining power versus the Government. Interestingly enough, this Office cannot formulate any suggestion on the policies themselves but only evaluate those proposed by the Government in a clear effort to remark the unique advisory role of this institution.3

A possible more active role for bureaucrats is that of actual decision makers, corresponding to the cases of delegation of the decision making power from elected officials to appointed bureaucrats. The interest of researchers has followed the increasing diffusion of Agencies, across both USA and Europe, following the example of the Swedish system where agencies are the main model of organisation of public administration.

The spread of these agencies, often endowed with large decision power, has stimulated a first wave of studies, analysing the optimal delegation level from Congress to Agencies in the form of an agency model, highlighting the trade off between the increase of expertise in the decision, and the decrease of control connected to a higher level of delegation (see Epstein & O’halloran, 1994, 1999; Gailmard, 2002). A second generation of contributions has enriched the study of this standard trade off taking into account the presence of interest groups and their interaction with both politicians and bureaucrats. For example Boehmke, Gailmard, and Patty (2005) study a model of informational lobby where the interest group can decide to lobby either the Congress or the Agency. More related to this work, Bennedsen and Feldmann (2006b) study how this decision about delegation changes when bureaucrats can be targeted by interest groups through payments, and the different incentives for delegation connected to different institutional organisation of the government.

However there is also a third possibility which has not yet been analysed by the literature but which has become extremely common especially at local level: it is the role of bureaucrats as watchdog of choices taken by incumbents.4

In other words, the role of bureaucrats is not simply to help politicians take the right decision, but also to make sure that they do.

These mechanisms are usually imposed by the central government, exploiting the fact that, in general, the accountability of local politicians is not only political but also administrative, since they might be obliged to refund the State for the

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3A similar institution exists in other countries, for example, the “Central Planning Bureau” in the Netherlands and the “High Council of Finance” in Belgium. The Italian Parliament has also recently established a similar office, under the name of Ufficio parlamentare di bilancio.

4For example a major reform of Italian municipalities in the 90s has introduced a large shift of powers from mayors to bureaucrats in relevant areas like decisions concerning public works, town planning and budgeting.
negative consequences of their wrong choices. In such a context, the approval of a given decision by the bureaucrats greatly reduces or even eliminates, the liability of the politicians who took that decision.\(^5\)

In the context of the Italian municipalities, for example, many expenditure decisions are subject to prior scrutiny by external auditors. Their negative judgement in no way prevents such decisions from being taken. However, in this case, politicians are personally liable for any financial damage caused by such decisions.

Nevertheless, if one admits that even if they do not have reelection motives, also bureaucrats can be captured by the action of special interest groups, it is not \textit{ex ante} guaranteed that the decision making process can benefit from the presence of a watchdog.

The aim of this paper is to study whether the introduction of a watchdog can actually improve the quality of the policies chosen when both politician and watchdog are sensitive to pressure exerted by an interest group.

To do so, starting from the standard framework of informational lobbying models (see for example Dahm & Porteiro, 2008b), this paper presents a model of policy decision under uncertainty in a context in which a special interest group offers a political contribution in exchange for a favourable decision.

An incumbent politician, who values both making good choices and receiving a contribution, has to take a single policy decision between two alternatives without knowing which is the correct one. The bureaucrat knows the correct policy choice and exercises the power of a watchdog by recommending a policy. The incumbent can decide to comply with the recommendation or not. In the latter case, the incumbent has to pay a cost for not complying, the size of which is proportional to the power of the bureaucrat.

However, the bureaucrat is not obliged to be sincere, consequently, the interest group can also offer a bribe to the bureaucrat in order to have recommended its favourite policy.

In order to evaluate whether or not the bureaucrat can have a positive impact in the process, the paper begins with the analysis of a benchmark case, in which only the interest group and the incumbent are present. In order to make a strong case, the benchmark is designed as a worst case scenario in which the interest group is always able to convince the incumbent to choose its favourite policy independently from the probability distribution of the states of the world or the actual realisation of it.

After that, the full game is solved by backward induction, characterising for a for a generic value of the cost, the optimal strategy of the interest group with respect to the probability distribution of the states and their actual realisation.

Then, assuming the perspective of a Social Planner, the paper investigates whether it is possible to set up the power of the bureaucrat, i.e. the cost for not complying with the policy recommended, in such a way that at least for some possible probability distributions of the states of the world, the incumbent eventually

\(^5\)In more rare cases bureaucrats are endowed with \textit{veto power} over the decisions of politicians. However, the use of this power from the bureaucrat is usually limited to the need to make sure that the incumbent does not violate the law, or to a very restricted set of decisions which require a very high degree of technical knowledge in order to be taken.
chooses the correct policy rather than the favourite project of the interest group.

The main result is the derivation of a sufficient condition for the value of the cost ensuring that such result is possible; therefore, in principle, a Social Planner could improve the situation of the benchmark case by appointing a bureaucrat with watchdog power.

However, this window of opportunity is rather small, therefore the improvement is limited to cases in which the \textit{ex ante} probability that the favourite project of the interest group is the correct choice is very small and the Social Planner knows the probability distribution of the game.

Moreover, the improvement in the quality of the decision making process comes at the cost of an increase in wasteful transfers of money from the interest group to the officials, added to a partial shift of power from an elected incumbent to an unelected bureaucrat.

The paper is organised as follows. Section 1.2 presents the model. Section 1.3 analyses the benchmark model, while section 1.4 characterises the solution of the game. Finally section 1.5 presents the main results of the paper, while section 1.6 concludes.

### 1.2 The model

There are two possible states of the world \( \{1, 0\} \), where 1 occurs with probability \( \rho \in (0, 1) \), while 0 with probability \( 1 - \rho \). The probability distribution is assumed to be common knowledge among all players.

There are two possible policies \( A \) or \( B \), where \( A \) is the optimal policy if the state of the world is 1, while \( B \) is the optimal one if the state of the world is 0.

The players of the game are the interest group, the incumbent and the bureaucrat.

**1.2.1 Interest group**

The interest group has a state independent preference; it always prefers policy \( B \), which hereafter will be referred to as “pet project”.

In order to ensure that its pet project is selected, the interest group has two instruments: the first one is a political contribution \( c \) to the incumbent and the second one is a bribe \( b \) to the bureaucrat.

In both cases, this is done through the offer of contribution schedules in line with the literature on lobbying as an agency problem (see Bernheim & Whinston, 1986; Grossman & Helpman, 1994). The main idea is that the principal (the interest group) proposes a schedule that assigns to each possible choice of agents, the amount of contribution that it is willing to pay if that choice is realised. Then the agent (the politician), given this schedule, selects the utility maximising policy, evaluating the political contribution as a mean for the future political campaign, which however is left outside of the model.

Even though the possibility that the interest group engages the incumbent with an informational lobby is discarded from the paper, the interest group is assumed to
have knowledge about the realised state of the world. This assumption is convenient since it rules out the possibility of strategic actions of the bureaucrat.

The interest group maximises its profit net of the contributions given to the bureaucrat and to the incumbent.

\[
\pi = \Pi - (b + c)
\]

where \(\Pi\), the gross profit deriving from the policy choice, is assumed to be equal to 0 if project \(A\) is chosen and to 1 if project \(B\) is chosen.\(^6\)

\[
\Pi = \begin{cases} 
1 & \text{if } i = B \\
0 & \text{if } i = A.
\end{cases}
\]

### 1.2.2 Incumbent

The incumbent has to choose a policy \(i\) among the options \(A\) and \(B\).

In line with the literature on lobbying, it is assumed that the incumbent is interested on the one hand in matching policies and states of the world, and on the other hand in receiving political contributions from the interest group.

Moreover, the incumbent does not have access to the realisation of \(\omega\). Therefore the choice of the state will rely on two elements. The first one is the knowledge of the probability distribution of the states (i.e. \(\rho\)). The second element is the message \(m\) that the incumbent receives from the bureaucrat about the right policy. The particular feature of this message, which also represents the novelty of this contribution, is that it does not impact on the politician’s beliefs (like in the standard literature on informational game, Crawford & Sobel, 1982), and it is not binding (as it would be in case of delegation) but imposes a cost on the incumbent that decides to ignore it.

The utility function \(G\) of the incumbent is additive and separable in all elements and takes the following form:

\[
G(i) = \begin{cases} 
\cancel{c(i)} + rp(i) - x & \text{if } i \neq m \\
\cancel{c(i)} + rp(i) & \text{if } i = m,
\end{cases}
\]

where \(p(i)\) is the probability that policy \(i\) is optimal given the realisation of \(\omega\), \(r\) is the fixed payoff premium that the incumbent obtains for the correct matching and \(x\) is the cost that the incumbent pays for ignoring the policy recommendation of the bureaucrat.

There are many possible interpretations for \(r\). For example, it may represent a political premium connected to voters’ appreciation for correct matching; therefore a good matching will, eventually, increase to some extent the probability of being reelected. Different interpretations can be found for example in Ferejohn (1986) where \(r\) measures the explicit compensation of holding the office, or in Maskin and

\(^6\)For example, if the policy to be chosen is the construction of a public infrastructure, then if project \(B\) is chosen, the interest group is rewarded with the contract to realise it, while if project \(A\) is chosen, the contract will be assigned to someone else.
Tirole (2004) where \( r \) is a measure of the *legacy motive*: the incumbent wants to be remembered for the good choices made.

In the proceeding of the paper it is assumed that the incumbent has a rather strong office holding motive, therefore that \( r \in [\frac{1}{2}, 1] \).

As for \( x \), whose range of values is assumed to be \([0, 1] \), it captures the fact that, at the local level, politicians are not only politically accountable to the voters but also administratively accountable to the central state. This means that local politicians are responsible both for decisions taken in the wrong way and for decisions that have proved to be wrong by economically damaging the institution they administer.

The simplest way to introduce this accountability into the model is through a cost that comes with the need to prove the goodness of the choice with respect to what has been reported by the bureaucrat. This is consistent with the already stated interpretation given that an administrative violation typically implies a fee.

It then follows that \( x \) can also be interpreted as a measure of the power of the bureaucrat. The higher the cost that the politician has to pay to defend the decision to defy the bureaucrat’s recommendation, the higher is the power of the bureaucrat.

It is worth noticing that even if the cost incurred by the incumbent is connected to the case in which the policy recommended happens to be right, \( x \) is independent of the actual realisation of \( \omega \), implying that the incumbent sustains the cost even if the decision to ignore the message of the bureaucrat is *ex post* right. This is not unreasonable because, in reality, even if the decision of the politician to ignore the recommendation of the bureaucrat turns out to be correct, demonstrating the rightness of this choice generally requires a long and expensive process.

### 1.2.3 Bureaucrat

The bureaucrat is a subject with technical skills concerning the policy issue, whose role is to observe the realised state of the world, and on that basis, to recommend a policy to the incumbent, through a message \( m \).

This message does not need to be truthful, even though, differently from the standard models, this is not due to differences in preference between the bureaucrat and the incumbent, but because of the influence of the interest group, who may be willing to bribe the bureaucrat in exchange for a more favourable recommendation.

However, this misreporting is costly. The bureaucrat suffers a cost, either in terms of loss in reputation, negative economic consequences or as a cost of “manipulating” the report, if the recommended policy is the wrong one.

Moreover, the fact that the probability distribution of the states of the world is common knowledge implies that the incumbent has a belief about what the correct policy might be. As a consequence the more a state of the world is likely *ex ante* the more costly will be for the bureaucrat to misreport the reality.

The bureaucrat has a utility function \( V \) which is additive and separable in the amount of the bribe and the lying cost.

Following the literature (see for example Kartik, 2009), the lying cost is assumed to be quadratic. In particular, it is equal to the quadratic difference between the expected value of \( \omega \) and the reported message \( m \).
Given the definition of $\omega$, it follows that

$$E[\omega] = \rho,$$

therefore

$$V = \begin{cases} 
  b(m) - (\rho - m)^2 & \text{if } m \neq \omega \\
  b(m) & \text{if } m = \omega .
\end{cases}
\quad (1.4)$$

It is worth noticing that both the incumbent and the bureaucrat give the same weight to all the addends in their utility function. In the lobbying literature, however, it is sometimes assumed a positive weight $\alpha$ for the contribution. The reasons for this choice and the consequences of a different one are discussed in Appendix 1.B.

Finally, given that the profits that the interest group obtains if policy $A$ is implemented are equal to 0, it follows that the interest group will offer a bribe to the bureaucrat and a contribution to the incumbent only in exchange of the recommendation or the choice, respectively, of its pet project. Hence $b(1) = c(A) = 0$. Therefore, in order to simplify the notation, from now on define $b(0) = b$ and $c(B) = c$.

### 1.2.4 Time of the game

The timeline of the game is as follows.

In the first stage, the interest group chooses the bribe schedule for the bureaucrat, $b$ in exchange for a favourable policy recommendation. In the second stage, given the bribe that the interest group offered, the probability distribution of states, and the realised one, the bureaucrat chooses which policy to recommend. In the third stage, the interest group decides its contribution schedule to offer to the incumbent. Finally the incumbent, given the contribution schedule and the policy recommended, selects the policy.

The timeline is depicted in Figure 1.1.

![Figure 1.1: Timeline](image)

The game is solved by backward induction.

### 1.3 Benchmark Case

The first step of this analysis is to consider a model where there are only an incumbent and the interest group.
With respect to the full model, in this case, the cost for not complying \( x \) is absent. The utility function of the incumbent then takes the following form

\[
G(i) = \begin{cases} 
  r \rho & \text{if } i = A \\
  c + r(1 - \rho) & \text{if } i = B
\end{cases}
\] (1.5)

while the profit function of the interest group is equal to

\[
\pi = \begin{cases} 
  0 & \text{if } i = A \\
  1 - c & \text{if } i = B
\end{cases}
\] (1.6)

Interest group’s objective function is simply the gross profit, net of the political contribution only.

\[
\pi = \Pi(i) - c.
\] (1.7)

The following lemma characterises the solution of this game, both in the case in which there is no lobbying and in the case in which lobbying is allowed.\(^7\)

**Lemma 1.1.** In the benchmark case,

(i) if there is no lobbying activity, the incumbent chooses project A if \( \rho > \frac{1}{2} \) and project B if \( \rho \leq \frac{1}{2} \);

(ii) if there is lobbying activity, the incumbent always chooses policy B, and the interest group pays a contribution equal to

\[
c = (2 \rho - 1)r,
\] (1.8)

which is positive only if \( \rho > \frac{1}{2} \).

The intuition behind this result is simple. If 0 is the state of the world more likely to happen, the incumbent would choose project B even without the presence of the interest group, therefore no contribution is needed. However, if the most likely state of the world is 1, without any contribution the incumbent would choose project A, which ensures a higher expected payoff. In this case, the interest group can interfere in the decision, and given that the incumbent cares enough about the campaign money, its action is always successful.

The profit of the interest group takes the following form:

\[
\pi = \begin{cases} 
  1 & \text{if } \rho \in (0, \frac{1}{2}] \\
  1 - (2 \rho - 1)r & \text{if } \rho \in (\frac{1}{2}, 1)
\end{cases}
\] (1.9)

Notice however that the presence of the interest group is not necessarily *ex ante* negative for the quality of the decision. As a matter of fact, if one measures the quality of the political process by the *ex ante* probability that the incumbent makes the wrong choice

\[
\Psi = \Pr(\omega = 1)\Pr(i = B) + \Pr(\omega = 0)\Pr(i = A)
\] (1.10)

\(^7\)All proofs are provided in Appendix 1.A

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Chapter 1. Contribution and bribe: lobbying in presence of a watchdog bureaucrat
it follows that, in the case without lobbying

\[ \Psi = 2\rho (1 - \rho) \]  

(1.11)

while in case of lobbying it is equal to

\[ \Psi = \rho. \]  

(1.12)

From this it follows that if \( \rho < \frac{1}{2} \) the presence of lobbying is actual welfare improving.

For this reason in the proceeding of the paper the state of the world 1 is assumed to be more likely, i.e. \( \rho > \frac{1}{2} \).

### 1.4 Full game

In the benchmark model, if the interest group is allowed to lobby the incumbent, the project A is never chosen. From the point of view of a central government interested in a good matching between policies and states of the world, this outcome could be considered undesirable.

There are of course many possible reforms that could be implemented trying to improve this situation. The one considered in this paper is the appointment of a bureaucrat with watchdog powers.

This section presents the full game, where all the three players are present.

Given that the contribution schedule offered by the interest group to the incumbent depends on the message transmitted by the bureaucrat, henceforth let \( c_m \) be the contribution schedule that the interest group offers, given the policy recommended by the interest group.

#### 1.4.1 Stage IV

In the last stage of the game, given the recommendation \( m \) received from the bureaucrat and the contribution schedule \( c \) of the interest group, the incumbent has to choose the policy \( i \).

Since the objective function of the incumbent is message dependent, two cases have to be taken into account. The campaign contribution will, therefore, be message dependent.

**Lemma 1.2.** Let \( \rho \equiv \frac{r + x}{2r} \), then

(i) if the bureaucrat recommends policy A (\( m = 1 \)), for any value of \( \rho \), the incumbent chooses the policy B if the contribution is equal to:

\[ c_1 = (2\rho - 1)r + x; \]  

(1.13)

(ii) if the bureaucrat recommends policy B (\( m = 0 \)), then if \( \rho > \rho \) the incumbent chooses the policy B if the contribution is equal to:

\[ c_0 = (2\rho - 1)r - x, \]  

(1.14)

while if \( \rho < \rho \) no contribution is needed.
Lemma 1.2 is an extension of Lemma 1.1 where the extra cost $x$ is included. If the bureaucrat has recommended policy $A$, the incumbent has to pay $x$ to choose $B$, and this extra cost has to be compensated by the interest group. Therefore, *ceteris paribus*, the contribution is higher. On the other hand, if the bureaucrat has recommended policy $B$, the incumbent will be more willing to comply with the request of the interest group because the alternative leads to higher cost. Therefore, *ceteris paribus*, the contribution is lower.

It is worth noticing that, with respect to Lemma 1.1, if the incumbent receives the policy recommendation $B$, the interest group needs to pay a positive contribution in a smaller set of cases, which is connected to the value of the cost $x$.

Therefore, if the cost for not complying is higher than the payoff premium for the right matching, the incumbent is willing to comply to the reported message, hence no contribution is needed from the interest group.

### 1.4.2 Stage III

In the third stage of the game, given the policy recommended to the incumbent, the interest group decides whether to offer a political contribution and in what amount.

Its objective function is

$$ \Pi = \pi(i) - (c_m + b), $$

(1.15)

where at this stage of the game $b$ is fixed, while the political contribution is built as a schedule for any possible choice.

Then, according to different messages, two cases are possible.

Lemma 1.3 characterises the solution of this stage of the game.

**Lemma 1.3.** Let $\bar{\rho}_1 \equiv \frac{1 + r - x}{2r}$, then:

(i) if the bureaucrat recommended policy is $A$ ($m = 1$), then, if $\rho < \bar{\rho}_1$, the interest group offers a positive contribution

$$ c_1 = (2\rho - 1)r + x, $$

(1.16)

whereas if $\rho > \bar{\rho}_1$ it does not offer anything.

(ii) if the bureaucrat recommended policy is $B$ ($m = 0$), then, if $\rho > \rho_0$ the interest group always offer a political contribution equal to:

$$ c_0 = (2\rho - 1)r - x, $$

(1.17)

while if $\rho < \rho_0$ the interest group does not offer any contribution.

The intuition of Lemma 1.3 is the following. If the bureaucrat recommends policy $A$, the interest group has to compensate the cost that the incumbent suffers for choosing its pet project. However, if the cost is too high, this choice would be unprofitable for the interest group. In that case, it would rather let the incumbent choose $A$ and obtain $0$.

If the bureaucrat recommends policy $B$, the incumbent pays the extra cost for
choosing policy \( A \), therefore the interest group takes advantage of this situation and offers a contribution only when needed. Moreover given that, in this case, the cost of the contribution is lower, the choice of offering a contribution is always profitable, since \( c_0 < 1 \) always.

To be precise at this stage of the game, if \( m = 0 \), it is possible that in the first stage of the game, the interest group had already paid a bribe to the bureaucrat to recommend the wrong policy. Therefore, the general condition over the contribution would be \( c_0 < 1 - b \) and it might be the case that, if \( b \) and \( \rho \) are large enough, the contribution needed to convince the incumbent to choose policy \( B \) would be too expensive for the budget of the interest group.

However, this possibility can be discarded. In fact, it would make no sense for the interest group to pay a bribe to the bureaucrat in exchange for nothing. Paying a bribe would be convenient if it ensured that the incumbent will choose policy \( B \), either by itself or combined with a contribution. If that is not the case, the interest group would be better off without paying the bribe.

Therefore if in the third stage of the game the recommended policy is \( B \), contribution is always feasible, either because no bribe has been paid or because the bribe paid is small enough for the budget constraint not to be binding.

Finally, notice that the actual binding of the budget constraint in the case \( m = 1 \) depends again on the value of \( x \).

1.4.3 Stage II

In the second stage of the game, the bureaucrat has to decide which signal to send to the incumbent, given the true state of the world observed and the contribution schedule \( b \) received from the interest group.

Lemma 1.4. (i) If \( \omega = a \), the bureaucrat recommends policy \( B \) in exchange of a bribe equal to

\[
b = \rho^2,
\]

and recommends policy \( A \) otherwise.

(ii) If \( \omega = b \) the bureaucrat always recommends policy \( B \), regardless of the bribe.

Lemma 1.4 has two main implications. The first one is that the interest group can always bribe the bureaucrat because the amount required is always lower than 1. The second one is that if \( \omega = 0 \) the bureaucrat has no incentive to report untruthfully, therefore there is no need for the interest group to pay a bribe.

1.4.4 Stage I

This section analyses the first stage of the game, hence the choice of the interest group over the bribe to the bureaucrat.

There are two reasons for the interest group to offer a bribe to the bureaucrat. In its best case scenario, ensuring that the policy recommended is \( B \) is sufficient to make sure that that one is the chosen policy. However, even if the policy recommendation of the bureaucrat is not sufficient by itself to convince the incumbent to choose \( B \),
it reduces the amount of the contribution required to do so. Therefore, the interest group could find more profitable to offer a bribe and a reduced contribution rather than only a single contribution.

At this point of the game, the assumption that the interest group knows \( \omega \) comes in handy because when deciding the bribe schedule, it allows the interest group to discriminate its offer according to \( \omega \). \(^8\)

If \( \omega = 0 \), the bureaucrat always recommends policy \( B \), therefore the interest group has no need to pay a bribe. Then the interest group has only to choose between two strategies:

- To offer a contribution to the incumbent
- Not to offer a contribution to the incumbent

Given the fact the policy recommended is \( B \), contribution is always a feasible strategy for the interest group.

The following lemma summarises the optimal strategy of the interest group if the state of the world is 0.

**Lemma 1.5.** If \( \omega = 0 \), the interest group never pays a bribe to the bureaucrat.

Lemma 1.5 is an immediate consequence of Lemma 1.4. If \( \omega = 0 \) the bureaucrat reports the correct (and favourable to the interest group) message without the need of any external incentive.

If, on the other hand, \( \omega = 1 \), there are four possible strategies for the interest group.

- the interest group does not offer any bribe to the bureaucrat while offers a contribution to the incumbent
- the interest group offers a bribe to the bureaucrat while does not offer any contribution to the incumbent
- the interest group offers both a bribe to the bureaucrat and a contribution to the incumbent
- the interest group offers neither a bribe nor a contribution

To clarify how the solution proceeds, Figure 1.2 provides a graphical representation of all the possible paths that the interest group can follow when deciding about the interference into the process.

**Lemma 1.6.** If \( \omega = 1 \), the interest group pays a bribe to the bureaucrat in the following cases:

(i) if \( \rho < \rho^0 \), the interest group pays a bribe as long as \( b < c_1 \);

(ii) if \( \rho > \rho^0 \), the interest group pays a bribe as long as \( b < \min\{c_1 - c_0, 1 - c_0\} \).

\(^8\)If the interest group does not know the true state of the world it can bargain with the bureaucrat only on the policy recommendation \( m \). Therefore what might happen is that the interest group pays a bribe to the bureaucrat even in cases in which no bribe would be necessary, i.e. if \( \omega = 0 \).

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Lemma 1.6 states that if the favourable recommendation of the bureaucrat is sufficient to convince the incumbent, the interest group will choose the cheapest option between the bribe and the political contribution.\footnote{Keep in mind that, on the one hand bribe is always feasible while, on the other hand, contribution is always sufficient to convince the incumbent.} If, however, the favourable recommendation of the bureaucrat is not sufficient to convince the incumbent, the interest group can choose to combine it with a political contribution. If it is the case, the interest group will choose this option if the combination of bribe plus political contribution is both feasible and cheaper than the single political contribution. It is worth to mention here that $b \in (c_1 - c_0, 1 - c_0)$ corresponds to the case in which both the strategies are feasible, but it is cheaper to just pay a higher political contribution rather than to pay both the bureaucrat and the incumbent, while $b \in (1 - c_0, c_1 - c_0)$ corresponds to the case where the interest group cannot convince the incumbent to choose policy $B$ with any instrument at its disposal.

### 1.5 The power of the bureaucrat

The previous section provided the characterisation of the game for a given level of $x$. This section deals with the key question of the paper, i.e. whether it is possible to set up a $x$ such that the final outcome of the decision making process would be better than in the benchmark case.

The impact of the watchdog is firstly analysed in the simpler, though insightful, case in which the state of the world is 0, then the analysis moves to the more complex case in which the state of the world is equal to 1.
1.5.1 The realised state of world is 0

As stated by Lemma 1.5, the interest group never pays any bribe if the realised state of the world is 0. Moreover Lemma 1.2 showed that if the policy recommended by the bureaucrat is \( B \), the interest group pays a contribution only if \( \rho > \rho' \). Therefore, the following proposition comes directly from these two results.

**Proposition 1.1.** In the case in which \( \omega = 0 \) the policy chosen is \( B \) regardless of the values of \( x \). In particular

(i) if \( x > r \), the interest group never pays a contribution;

(ii) if \( x < r \), the interest group pays a contribution for values of \( \rho > \rho' \).

The intuition behind Proposition 1.1 is that if the state of the world is favourable to policy \( B \), then the interests of all players are aligned, therefore the final outcome will always be policy \( B \). In particular, if \( x \) is higher than the payoff for the good matching, the incumbent has nothing to gain by overlooking the recommendation of the bureaucrat independently from \( c \), therefore the interest group does not need to pay any contribution. On the other hand, if \( x < r \), then it might still be the case that, if \( \rho \) is sufficiently large, the incumbent could decide to refuse the recommendation. In this case, the interest group has to offer a contribution to convince the incumbent to choose policy \( B \) anyway, and this action always yields a positive profit for the interest group.

1.5.2 The realised state of world is 1

If the realised state of the world is 1, the analysis is much more complicated, given the three possible alternative strategies of the interest group. However, given that the interest of the paper is to understand whether or not there exist a value of \( x \) such that the interest group is not always able to make the incumbent choose \( B \), the analysis can be simplified through the following condition.

**Proposition 1.2.** If \( x < 1 - r \), policy \( A \) is never the outcome of the game.

The intuition behind this condition is that if the power of the bureaucrat is not sufficiently strong, the interest group will always have enough resources to buy the incumbent.

Given that the result of Proposition 1.2 is a necessary condition, it might be the case that the actual choice of the interest group is different from the single contribution to the incumbent. In fact, for values of \( x \) sufficiently high the interest group might find more convenient to mix the instruments. However, if the power of the bureaucrat is particularly weak, a single contribution is the only strategy played as stated by the following corollary.

**Corollary 1.1.** If \( x < r(1 - r) \)

\[
c_1 < b, \forall \rho \in \left( \frac{1}{2}, 1 \right),
\]

(1.19)

therefore from the point of view of the interest group, not only a single contribution is always sufficient, but it is also cheaper than bribing.
If for the politician the cost of not complying with the recommended policy
is too low, the bureaucrat has no real power: the interest group will not change
its behaviour with respect to the benchmark case, it will just increase its offer to
compensate the incumbent for the extra cost.

This result is very intuitive but meaningful: a watchdog without any real power
is not only unable to prevent any malpractice of the incumbent, but at most, it will
increase the amount of money present in the decision making process.

Proposition 1.2 defines a lower bound for the power of the bureaucrat, under
which there is no purpose to appoint a watchdog since it would be ineffective. How-
ever, as stated by the following proposition, there also exists an upper bound for
x.

Proposition 1.3. If \( x > r \), the interest group only pays the bribe to the bureaucrat
and, \( \forall \rho \in [0, 1] \), project A is never chosen. Moreover, the amount of the bribe is
always higher than the amount of the contribution in the benchmark case.

In other words, if the bureaucrat is too powerful, practically the incumbent is the
actual policy maker, consequently becoming the objective of the lobbying activity.

Moreover, the switch from the political contribution to the bribe turns out to be
always more costly for the interest group. However, this can hardly be considered as
an improvement with respect to the benchmark case. In fact, the reduction in profits
is needed to finance a wasteful transfer to the bureaucrat. Finally, there is also the
additional hidden cost of the shifting of power from an elected representative to an
unelected official.

Given propositions 1.2 and 1.3, the only viable interval for \( x \), where it might be
possible to improve the quality of the decision making process, is \((1 - r, r)\). In fact,
it is possible to demonstrate that in this interval an improvement with respect to
the benchmark situation is always possible.

Proposition 1.4. For any \( x \in (1 - r, r) \) there always exists \( \rho^* < 1 \), such that,
\( \forall \rho \in (\rho^*, 1) \), the policy chosen is A.

As clearly stated by Proposition 1.4, if \( x \) lies in the interval \((1 - r, r)\), then for
sure there is at least one probability distribution for which the interest group is not
able to convince the incumbent to choose policy B. This means that if the power of
the bureaucrat is sufficiently large, but not too large, and if the \textit{ex ante} probability
that policy A is sufficiently high, all the instruments that the interest group can
deploy are either insufficient or unaffordable.

1.6 Final Remarks

This paper has shown that, under some circumstances, the presence of a bureaucratic
watchdog may actually improve the quality of the decision, even though to a limited
extent. Making the case for those who, in the public debate, support an increasing
role of bureaucracy to control politicians’ misbehaviour

In a model where an incumbent has to choose a dichotomous policy under the
influence of a special interest group and a watchdog bureaucrat, in principle it is
possible to assign to the bureaucrat an amount of power sufficiently high not to be irrelevant but not large enough to completely replace the incumbent as the actual policy maker. In this case, for values of $\rho$ sufficiently high, the presence of the bureaucrat allows policy $A$ to be chosen.

In fact, if the bureaucrat has not enough power, then the interest group will behave exactly like if there is no watchdog at all, except for the need to slightly increase the amount of its payment to the incumbent. On the other hand, if the bureaucrat is too powerful, the interest group will switch its attention from the incumbent to the bureaucrat, choosing always to bribe rather than to offer political contributions.

However, the paper has also shown that the effectiveness is quite difficult to implement. Indeed, while the presence of the watchdog always implies an increase in money transfers, this increase in cost is effective in reducing bad matching due to lobbying only for rather large values of $\rho$. This provides a possible explanation of the unclear results obtained, for example, with the reforms of municipalities in Italy. It also raises the question of how much it is worth reducing the level of accountability in exchange for benefits that are difficult to implement.

Finally, given the fact that many issues are left outside this simple model, there is a great space for further research.

In particular, in this model the policy recommendation from the bureaucrat does not affect at all the beliefs of the incumbent but only imposes an additional cost. This is clearly a limitation of the model, and a more comprehensive framework that entails both effects would be appropriate.

Moreover, even though it is quite common in literature, there is no \textit{a priori} reason to consider $r$ as an exogenous parameter. In fact, another possibility would be to consider a framework where $r$ is endogenous into the game. This choice would open the possibility to introduce another way to influence the choice of the incumbent, enriching the analysis with different interest groups, that could act on the political side of the choice, rather than through direct economic contributions, which is usually the strategy of NGOs and environmental groups (see for example Yu, 2005).
Appendix

1.A Proofs

Proof of Lemma 1.1.

If lobbying activity is absent the result is immediate.

If the lobbying activity is allowed, the incumbent chooses to implement policy $B$ only if $G(B) \geq G(A)$ that, given equation (1.5), implies:

$$c + r(1 - \rho) \geq r\rho,$$

from which it follows that

$$c^* \equiv (2\rho - 1)r,$$

which is positive for any $\rho > \frac{1}{2}$.\(^{10}\)

The optimal contribution schedule of this game is therefore equal to

$$c^* = \begin{cases} 
0 & \text{if } \rho \in (0, \frac{1}{2}] \\
(2\rho - 1)r & \text{if } \rho \in (\frac{1}{2}, 1) 
\end{cases},$$

(1.A.3)

In addition, given that the profit of the interest group, in this case, is equal to 1, the contribution has to fulfil the constraint $c < 1$, implying that:

$$(2\rho - 1)r < 1,$$

(1.A.4)

which is satisfied as long as

$$\rho < \frac{1+r}{2r} = \frac{1}{2} + \frac{1}{2r},$$

(1.A.5)

which is always greater than one for values of $r \in (\frac{1}{2}, 1)$. Therefore the constraint is never binding, meaning that the interest group can always lobby the incumbent and obtain a positive profit.

Proof of Lemma 1.2.

If the recommended policy is $A$ ($m = 1$), the utility function of the incumbent takes the form

$$G = \begin{cases} 
r\rho & \text{if } i = A \\
c_1 + r(1 - \rho) - x & \text{if } i = B 
\end{cases}.$$

(1.A.6)

\(^{10}\)The usual underlying assumption is that, when indifferent, the incumbent chooses the preferred outcome of the interest group.
In this case policy $B$ is chosen if $G(B) > G(A)$:

$$c_1 + r(1 - \rho) - x > r\rho,$$

from which

$$c_1 = (2\rho - 1)r + x. \quad (1.A.7)$$

Moreover, for values of $\rho \in (\frac{1}{2}, 1)$, the contribution is always positive, being the sum of two positive quantities.

If the recommended policy is $B$ ($m = 0$) the utility function takes the form:

$$G = \begin{cases} r\rho - x & \text{if } i = A \\ c_0 + r(1 - \rho) & \text{if } i = B \end{cases}. \quad (1.A.8)$$

Project $B$ is chosen if $G(B) > G(A)$ which implies

$$c_0 + r(1 - \rho) > r\rho - x, \quad (1.A.9)$$

and then

$$c_0 = (2\rho - 1)r - x. \quad (1.A.10)$$

Notice that $c_0 > 0$ implies $(2\rho - 1)r - x > 0$, from which

$$\rho > \frac{r + x}{2r} \equiv \bar{\rho}. \quad (1.A.11)$$

**Proof of Lemma 1.3.**

The first part of the proof deals with the case $m = 1$.

In this case, given the analysis of the previous stages, and recalling that if $m = 1$ it follows immediately that $b = 0$, the profit function of the interest group is equal to

$$\Pi(i) = \begin{cases} 1 - c_1 & \text{if } c_1 \neq 0 \\ 0 & \text{if } c_1 = 0 \end{cases}. \quad (1.A.12)$$

The interest group will offer a contribution only if its profit is positive, i.e. if $c_1 < 1$ which is true if

$$\rho < \frac{1 + r - x}{2r} \equiv \bar{\rho}_1. \quad (1.A.13)$$

Therefore, the contribution schedule takes the following form

$$c_1 = \begin{cases} (2\rho - 1)r + x & \text{if } \rho \in \left(\frac{1}{2}, \bar{\rho}\right) \\ 0 & \text{if } \rho \in (\bar{\rho}, 1) \end{cases}. \quad (1.A.14)$$

The second part of the proof deals with the case $m = 0$.

The main differences, in this case, are that $b$ may be different from zero and that for $\rho < \rho$ contribution is not needed since $m = 0$ is per se enough to convince the incumbent to choose policy $B$. However, if $\rho > \rho$ this is not true, and a contribution is needed to ensure that policy $B$ is the chosen one even in presence of a favourable recommendation by the bureaucrat.
The profit function then takes the form:

$$\Pi(i) = \begin{cases} 
1 - b - c_0 & \text{if } c_0 \neq 0 \\
1 - b & \text{if } c_0 = 0 \text{ and } \rho \in \left(\frac{1}{2}, \hat{\rho}\right) \\
-b & \text{if } c_0 = 0 \text{ and } \rho \in (\hat{\rho}, 1) 
\end{cases}$$  \quad (1.A.15)

As in the previous case, the interest group is willing to contribute as long as it is profitable. In this case however, the bribe possibly paid to the bureaucrat has to be taken into account. Therefore the budget constraint is $c_0 < 1 - b$.

This requires:

$$\rho < \frac{1 - b + r + x}{2r}.$$  \quad (1.A.16)

Even if this is not true in general, it is immediate to recognise that, if no bribe has been paid in the previous stage, i.e. if $b = 0$, the political contribution is always a feasible option. However it has already been discussed why either $b = 0$ or it is such that $b + c_0 < 1$ for any value of $c_0$.

The contribution schedule assumes the following form:

$$c_0 = \begin{cases} 
0 & \text{if } \rho \in \left(\frac{1}{2}, \hat{\rho}\right) \\
(2\rho - 1)r - x & \text{if } \rho \in (\hat{\rho}, 1) 
\end{cases}$$  \quad (1.A.16)

**Proof of Lemma 1.4.**

If $\omega = a$ the objective function of the bureaucrat takes the following form.

$$V(m) = \begin{cases} 
0 & \\
b - \rho^2 & 
\end{cases}$$  \quad (1.A.17)

The bureaucrat will choose $m = 0$ if $V(b) \geq V(a)$, which implies:

$$b - \rho^2 \geq 0,$$  \quad (1.A.18)

from which it follows that:

$$b = \rho^2,$$  \quad (1.A.19)

which is always lower than 1.

If $\omega = b$, the objective function of the bureaucrat is equal to:

$$V(m) = \begin{cases} 
-(1 - \rho)^2 & \\
b & 
\end{cases}$$  \quad (1.A.20)

In this case then, $V(a) < 0$. Therefore the interest group prefers to recommend policy $B$ as long as $b$ is non negative. The minimum bribe necessary to obtain $m = 0$ is therefore:

$$b = 0.$$  \quad (1.A.21)
Proof of Proposition 1.1.
It has already been showed that in the case $\omega = 0$ the interest group needs to interfere with the decision process through a contribution only if $\rho > \bar{\rho}$.

However, if $x > r$, it follows that $\rho > 1$; therefore there is never the need for the interest group to offer a contribution to the incumbent.

In this case, it follows that

$$b = c_0 = 0,$$

with associated final profits for the interest group

$$\pi = 1.$$  \hspace{1cm} (1.A.22)

On the other hand, if $x < r$, it follows that $\rho$, therefore there are cases in which the interest group has to offer a positive contribution to the incumbent.

$$c_0 = \begin{cases} 
0 & \text{if } \rho \in \left(\frac{1}{2}, \bar{\rho}\right], \\
(2\rho - 1)r - x & \text{if } \rho \in (\bar{\rho}, 1).
\end{cases}$$  \hspace{1cm} (1.A.24)

with associated profit

$$\pi = \begin{cases} 
1 & \text{if } \rho \in \left(\frac{1}{2}, \bar{\rho}\right], \\
1 - (2\rho - 1)r + x & \text{if } \rho \in (\bar{\rho}, 1).
\end{cases}$$  \hspace{1cm} (1.A.25)

Proof of Proposition 1.2.
The necessary condition can be derived directly from the feasibility of $c_1$. In fact, as stated by Lemma 1.2, the direct contribution to the incumbent is always sufficient for the interest group to obtain $B$ as long as its cost is lower than 1, i.e. as long as

$$\rho < \bar{\rho}_1.$$  \hspace{1cm} (1.A.26)

Therefore if $\bar{\rho}_1 > 1$ contribution is always feasible, meaning that policy $B$ will always be selected, from which the necessary condition:

$$\frac{1 + r - x}{2r} > 1 \iff x < 1 - r$$  \hspace{1cm} (1.A.27)

Proof of Corollary 1.1.
First of all the condition $c_1 < b$ requires:

$$(2\rho - 1)r + x < \rho^2,$$  \hspace{1cm} (1.A.28)

which implies

$$\rho^2 - 2r\rho + r - x > 0.$$  \hspace{1cm} (1.A.29)

The associated $\Delta$ is equal to

$$\Delta = 4(r^2 - r + x).$$  \hspace{1cm} (1.A.30)
Therefore, if \( x < r(1 - r) \), \( \Delta < 0 \) and hence either contribution or bribe is always the cheaper alternative; to prove that the former is the case, assume \( x = r(1 - r) - \epsilon \), inequality (1.A.29) is then equal to
\[
\rho^2 - 2r\rho + r - (r(1 - r) - \epsilon) = (\rho - r)^2 + \epsilon > 0.
\] (1.A.31)
which is clearly always true.

**Proof of Proposition 1.3.**

As already proved in Proposition 1.1, if \( x > r \), the incumbent always comply with the policy recommended by the bureaucrat, therefore no additional contribution is needed.

It then follows that the interest group has to decide between the single contribution or the bribe, and it will choose the cheaper.

Given that, in this case, \( \Delta > 0 \), equation (1.A.29) implies that \( b < c_1 \) if \( \rho \in [\hat{\rho}_1, \hat{\rho}_2] \), where
\[
\begin{cases}
\hat{\rho}_1 = r - \sqrt{r^2 - r + x} \\
\hat{\rho}_2 = r + \sqrt{r^2 - r + x}
\end{cases}
\] (1.A.32)

However, for \( x > r \), if follows that
\[
\begin{cases}
\hat{\rho}_1 < 0 \\
\hat{\rho}_2 > 1
\end{cases}
\] (1.A.33)
therefore
\[
b < c_1 \quad \forall \rho.
\] (1.A.34)

To prove the last part of the proposition, recall that the contribution that the interest group pays in the benchmark case is equal to \( (2\rho - 1)r \). Therefore, the following inequality has to be studied.
\[
\rho^2 \geq (2\rho - 1)r.
\] (1.A.35)
The associated \( \Delta = 4r(r - 1) \) is always negative.
Then one can simply notice that for any couple of values of \( (\rho, r) \) in their allowed space of value, the condition holds with strict inequality, for example, if \( \rho = \frac{1}{2} \) the LHS is positive while the RHS is null.

**Proof of Proposition 1.4.**

In the case in which \( \omega = 1 \), in order to convince the incumbent to choose policy \( B \), the interest group has in general three instruments: it can decide to offer a single contribution to the incumbent, a single bribe to the bureaucrat, or to mix the instruments.

The proof of the sufficient condition requires the existence of a \( \rho^* \) such that for \( \rho > \rho^* \) only to bribe the bureaucrat is not sufficient to ensure that policy \( B \) is selected while both \( c_1 \) and \( b + c_0 \) are greater than one. The first situation corresponds to the condition \( \rho > \bar{\rho}_1 \), while the second one to the condition \( \rho > \bar{\rho}_2 \).
The condition \( b + c_0 > 1 \) can be written as:

\[
(2\rho - 1)r - x + \rho^2 > 1, \quad (1.A.36)
\]
\[
\rho^2 + 2r\rho - r - x - 1 > 0, \quad (1.A.37)
\]

the corresponding equation has two solutions

\[
\begin{align*}
\rho &= -r - \sqrt{r^2 + r + x + 1} \\
\rho &= -r + \sqrt{r^2 + r + x + 1}
\end{align*}
\]

where the first one is always negative.

Therefore, defining the second solution as \( \bar{\rho}_0 \), it follows that \( b + c_0 > 1 \quad \forall \rho > \bar{\rho}_0 \).

In order to prove the proposition, it has to be proved that, for \( x \in (1 - r, r) \), there exist \( \rho^* \) such that for \( \rho > \rho^* \). This is equivalent to proving that for \( x \in (1 - r, r) \):

\[
\begin{align*}
\bar{\rho}_1 &< 1 \\
\bar{\rho}_0 &< 1 \\
\rho &< 1
\end{align*}
\]

substituting, solving and rearranging with respect to \( x \)

\[
\begin{align*}
x &> 1 - r \\
x &< r \\
x &< r
\end{align*}
\]

Therefore \( \forall x \in (1 - r, r) \) it follows that it is always possible to define \( \rho^* \) in the following way:

\[
\rho^* = \max\{\bar{\rho}_1, \bar{\rho}_0, \rho\} \quad (1.A.40)
\]
such that \( \forall \rho > \rho^* \), policy \( A \) is selected.

### 1.B The weight of the contribution

In the literature it is often the case that the value of contributions are weighted through a parameter \( \alpha \), (see Bennedsen & Feldmann, 2006b), which can be interpreted as the relative value that each agent places to the transfer. In this context, this would be a measure of the greediness of each agent.

Formally by fixing at 1 the cost of contributions made by the interest group,\(^{11}\), it is obtained

\[
G(i) = \begin{cases} 
\alpha c(i) + rp(i) - x & \text{if } i \neq m \\
\alpha c(i) + rp(i) & \text{if } i = m 
\end{cases},
\]

and

\[
V = \begin{cases} 
\beta b(m) - (\rho - m)^2 & \text{if } m \neq \omega \\
\beta b(m) & \text{if } m = \omega 
\end{cases}.
\]

\(^{11}\)Event though in principle it can also be different from 1.
Since, in principle, there is no reason to assume that the incumbent and the bureaucrat attach the same weight to contributions.

Notice that these parameters have a relevant impact on the model: if $\alpha$ and $\beta$ are sufficiently low, then it may be the case that, for sufficiently high levels of $\rho$, the interest group is not able to move the final decision in favour of its own pet project neither by offering a contribution, nor with a bribe, hence improving the situation for all cases.

However, in order to obtain this outcome, both $\alpha$ and $\beta$ need to be lower than 1. On the other hand, if they are larger than 1, then it would be almost impossible to set up a mechanism that would avoid the interest group to be successful in its objective.

In conclusion, there is no \textit{a priori} reason to assume these parameters to be higher or lower than 1. On the other hand, the assumption of $\alpha = \beta = 1$ is totally plausible and has the advantage of greatly simplifying computations.
Chapter 2

Partial decentralisation and inter-governmental electoral competition

2.1 Introduction

In the last decades decentralisation reforms in public good provision and management have been implemented all over across countries, both in developed countries and developing ones, aiming to increase overall government accountability, in complete agreement with the famous decentralisation theorem (Oates, 1972).

Yet the empirical evidence is far from conclusive about the positive results of decentralisation reforms implemented across countries. On the contrary, results of many different studies support the opposite outcome. An example coming from the history of the United States can be found in Wallis, Fishback, and Kantor (2006); they show how, during the New Deal, the direct intervention of the Federal Government in substituting local government in the management of welfare programs dramatically reduced the corruption that at the time affected those programs. A similar result can be found in Fan, Lin, and Treisman (2009) who, exploiting a cross-national dataset, show that in countries with higher tiers of government the reported corruption is consistently higher.

Summarising these contradictory results, Treisman (2007) notices that “Empirical studies have found almost no solid, general result about the consequences of decentralisation. Decentralising government in a particular place and time is very much a leap in the dark”.

One of the most common explanations for this lack of results is that decentralisation reforms are often incomplete. In some cases, even after the provision of a public good has been formally decentralised, the central government \textit{de facto} remains deeply involved in its provision. In other cases even if the decisions about public good provision are really decentralised, the central government provides a large share of funds.\footnote{The first situation is more common in developing countries, where the quality of institutions is...}

\footnote{For example, in its \textit{World Development Report} (World Bank, 2003) the World Bank has proposed decentralisation reforms as an effective way to reduce corruption in developing countries.}
However, in spite of the different characteristics of each case, the common result is a weakening of political accountability, which typically requires the possibility for voters to easily assign a policy outcome to a given incumbent official, thus undermining the very reason for which decentralisation was implemented in the first place.

In the economic literature, these situations fall under the concept of *partial decentralisation*, defined by Devarajan, Stuti, and Shah (2009) as an “attempt at decentralisation that has not led to citizens’ being able to hold local governments accountable for budgetary allocations and their outcomes”.

The contribution of this paper is to develop a model taking into account the possibility that, despite the fact that they cannot observe the single contribution of each single government tier to the provision of a public good, voters still have an opinion about it, and since they condition their electoral choice on that opinion politicians will have the incentive to influence it.

Two tiers of government, central and local, are involved in the provision of a public good. The local government is directly involved since it is the actual provider, while the central government is indirectly involved since it provides resources. Incumbents are rent-seeking politicians with career concerns. Voters form an opinion on the relative contribution of each tier onto which they base their electoral choices for both incumbents. The local incumbent can divert part of the budget to spend on communication aiming to increase the perceived relative contribution of the local government at the expense of the central incumbent.

The main feature of this model is that, since the credit assigned for the provision of the public good is a *zero sum game*, there is a conflict of interest between the incumbents that affects both the amount of resources transferred and the final expenditure for the actual provision of the public good. This strategic interaction between the two governments could be considered as a proxy for the real forms of a bargaining about the allocation of resources from the central state to regions and provinces, typical of many developed countries where partial reforms of decentralisation took place.

To the best of the author’s knowledge, this is the first attempt to take this feature into account. The usual assumption found in literature is that voters only observe the total amount of public good provided (see for example Joanis, 2014).

The analysis highlights how this conflict of interest damages the provision of a public good, allowing to underline what are the main drivers of the central governments’ decision to transfer. In particular, the paper shows how party alignment between local and central incumbent increases both transfer and public good provision.

The paper is organised as follows. A survey of the relevant literature is carried out in section 2; section 3 introduces the stylised theoretical framework and section 4 presents the main results. Section 5 explores an extension of the model taking into account the presence of two local authorities rather than a single one. Finally, section 6 concludes the paper by discussing limits and possible extensions.
2.2 Related Literature

Decentralisation has been an increasing topic in the last decades, in particular with the growth of the “second generation fiscal federalism” literature (Oates, 2005; Weingast, 2009). In order to explain the unsatisfactory results of decentralisation reforms, this strand of literature has abandoned the benevolent social planner assumption of the first generation of fiscal federalism models, and focuses more on political economy and corruption problems related to centralisation. 3

As a matter of fact, the definition provided by Devarajan et al. (2009) is very broad and leaves space for ambiguity, and indeed many refinements have been provided in the literature. The one mainly connected with this paper distinguishes whether the incompleteness of the decentralisation is related to the collection of revenues or the expenditure of such revenues. The first case falls under the definition of partial fiscal decentralisation, defined by Brueckner (2009) as a regime in which the spending authority is devolved to the subnational level while financing relies on transfers from the central government. The second case is defined as partial expenditure decentralisation, described by Joanis (2014) as a situation where after the implementation of decentralisation reforms, the central government remains involved in the provision of the local public good targeted by decentralisation.

The model developed in this paper analyses a situation of partial fiscal decentralisation (fulfilling the definition of Brueckner, 2009), but follows Joanis (2014) in assuming that voters are aware that the public good provision depends on the contribution of both authorities but they are not able to measure the exact contribution of each government.

Despite the relatively scarce theoretical literature, both directly and indirectly, partial decentralisation is the subject of a large and still growing empirical literature. The distinction between fiscal and expenditure forms of partial decentralisation applies also to the empirical studies. Moreover, the occurrence of one case or the other seems to be correlated with the degree of development of countries.

Partial expenditure decentralisation is more common in developing countries, where, even though large decentralisation reforms have been de jure implemented, they are often incomplete because de facto central government has still an active role. This phenomenon has generated a large number of case studies, triggering a growing empirical literature.

For example, Estache, Garsous, and Seroa da Motta (2016) studied sanitation policies in the state of São Paulo in Brazil, where the Constitution divides the mandate related to water pollution among mayors and governor, assigning to the former the control of the local sanitation services, specifically sewage treatment, and to the latter the responsibility for the quality of water bodies. In studying the case of Bolivian decentralisation reforms, Inchauste (2009) argues that the lack of a clear improvement for either the poorest segments of the population and the poorest municipalities can be linked to the misalignment of responsibilities between different levels of government. Caldeira, Foucault, and Rota-Graziosi (2012) describe a similar pattern in decentralisation reforms in Benin, where some of the most important

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3For a comprehensive discussion of the literature on political decentralisation see Lockwood (2006), Mookherjee (2015)
public services, like education and water management, are provided in a regime of “shared competency”.

Partial fiscal decentralisation is more common in developed countries, and particularly in European countries where, following the emergence of the vertical subsidiarity paradigm, important expenditure decentralisation reforms have been implemented, however not always followed by clear or complete decentralisation on the financing side. As a consequence, despite the full autonomy in the expenditure and regulatory functions, central governments still play a key role through the fiscal channel (see for example Ambrosiano & Bordignon, 2006).

This phenomenon is mainly captured by the large literature studying the dynamic of intergovernmental grants in response to political economy factors. In particular, numerous studies conducted on several countries have consistently highlighted a positive correlation between political alignment and intergovernmental grants. For example, see the works of Levitt and Snyder Jr (1995) for the U.S., Solé-Ollé and Sorribas-Navarro (2008) for Spanish municipalities, Case (2001) for Albania, Migueis (2013) for Portugal. More recently Bracco, Lockwood, Porcelli, and Redoano (2015) studied the case of Italian municipalities, arguing that the main reason for this positive relationship is that the national government will use grants to increase the reelection probability of its aligned Mayors. Closely related to these studies, Joanis (2016) studied the evolution of Canada’s intergovernmental fiscal arrangements in spite of changes in federal and provincial electoral variables, showing a strong influence of political consideration on the evolution of those arrangements.

2.3 The model

There are two levels of government, a “central government” (indexed by the subscript $c$) and a “local government” (indexed by the subscript $l$) each of them controlled by an incumbent, who is a rent-seeking politician also caring about being reelected. The local government is formally in charge of the provision of the local public good $\theta$, while the central government provides a relevant part of the funding through direct transfers.

Since the focus of this work is on the allocation of spending, the paper assumes that the aggregate public revenue is given by $B$ that, without loss of generality, can be normalised to 1; let $\theta$ be measured by the monetary amount of public resources devoted to its provision.$^5$

The local government can spend its resources in two ways: it can assign a share $g$ to the actual provision of the local public good, and a share $m$ to influence voters’ perception about its own contribution to the provision of $\theta$.

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$^4$Another issue related to the decrease in accountability due to a partial fiscal decentralisation is the reduction in transparency over tax decisions by different governments. The intuition is that partial decentralisation may reduce voters’ ability to hold the right politician accountable for the amount of taxes they pay in exchange for the number of services they receive. In the end, they will be less able to select good politicians, undermining the assumption of Oates’ theorem (see for example Bordignon, Grembi, & Piazza, 2017, who provide an interesting analysis for the Italian municipal taxation).

$^5$Or similarly that the production function of $\theta$ is linear in the amount of funds.
At the beginning of the period, the local government is endowed with a share $b \in (0, 1)$ of the total revenue to which is then added the amount $t$ that it receives from the central government as a direct unconstrained transfer.\(^6\)

Then, the budget constraint of the local government can be written as:

$$g + m \leq b + t. \quad (2.1)$$

The reason to distinguish between $g$ and $\theta$ is due to a particular feature of this model. In fact, this paper assumes that incumbents extract a rent from each monetary unit spent in the provision of public goods onto which they retain total control. This means that the local incumbent extracts a rent from any monetary unit spent in the provision of $\theta$. Thus the level of the local public good actually provided to citizens is lower than the amount of resources devoted to its production.

Let $\alpha$ be the rent that the incumbent extracts from any monetary unit spent to provide $\theta$. It follows that

$$\theta = (1 - \alpha)g. \quad (2.2)$$

Therefore, on the one hand, the rent-extracting motive of the local incumbent is defined by the share $\alpha$ of the total expenditure in public good. On the other hand, the incumbent has a holding office motive, that can be considered as a mixture of economic benefits connected to the position and ego motives, and it is measured by $r \in (0, 1)$ (see for example Ferejohn, 1986; Maskin & Tirole, 2004).

As it is standard in the literature, by assuming a linear utility function for the local incumbent, it follows:

$$U_l = \alpha g + p(\theta)r, \quad (2.3)$$

where $p(\theta)$ is the probability of being reelected at the end of the game.

The central government is endowed with the remainder of the total revenues $(1 - b)$. The incumbent can spend this budget in two ways: it can transfer a share $t$ to the local government to finance the provision of $\theta$ and a share $G$ to finance a generic public expenditure at the central level.\(^7\)

The budget constraint of the central government is then

$$G + t \leq 1 - b. \quad (2.4)$$

Since the amount of resources transferred to the local government is under the control of the local incumbent, it follows that the central incumbent can extract a rent only out of the expenditure in $G$.

Therefore, let $\beta$ be the equivalent of $\alpha$ for the central incumbent and, for the sake of simplicity, assume that the pay off premium for being in office is equal for both levels.\(^8\)

\(^6\)For a discussion on the assumption of unconstrained transfers and its implication for the model see Appendix 2.E.

\(^7\)In this paper, $G$ will be treated as unproductive public expenditure, meaning that, when they need to decide about the reelection of the central incumbent, voters do not take into account the level of public expenditure at the central level.

\(^8\)It is not unreasonable to think that they might be different, and in particular that the benefit of holding the office at the central level is higher, Appendix 2.C relaxes this assumption.
Assuming that the central incumbent too has a linear utility function, it follows:

\[ U_c = \beta G + P(\theta) r, \]  

(2.5)

where \( P(\theta) \) is the central incumbent’s probability of being reelected.

### 2.3.1 Reelection probability

At the end of the game, both a central and a local election take place, and voters decide whether or not to reelect the incumbents on the basis of the level of the local public good they have received.

Following the literature on partial decentralisation (Joanis, 2014), assume that voters can only observe the value of \( \theta \) without any knowledge about how much local and central governments contribute. Moreover, this paper extends this idea by assuming that they form a belief about the relative governments’ contributions, which are endogenous.

In order to keep the analysis simple, an exogenous functional form for the reelection probability is assumed, such that it is increasing in \( \theta \) and takes into account the belief about the relative contribution of different branches. Defining \( \sigma \) as the voters’ belief about the share of local government’s contribution to \( \theta \), let \( p(\theta) \) take the form:

\[ p(\theta) = \phi_l + \sigma \theta, \]  

(2.6)

and \( P(\theta) \) the form:

\[ P(\theta) = \phi_c + (1 - \sigma) \theta. \]  

(2.7)

The first terms \( \phi_i \) represent a constant baseline reelection probability independent from the provision of \( \theta \). This considers the fact that, in general, the provision of \( \theta \) would not be the only issue at stake. Assume it to be exogenous, specific for each level of government, and such that \( \phi_l \in [0, \Phi_l] \) and \( \phi_c \in [0, \Phi_c] \).\(^9\)

### Voters belief about contribution

Voters can observe the level of the public good provided, \( \theta \), but they cannot observe the exact contribution of each governmental authority. However, they may form an opinion about the contributions. The formation of this opinion is in reality closely connected to political actions between the parties involved; these actions may involve political debates, but also public expenditure to influence voters.

This paper takes into account the case in which only the local government can directly affect the formation of this opinion. Despite being a simplification, this assumption is not so unreasonable for two reasons.

The first reason is that the local authority is the one that actually provides \( \theta \), which combined with the difficulties in understanding public budgets, makes it easier for voters to recognise the economic effort of the one that actually spent the money rather than the effort of the one who just transferred it. The second reason is that local politicians are much closer to voters than national ones, therefore it is much easier for them to convey their message to voters than it is for national politicians.

\(^9\)See Appendix 2.B for a detailed discussion.
Summing up, even if it may be a simplification to assume that only the local politicians are able to influence voters’ perception about contribution, it seems quite reasonable to think that, ceteris paribus, they are more effective.

Local government can influence voters’ perception of its own contribution to the provision of $\theta$ by spending a share $m$ out of its budget to increase the perception of its contribution. The expenditure in $m$ is directed to the financing of a “side good”, defined as a good which does not increase voters utility, but since it is connected to the provision of $\theta$, it works as a signal for voters about the contribution of the local government. This is a broad definition and can include different ways to influence voters; the simplest case of side good is the political advertisement to voters, to inform them that a public good is provided, thanks to the local government. More sophisticated situations can also fit this definition, such as the inclusion of accessory features that, while not increasing the functionality of a public good, indicate the cost and economic commitment of local government; this second case is more common if the public good is, for example, an infrastructural project.

However, the effectiveness of $m$ is not constant and may depend on the institutional situation of the specific constituency. In particular, it seems reasonable to think that if there is party alignment between the local and the central incumbent, the expenditure in $m$ will be less effective with respect to the case in which there is no alignment. To understand why this could be the case, one may think that since voters cannot observe the amount of money transferred from the central government to the local one, the local incumbent could have the incentive to pretend that the transfer has been lower than in reality, claiming that despite the cut from the central government, the local government guaranteed the provision of $\theta$. This message is more effective when the two incumbents are political opponents rather than allies.

In order to capture this alignment, suppose that the political spectrum can be represented by a unitary segment, where 0 corresponds to the most leftwing position and 1 corresponds to the most rightwing position. Then the political position of each incumbent corresponds to a point in this segment.

Let $\delta \in [0, 1]$ be the measure of the distance between the two points representing the incumbents’ political positions. This means that $\delta$ measures the degree of political misalignment between the two incumbents. Thus if $\delta = 0$, incumbents are perfectly aligned, while if $\delta = 1$ incumbents are at the extreme of the political space.

Hence $\sigma$ can be written as:

$$\sigma(m) = \delta m.$$  \hspace{1cm} (2.8)

Local government does not spend at all in the side good $m$ and its reelection probability is equal to $\phi_l$.\(^{10}\)

### 2.3.2 Time of the game

The game is divided into two stages. In the first stage of the game, the central incumbent decides how to allocate the share of public revenue assigned to the central

\(^{10}\)This is clearly a simplification, Appendix 2.D discusses the case in which $\sigma$ is partially exogenous.
government (1-b) between discretionary spending (G) and intergovernmental transfer (t).

In the second stage, given the total amount of resources available (b + t), the local incumbent decides how much to spend for the provision of the local public good (θ) and how much for the provision of the side good (m).

After the end of the game, both incumbents eventually face an election.

2.4 Solution of the game

2.4.1 Equilibrium spending of the local government

In the last stage of the game, given the amount of transfers t received from the national government, the local incumbent decides how much of its budget to devote to the production of θ, and how much to invest on voters’ beliefs.\footnote{All proofs are provided in Appendix 2.A.}

\[
\begin{align*}
\max_{U_l} & \quad U_l = \alpha g + r(\phi_l + \sigma \theta) \\
\text{s.t.} & \quad b + t = g + m \\
& \quad \theta = (1 - \alpha) g \\
& \quad \sigma(m) = \delta m
\end{align*}
\]

(2.9)

**Proposition 2.1.** Consider the maximisation problem of equation (2.9) and let \( \hat{t} \equiv \frac{\alpha}{sr(1-\alpha)} - b \). Two cases are possible.

(i) If \( t \geq \hat{t} \), the reaction functions of the local incumbent are:

\[
g^*(t) = \frac{b + t}{2} + \frac{\alpha}{2 \delta r (1 - \alpha)}
\]

\[
m^*(t) = \frac{b + t}{2} - \frac{\alpha}{2 \delta r (1 - \alpha)}
\]

(2.10)

(ii) If \( t < \hat{t} \), the reaction functions of the local incumbent are:

\[
g^*(t) = b + t
\]

\[
m^*(t) = 0
\]

(2.11)

Therefore, if the local government receives enough transfers from the central government, the local incumbent will divert part of these resources to influence the opinion of voters. On the other hand, if resources transferred by the central government are extremely scarce, the local incumbent prefers to assign all the budget to the provision of the local public good, even at the cost of minimising the probability of being reelected in the following period.
The reason is that while both $g$ and $m$ positively affect the reelection probability of the incumbent, only $g$ provides the rent; therefore, if the local budget is extremely scarce, the increase in the reelection chances due to a positive expenditure in $m$ is very small compared to the amount of rent that can be extracted from the provision of the public good.

This situation is depicted in Figure 2.1.

However, this suggests that, in order for this corner solution to appear, the rent extraction motive, $\alpha$, has to be sufficiently high. Indeed, the following relations can be derived.

**Corollary 2.1.** Given the result of Proposition 2.1, two extreme cases are possible:

(i) if $\frac{\alpha}{1-\alpha} \geq \delta r$, the solution is never interior;

(ii) if $\frac{\alpha}{1-\alpha} \leq \delta rb$, the solution is always interior.

In order to avoid the possibility of corner solutions, the underlying assumption will be $\alpha \leq \frac{\delta rb}{1+\delta rb}$. This assumption simplifies the analysis and it appears reasonable because, since $\alpha$ measures the share of public expenditure that is captured as rent, values of $\alpha$ too high would be unrealistic.

This situation is depicted in Figure 2.2.
Given equation (2.10) it directly follows that an increase in \( b \) and \( t \) has a positive effect on both \( g \) and \( m \). This is evident since, \textit{ceteris paribus}, the higher the local budget, either due to its own resources or due to an increase in transfers, the higher the expenditure in all sectors.

Conversely, the effect of the other parameters has an opposite sign on \( g \) and \( m \), as highlighted in the following Corollary.

**Corollary 2.2.** The expenditure in public good provision \( g(t) \)

(i) is higher when rent and political alignment are higher \((\partial g^*(t) / \partial \alpha > 0, \partial g^*(t) / \partial \delta < 0)\),

(ii) is lower when the office holding motive is higher \((\partial g^*(t) / \partial r > 0)\).

The converse holds for \( m(t) \).

The effect of rent extraction on \( g(t) \) is clear: since the local incumbent is rent-seeking, and the rent can be extracted only out of the provision of \( \theta \), the higher \( \alpha \) the higher the incentive for the local incumbent to increase the share of the public budget devoted to the local public good.

The positive effect of party alignment over the provision of the public good is due to the fact that the higher the alignment (i.e. the higher \( \delta \)), the less effective is the effort to persuade voters that the larger part of the credit for the provision of \( \theta \) should be recognised to the local government. Therefore the local incumbent will have the incentive to increase the spending \( g \). This can also be seen in the other way around. If the local incumbent belongs to a different party than the central one, voters will be more willing to believe that a lower level of \( \theta \) is due to the lack of transfer from the central government rather than to an opportunistic behaviour of the local one; this provides to the local incumbent the incentive to divert resources from \( g \) to \( m \).

Finally, an increase in the office holding motive of the local incumbent has a negative impact on the quantity of public good provided. This result may seem odd since, in principle, voters react positively to an increase of \( \theta \). The explanation is that without any rent motive, the “optimal” electoral motivated choice for the local incumbent would be to exactly split the budget between \( m \) and \( \theta \), since that maximises \( p(\theta) \). Because of the possibility to extract the rent, the local incumbent partly sacrifices the reelection chances devoting extra resources to the production of \( \theta \). Thus \( r \) represents the opportunity cost of extracting that rent. Therefore, the more the local incumbent cares about reelection, the closer will be the budget allocation to the “probability maximising” one.

### 2.4.2 Central Government

Consider now the first stage of the game, in which the central incumbent knows the reaction function of the local one and assigns its budget deciding the amount to transfer to the local authority.

The maximisation problem of the central incumbent is:

\[
\begin{align*}
\max_U & \quad U_c = \beta G + r[\phi_c + (1 - \sigma)\theta] \\
\text{s.t.} & \quad 1 - b = G + t
\end{align*}
\]  

(2.12)
Proposition 2.2. Consider the maximisation problem of equation (2.12). The optimal transfer \( t^* \) is equal to

\[
t^* = \frac{1}{\delta} - b - \frac{2\beta}{\delta r(1 - \alpha)}.
\] (2.13)

Two corner solutions are possible:

(i) if \( \beta < \beta \), the optimal transfer is equal to \( 1 - b \);

(ii) if \( \beta > \beta \), the optimal transfer is equal to \( 0 \).

Analogously to what done with the local government, conditions are derived with respect to the rent-extraction parameter \( \beta \). If the rent extraction incentive is sufficiently low, the central incumbent prefers to maximise the reelection probability, thus transferring all the budget to the local government. On the other hand, if the rent that can be extracted is sufficiently high, the central incumbent prefers not to transfer anything to the local authority, maximising the rent extracted, at the cost of a low reelection probability.\(^{12}\)

Consistently with what done for \( \alpha \), in the proceedings of the paper it will be assumed that \( \beta \in [\beta, \beta] \).

Comparative statics

The first thing that is worth to notice when analysing equation (2.13) is that, when deciding the amount to transfer, the central incumbent takes perfectly into account the budget of the local one and reduces the transfer accordingly. Therefore, ceteris paribus, the final budget of the local authority is constant with respect to \( b \).

Corollary 2.3. The amount of transfers to the local government, \( t^* \)

(i) is higher when career concern and alignment are higher (\( \frac{\partial t^*}{\partial r} > 0, \frac{\partial t^*}{\partial \delta} < 0 \)),

(ii) is lower when rent extraction is higher (\( \frac{\partial t^*}{\partial \alpha} < 0 \) and \( \frac{\partial t^*}{\partial \beta} < 0 \)).

For the central incumbent rent extraction and career concern act in the opposite way with respect to the case of the local one. Since the rent is extracted from public expenditure under the direct control of the incumbent, there are two consequences. On the one hand, the higher the incumbent can extract, \( \beta \), the higher the incentive to sacrifice reelection probability in exchange for a higher direct public expenditure \( G \). Then resources transferred will be lower.

On the other hand, since the office holding motive is the opportunity cost of extracting that rent and since higher rent means lower chances of being reelected, the stronger the career concern of the incumbent, the larger the transfer.

The effect of political alignment is the one expected from the literature: the more the local incumbent is aligned to the central one (i.e. the lower \( \delta \)), the higher

\(^{12}\)It is worth noting that, even if the central incumbent does not transfer anything to the local authority, the reelection probability is always greater than the baseline \( \phi_c \), because, given equation (2.10), the local incumbent will spend the entire budget \( b \) in the provision of \( \theta \). It then follows that \( P(\theta) = \phi_c + (1 - \alpha)b \), while \( p(\theta) = \phi_l \).
is the transfer received. What differs from the literature is the motivation for this result. *Ceteris paribus*, the central incumbent is willing to provide a higher grant to an aligned local incumbent knowing that this money will be spent more for the production of $\theta$ rather than $m$, from which the central incumbent can benefit in terms of reelection probability.

Therefore, a constituency whose local incumbent is more aligned to the central government receives a larger transfer. The reason is that the money transferred to a more aligned region will be spent more for the production of $\theta$ rather than $m$, hence the central incumbent will be less damaged. This result is coherent with the empirical literature that finds a consistent alignment bias in the analysis of inter-governmental transfers. Finally, a remarkable result is that the central incumbent always prefers an honest local official rather than a dishonest one. *Ex ante* this could be unexpected because from equation (2.10) the central incumbent knows that a more corrupt local politician will spend more in $g$ and less in $m$. Therefore one might expect that, at least in some cases, the amount of transfer would increase the higher $\alpha$.

However, this is never the case, meaning that the negative impact of a more corrupt local incumbent, namely that more resources are lost due to rent extraction, is always stronger than the positive one.

### 2.4.3 Optimal public good provision

**Proposition 2.3.** Given the regularity condition over $\alpha$ and $\beta$, the level of public good provided is equal to

$$\theta^* = \frac{r(1 - \alpha) + \alpha - 2\beta}{2\delta r}.$$  \hspace{1cm} (2.14)

The provision of the public good is higher the higher the political alignment between incumbents ($\frac{\partial \theta^*}{\partial \delta} < 0$) and the higher the rent extracted by the local incumbent ($\frac{\partial \theta^*}{\partial \alpha} > 0$).

The provision of the public good is lower the larger the rent of the central incumbent ($\frac{\partial \theta^*}{\partial \beta} < 0$).

The effect of a variation in the office-holding motive is ambiguous and depends on the relative strength among the rent extraction of each incumbent ($\frac{\partial \theta^*}{\partial r} \gtrless 0$ if $\beta \gtrless \frac{\alpha}{2}$).

The comparative statics for the level of public good highlights some insightful effects. Two of them, the effect of $\delta$ and $\beta$, were largely expected given the previous results.

Political alignment is confirmed to have a positive effect on the final public good provision. This result is in line with the literature, both theoretical and empirical, but the underlying motive is different from standard interpretations. The reason for which the aligned local government spends less in influencing the belief of voters is not that it internalises the damage caused to its aligned central incumbent, but just that the expenditure in $m$ is less credible and then less effective because voters are aware of their alignment.

The negative impact of $\beta$ over the public good provision comes directly from its negative effect over the transfer from the central to the local government.
The overall impact of $\alpha$, on the other hand, was *ex ante* less obvious. On the one hand $\alpha$ has a positive direct impact over the expenditure in the provision of the public good from the local incumbent but, on the other hand, it has an indirect negative impact on the amount of money that the local government receives from the central incumbent.

The intuition for this neat result is that while the local incumbent has a strong incentive in increasing the expenditure in $g$ due to the rent extraction, the central incumbent has a much weaker incentive to punish a more corrupt local incumbent. After all, despite the waste in public resources due to the higher rent extraction, a more corrupted local incumbent will, *ceteris paribus*, spend more for the public good and less for influencing voters, thus increasing the reelection chances of the central incumbent.

A particular attention is required in discussing the effect of the office holding motive, $r$.

Similarly to the case of $\alpha$, this ambiguous net outcome is due to the opposite effects that $r$ has on incumbents, discussed earlier in the paper. For the local incumbent, an increase in $r$ has a negative effect on the expenditure for $\theta$. Spending less in $m$ to increase the expenditure in $g$ is not a probability maximising behaviour, therefore the higher $r$ the higher the opportunity cost of doing it.

For the same reason it has a positive effect on the central incumbent: the higher the office holding motive, the higher will be the transfer to the local government.

However, this result firstly depends on the simplifying assumption that incumbents have the same payoff premium. If the payoffs differ for different offices, the two effects are separated, and each of them has an unambiguous impact.\(^{13}\)

### 2.5 The case of two constituencies

The results derived in the previous section rely on the assumption that the two incumbents share exactly the same voters. This situation is clearly unrealistic since there would be no reason to have two different tiers of government covering exactly the same territorial area.

In reality, a higher tier of government is in charge of several lower ones, therefore, given the budget of that tier, a higher transfer to a given local authority often implies a lower transfer to another one.

This section provides an extension of the model, in which the central government is in charge of an election district divided into two constituencies, each of them controlled by a local government.

Since the reelection probability, the central incumbent now does not depend on the level of public good provided by just one local incumbent. Moreover, if regions are not symmetric, the opportunity for the central incumbent to behave opportunistically increases.

In particular, the expected result is that *ceteris paribus*, the incentive to transfer to a given constituency will be higher, the larger the share of the population in that region and the more the local incumbent is aligned to the central incumbent.

\(^{13}\)See Appendix 2.C for an extended discussion of heterogeneous payoffs.
2.5.1 The environment

With respect to the basic model, assume now that there are two constituencies, \{1, 2\}, with a different population. In particular let \( \lambda \in (0, 1) \) be the share of the total population living in the constituency 1, and \((1 - \lambda)\) the share of population living in constituency 2.

In order to keep the analysis simple assume that there is no yardstick competition neither between voters nor between local incumbents. The first assumption has two implications. On the one hand, the reelection probability in each constituency depends exclusively on the level of the local public good provided in that constituency. On the other hand, voters’ belief about the local government’s contribution depends only on the level of expenditure \( m \) of their constituency. The second assumption implies that each local incumbent takes into account only the amount of transfer received.

Combining these two assumptions implies that each local incumbent faces the same maximisation problem of the simpler model. Given the amount of resources received from the central government, the local budget is allocated independently from what happens in the other constituency.

The time of the game is analogous as before. The sole difference is that now in the first stage the central incumbent has to decide not only how much to transfer to local governments, but also how much to transfer (if any transfer occurs) to each local government.

A representation of the temporal structure of the game is provided in figure 2.3.

![Figure 2.3: Structure of the Game](image)

It is now relevant how to consider these dual channels into the reelection probability. It is reasonable to assume that politicians care about their chances to be reelected in both regions, and even though the focus may be stronger on regions with larger population, the interest for the smaller ones will be rarely zero. The same is true for the reelection probability of the incumbent government in any region: no matter the decision of a central government, it will never be zero.

Given these observations, let \( P(\theta_1, \theta_2) \) be the central incumbent’s probability of winning the majority in the district. Let also \( P(\theta_1) \) and \( P(\theta_2) \) be the central incumbent’s probability of winning the majority in constituency 1 and 2 respectively.
Due to the absence of yardstick competition between voters, \( P(\theta_1) \) and \( P(\theta_2) \) are independent from each other and can be then defined as in equation (2.7).\(^{14}\)

In order for the central incumbent to win the election in the district, it is not necessary to win the majority in each constituency, therefore winning the majority in a more populated constituency will increase the chance of winning more than winning the majority in a less populated area.

In this case, \( \lambda \in (0, 1) \) can be exploited to weight the importance of each constituency in increasing the chances of winning of the central incumbent.

Thus the central incumbent’s probability of winning the reelection, \( P(\theta_1, \theta_2) \), can be written as

\[
P(\theta_1, \theta_2) = \lambda P(\theta_1) + (1 - \lambda) P(\theta_2).
\]

(2.15)

Therefore the one constituency case of the previous section corresponds to one of the two extreme cases in which either \( \lambda = 1 \) or \( \lambda = 0 \).

Given that reelection probability depends on the level of the public good provided and voters’ belief about central government’s contribution in each region, it follows by equation (2.7) that \( P(\theta_1, \theta_2) \) takes the form:

\[
P(\theta_1, \theta_2) = \lambda (1 - \sigma_1) \theta_1 + (1 - \lambda)(1 - \sigma_2) \theta_2.
\]

(2.16)

### 2.5.2 Local governments

As previously stated, given the no yardstick competition assumption, the game has not changed for each local incumbent.

Given the amount of transfer \( t_i \) received from the central government, the share \( b_i \) of total revenues and the rent extraction parameter \( \alpha_i \), the reaction functions of the local incumbent of constituency \( i \) are analogous to equation (2.10):

\[
g^*_i(t) = \frac{b_i + t_i}{2} + \frac{\alpha_i}{2 \delta_i r (1 - \alpha_i)}, \tag{2.17}
\]

and

\[
m^*_i(t) = \frac{b_i + t_i}{2} - \frac{\alpha_i}{2 \delta_i r (1 - \alpha_i)}, \tag{2.18}
\]

where all the observations made in previous sections are still valid.

### 2.5.3 Central Government

Consider now the first stage of the game. The central incumbent has to decide how much to transfer to local authority 1 and 2, \( t_1 \) and \( t_2 \) respectively, and how much to hold for the rent extraction motive.

Taking into account the reaction functions, the relative dimensions of the two constituencies and the share of total revenues of the central government, the opti-\(^{14}\)For the sake of simplicity assume the baseline probabilities in both constituencies to be equal to 0.
misisation problem of the central incumbent can be written as:

\[
\begin{cases}
\max U_c = \beta G + r[\phi_c + \lambda(1 - \sigma_1)\theta_1 + (1 - \lambda)(1 - \sigma_2)\theta_2] \\
\text{s.t.} & 1 - (b_1 + b_2) = G + (t_1 + t_2) \\
& t_1 + t_2 \leq 1 - B \\
& t_1, t_2 \geq 0
\end{cases}
\]  

(2.19)

Given that constrains form a close and bounded region, a maximum always exists, however, the focus is now moved to the interior solution, conditioning the existence of such maximum on the relationship between the degree of party alignment between incumbents and the share of the population in the region.

Since the focus is on the relationship between $\delta_1$, $\delta_2$ and $\lambda$, assume, for the sake of simplicity, that both local incumbents extract the same rent and that local regions are equally endowed.

The following proposition characterises the existence of an interior equilibrium.

**Proposition 2.4.** Let $H$ be the problem defined by equation (2.19), and let $\alpha_1 = \alpha_2 = \alpha$ and $b_1 = b_2 = b$.

If

(i) $\lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]$,

(ii) $\beta \in [\beta_{\text{min}}, \min\{\beta_{1,\text{max}}, \beta_{2,\text{max}}\}]$,

then, the global maximum of $U_c$

\[
t^*_1 = \frac{1}{\delta_1} - b - \frac{2\beta}{\lambda_1 r(1 - \alpha)} \\
t^*_2 = \frac{1}{\delta_2} - b - \frac{2\beta}{(1 - \lambda)\delta_2 r(1 - \alpha)}
\]  

(2.20)

is an interior solution of $H$.

Figure 2.4, provides a representation of Proposition 2.4.

Figure 2.4: The case of two regions: optimal and total transfers  
($\alpha = 0.2, \beta = 0.1, b = 0.1, r = 0.75, \delta_1 = 0.7, \delta_2 = 0.8$)

Figure 2.4 represents the amount of money transferred by the central government to each local government. If the share of voters in a region is too small, the central
incumbent would rather eliminate the transfer to that region. On the contrary, it might be the case that the optimal transfer region is too large; in that case a corner solution arises (in this case it happens with \( t^*_2 \)). Figure 2.4b represents the corresponding total amount of budget that the central government transfers to local one. It is worth to notice that at the extremes of \( \lambda \) the figures corresponds, given that for extreme values of \( \lambda \) only one region receives a positive transfer. Moreover it can be noticed that the central government budget can be better off if transfers are granted to both regions, rather than to just one region.

It is possible to recognise that all the comparative statics done for the one region case are still valid. Focus now the analysis on the impact of changes of \( \lambda \) and \( \delta_i \) on the optimal transfers.
The first effect is that the higher the share of population present in a region, the higher the transfer that it receives from the central government.

\[
\frac{\partial t^*_1}{\partial \lambda} = \frac{2\beta}{\lambda^2(1-\alpha)\delta_1 r} > 0
\]
\[
\frac{\partial t^*_2}{\partial \lambda} = \frac{-2\beta}{(1-\lambda)^2(1-\alpha)\delta_2 r} < 0
\] (2.21)

Focusing on the relation between \( \delta_i \) and \( \lambda \), it is easy to see that, \textit{ceteris paribus}, a higher party alignment can compensate a reduction in size. In fact, on the one hand, given the same size, more aligned regions receive a larger transfer, and, on the other hand, more aligned incumbents receive a positive transfer for a smaller relative dimension.

The simplest way to show this result is to analyse the corner solution \( t^*_1 = 0 \).\(^{15}\) \( t^*_1 = 0 \) implies:

\[
-b + \frac{1}{\delta_1} - \frac{2\beta}{\delta_1 \lambda r (1-\alpha)} = 0
\] (2.22)

which gives the condition:

\[
\delta_1 = \frac{1}{b} - \frac{2\beta}{\lambda br (1-\alpha)} \equiv \delta_1^{\text{max}}
\] (2.23)

where \( \delta_1^{\text{max}} \) is the maximum value of \( \delta_1 \) (so the minimum level of political alignment) that, given the population of region 1, guarantees a non negative transfer. It is immediate to see that higher \( \lambda \) the higher \( \delta_1^{\text{max}} \).
The implication is that if a region is relatively small, it is likely that it will receive a grant from the government only if it is very aligned, while if a region is big, it will receive a grant even if the local incumbent is not very aligned with the central one. This may provide an explanation for the fact that big municipalities are always able to intercept grants from regions and central government, despite the political alignment of the Mayor.

\(^{15}\)The analysis is symmetric for region 2.
2.6 Discussion and concluding remarks

Political decentralisation is a key topic which is basically focused on the hierarchical structure of governments. The key question is always whether this multi-level structure and shared responsibility are improving the efficiency of local public good provision and there are several ways to focus on this efficiency.

A relative unexplored efficiency issue is the information problem that emerges among voters on the spending side due to the shared responsibility. Conceptually this paper builds on the idea of the informational problem caused by a partial expenditure decentralisation of Joanis (2014), though treating this voters’ uncertainty as a strategic variable of the model.

In each constituency, a local and a central incumbent seek reelection by the same voters, even though in different elections, through the provision of a local public good. Local government provides the public good out of its budget spending, while the central government finances that budget through a transfer out of its resources. Voters observe only the final level of the public good provided and form a belief about the contribution of each government. The local authority can influence this belief by diverting a share of the public budget for advertisement.

This partial decentralisation creates a diverging incentive for the incumbents. By characterising the optimal level of both transfer and spending in the local public good, the paper highlights how party alignment is a key factor in both. In particular, it shows that a high party alignment between incumbents is beneficial in terms of resources transferred and spending in the local public good. Even though this result is consistent with the existing literature, the driver of this result is not an altruistic behaviour of politicians in the same party, rather the lower capacity of the local government to influence voters’ belief in its favour. In fact, the greater the alignment, the less voters are willing to believe in a conflict between the two tiers, thus the local government’s effort to increase its consensus at the expense of the central government is less effective. Therefore, ceteris paribus, the local incumbent prefers to increase the expenditure in the provision of the public good.

The model is then extended to include two asymmetric constituencies to take into account the incentive for an opportunistic redistribution of transfers by the central incumbent.

Finally, due to the simplicity of the model many issues have been left aside. In particular, the inefficiency at the heart of this process might be compensated by the standard advantages of decentralisation well identified by the literature. Moreover the framework set up in this paper leaves room for theoretical refinements: in particular, the model could be extended to provide a micro foundation of voters’ behaviour and to allow a comparison of this model with a baseline non-decentralised model.
Appendix

2.A Proofs

*Proof of Proposition 2.1.*

Equation (2.10) is derived from the maximisation of $U_l$ under the binding budget constraint. The reaction functions have also to fulfil the non-negativity constraint for both $g$ and $m$.

(i) $g \geq 0$ implies:

$$
\frac{b + t}{2} + \frac{\alpha}{2\delta r(1 - \alpha)} \geq 0,
$$

(2.A.1)

which is true since it is the sum of two non negative quantities.

(ii) $m \geq 0$ implies:

$$
\frac{b + t}{2} - \frac{\alpha}{2\delta r(1 - \alpha)} \geq 0,
$$

(2.A.2)

which is true only if

$$
t \geq \frac{\alpha}{\delta r(1 - \alpha)} - b \equiv \hat{t}
$$

(2.A.3)

Thus for $t \geq \hat{t}$ the maximisation of $U_l$ under the budget constraint fulfils both non negativity constraints, hence the reaction functions are well defined. If $t < \hat{t}$ the result is a corner solution where $g = b + t$ and $m = 0$.

*Proof of Corollary 2.1.*

On the one hand, condition (i) can be immediately derived by imposing $\hat{t} > 1 - b$. If this is the case, then for every possible $t$, it is always the case that $t < \hat{t}$. Hence there are only corner solutions.

On the other hand, condition (ii) directly derives by imposing $\hat{t} < 0$. If this is the case, then for every $t$, it is always the case that $t > \hat{t}$. Therefore there are only interior solutions.
Proof of Corollary 2.2.

Given equation (2.10), the first derivatives of $g(t)$ w.r.t $\alpha$, $r$ and $\delta$ are respectively:

\[
\begin{align*}
\frac{\partial g^*(t)}{\partial \alpha} &= \frac{1}{2\delta r(1-\alpha)^2} > 0, \\
\frac{\partial g^*(t)}{\partial r} &= -\frac{\alpha}{2\delta(1-\alpha)r^2} < 0, \\
\frac{\partial g^*(t)}{\partial \delta} &= -\frac{\alpha}{2\delta^2(1-\alpha)r} < 0,
\end{align*}
\] (2.A.4)

while partial derivatives of $m(t)$ have exactly the opposite sign:

\[
\begin{align*}
\frac{\partial m^*(t)}{\partial \alpha} &= -\frac{1}{2\delta r(1-\alpha)^2} < 0, \\
\frac{\partial m^*(t)}{\partial r} &= \frac{\alpha}{2\delta(1-\alpha)r^2} > 0, \\
\frac{\partial m^*(t)}{\partial \delta} &= \frac{\alpha}{2\delta^2(1-\alpha)r} > 0.
\end{align*}
\] (2.A.5)

Proof of Proposition 2.2.

Given equation (2.13), that immediately derives from the constrained maximisation of $U_c$, the two conditions on $\beta$ follow by imposing that $t^*$ belongs to the allowed space $[0, \hat{t}]$.

$t^* \geq 0$ requires $\frac{1}{\delta} - b - \frac{2\beta}{\delta r(1-\alpha)} \geq 0$, that rearranged with respect to $\beta$ gives

$$\beta \leq \frac{1}{2\delta}r(1-\alpha)(1-\delta b) \equiv \bar{\beta}.$$ (2.A.6)

Therefore for $\beta > \bar{\beta}$, $t^*$ assumes negative values.

On the other hand, $t^* \leq 1 - b$ requires $\frac{1}{\delta} - b - \frac{2\beta}{\delta r(1-\alpha)} \leq 1 - b$, and rearranging with respect to $\beta$,

$$\beta \geq \frac{1}{2\delta}r(1-\alpha)(1-\delta) \equiv \underline{\beta}.$$ (2.A.7)

Therefore for $\beta < \underline{\beta}$, $t^*$ assumes values greater than $1 - b$.\footnote{It is worth noting that, as long as $b \leq 1$, $\beta \leq \bar{\beta}$. Thus for every combination of the parameters, it always exists a value of $\beta$ such that $t^*$ is an interior solution.}

We can notice that, since $\underline{\beta} \leq \bar{\beta}$ as long as $b \leq 1$, an internal solution for the problem always exists.

To better underline the impact of the assumption made over $\alpha$, reconsider now the condition for an internal solution derived in the Corollary 2.1 by imposing the additional condition $t^* \geq \hat{t}$.

$$\frac{1}{\delta} - b - \frac{2\beta}{\delta r(1-\alpha)} \geq -b + \frac{\alpha}{\delta(1-\alpha)r}$$
The result is an additional condition over $\beta$:

$$\beta \leq \frac{1}{2} r(1 - \alpha) - \frac{\alpha}{2} \equiv \hat{\beta}. \tag{2.A.8}$$

It can be easily seen that if $\alpha \leq \frac{\delta rb}{1 + \delta rb}$ then $\beta < \hat{\beta} < \bar{\beta}$, meaning that if $\beta$ is such that it guarantees an interior solution for $t^*$, it also guarantees an interior solution for $g(t)$.

For values of $\alpha \in (\frac{\delta rb}{1 + \delta rb}, \frac{\delta r}{1 + \delta r})$, $\beta < \hat{\beta} < \bar{\beta}$, meaning that the absence of a corner solution for $t^*$ does not guarantee that there is not a corner solution for $g(t)$.

Finally, if $\alpha \geq \frac{\delta r}{1 + \delta r}$ it follows that $\hat{\beta} < \beta < \bar{\beta}$, and consequently either there is a corner solution for $t^*$, or there is a corner solution for $g(t)$.

**Proof of Corollary 2.3.**

Given equation (2.13), the first derivatives of $t^*$ w.r.t $\alpha, r$ and $\delta$ are respectively:

$$\frac{\partial t^*}{\partial \alpha} = -\frac{2\beta}{\delta(1 - \alpha)^2 r} < 0,$$

$$\frac{\partial t^*}{\partial r} = \frac{2\beta}{\delta(1 - \alpha)r^2} > 0,$$

$$\frac{\partial t^*}{\partial \beta} = -\frac{2}{\delta(1 - \alpha)r} < 0,$$

$$\frac{\partial t^*}{\partial \delta} = -\frac{2\beta - r(1 - \alpha)}{\delta^2(1 - \alpha)r} < 0. \tag{2.A.9}$$

For the sake of precision, if $\beta > \frac{1}{2} r(1 - \alpha)$ the derivative of $t$ with respect to $\delta$ is positive. However, for these values of $\beta$ the problem has not an internal maximum, so this case can be ruled out.

**Proof of Proposition 2.3.**

The first step of the proof is to derive equation (2.14).

Assuming regularity condition on $\alpha$ and $\beta$, by putting together equations (2.10) and (2.13) it follows that

$$g^* = \frac{1}{2\delta} - \frac{2\beta - \alpha}{2\delta r(1 - \alpha)} \tag{2.A.10}$$

and

$$m^* = \frac{1}{2\delta} - \frac{2\beta + \alpha}{2\delta r(1 - \alpha)}. \tag{2.A.11}$$

Equation (2.14) follows directly by equations (2.A.10) and (2.2).

The second step is to analyse the different comparative statics on $\theta^*$. The partial derivative of $\theta^*$ with respect to $\delta$ is equal to

$$\frac{\partial \theta^*}{\partial \delta} = -\frac{r(1 - \alpha) - 2\beta + \alpha}{2\delta^2 r}. \tag{2.A.12}$$

$\frac{\partial \theta^*}{\partial \delta} < 0$ implies $\beta \leq \frac{1}{2} r(1 - \alpha) + \frac{\alpha}{2}$, which under the regularity conditions of $\alpha$ and $\beta$ is always true.\(^{17}\)

\(^{17}\)One way to immediately verify this claim is to confront this condition with the one given by equation (2.A.8).

---

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The partial derivative of $\theta^*$ with respect to $\alpha$ is equal to

$$\frac{\partial \theta^*}{\partial \alpha} = \frac{1-r}{2\delta r},$$

(2.A.13)

which is always positive.

The partial derivative of $\theta^*$ with respect to $\alpha$ is equal to

$$\frac{\partial \theta^*}{\partial \beta} = -\frac{1}{\delta r},$$

(2.A.14)

which is always negative.

Finally a particular attention is required to analyse the effect of $r$. The partial derivative is equal to:

$$\frac{\partial \theta^*}{\partial r} = \frac{2\beta - \alpha}{2\delta r^2},$$

(2.A.15)

where it is easy to see that if $\beta > \frac{\alpha}{2}$ then $\frac{\partial \theta^*}{\partial r} > 0$ while if $\beta < \frac{\alpha}{2}$ then $\frac{\partial \theta^*}{\partial r} < 0$.

**Proof of Proposition 2.4.**

The first step is to prove that equation (2.20) is the global maximum of $U_c$, i.e. that $U_c$ is concave. The gradient of $U_c$ is

$$\nabla U_c(t_1, t_2) = \begin{pmatrix} \frac{1}{2}\lambda r(1 - \alpha_1)(1 - (b_1 + t_1)\delta_1) - \beta \\ \frac{1}{2}(1 - \lambda)r(1 - \alpha_2)(1 - (b_2 + t_2)\delta_2) - \beta \end{pmatrix},$$

(2.A.16)

from which can be derived the corresponding Hessian matrix:

$$H_{U_c}(t_1, t_2) = \begin{pmatrix} -\frac{1}{2}\lambda\delta_1 r(1 - \alpha_1) & 0 \\ 0 & -\frac{1}{2}(1 - \lambda)\delta_2 r(1 - \alpha_2) \end{pmatrix}.$$  

(2.A.17)

It is possible to see that the Hessian is negative-definite, then $U_c$ is concave. It follows that the necessary condition for the maximum

$$\nabla(U_c) = 0,$$

(2.A.18)

is also sufficient.

Writing the corresponding system

$$\begin{cases} \lambda r(1 - \alpha_1)(1 - (b_1 + t_1)\delta_1) \\ (1 - \lambda)r(1 - \alpha_2)(1 - (b_2 + t_2)\delta_2) \end{cases} = \beta,$$

(2.A.19)

and imposing the simplification $\alpha_1 = \alpha_2 = \alpha$ and $b_1 = b_2 = b$, it follows:

$$t_1^* = \frac{1}{\delta_1} - b - \frac{2\beta}{\lambda\delta_1 r(1 - \alpha)}$$

$$t_2^* = \frac{1}{\delta_2} - b - \frac{2\beta}{(1 - \lambda)\delta_2 r(1 - \alpha)}.$$  

(2.A.20)
The second step is to derive the regularity condition under which the global maximum is an interior solution of $H$.

In order to be an interior solution, the couple $(t_1^*, t_2^*)$ has to fulfil both the non negativity constraints and the budget constraint, i.e:

\[
\begin{align*}
    t_1 &\geq 0 \\
    t_2 &\geq 0 \\
    t_1 + t_2 &\leq 1 - 2b
\end{align*}
\]  

The non negativity constraints imply:

\[
\beta \leq \frac{\lambda r(1 - \alpha)(1 - b \delta_1)}{2} \equiv \beta_1^{\max},
\]

\[
\beta \leq \frac{(1 - \lambda) r(1 - \alpha)(1 - b \delta_2)}{2} \equiv \beta_2^{\max}.
\]  

The budget constrains can be rewritten as:

\[
\frac{1}{\delta_1} - \frac{2\beta}{\lambda \delta_1 r(1 - \alpha)} + \frac{1}{\delta_2} - \frac{2\beta}{(1 - \lambda) \delta_2 r(1 - \alpha)} \leq 1,
\]

from which the third condition is obtained:

\[
\beta \geq \frac{\lambda(1 - \lambda)r(1 - \alpha)(\delta_1 + \delta_2 - \delta_1 \delta_2 b)}{2(\lambda \delta_1 + (1 - \lambda) \delta_2)} \equiv \beta^{\min}.
\]  

In order for the global maximum to be an interior solution, the following condition must be satisfied:

\[
\beta^{\min} \leq \min\{\beta_1^{\max}, \beta_2^{\max}\}.
\]  

Condition $\beta^{\min} \leq \beta_1^{\max}$ implies

\[
\lambda \geq \frac{1 - (1 - b) \delta_2}{2 - b \delta_1 - (1 - b) \delta_2} \equiv \lambda^{\min},
\]

while condition $\beta^{\min} \leq \beta_2^{\max}$ implies

\[
\lambda \leq \frac{1 - b \delta_2}{2 - (1 - b) \delta_1 - b \delta_2} \equiv \lambda^{\max}.
\]  

These conditions imply that there exist values of $\lambda$ for which the global maximum is interior if and only if

\[
\lambda^{\min} \leq \lambda^{\max}.
\]  

This condition is true for $b \leq \frac{1}{2}$ which is always true.

Then for $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ the regularity condition for $\beta$ guarantees that the global maximum is an interior point.
2.B Regularity condition

Given $g^*$ and $m^*$, it follows that the reelection probabilities of incumbents are equal to:

$$p = \phi_l + \frac{(r(1 - \alpha) - 2\beta)^2 - \alpha^2}{4\delta r^2(1 - \alpha)}, \quad \text{(B.1)}$$

and

$$P = \phi_c + \frac{(r(1 - \alpha) + \alpha)^2 - 4\beta^2}{4\delta r^2(1 - \alpha)}. \quad \text{(B.2)}$$

in order to avoid corner solutions, conditions $p \leq 1$ and $P \leq 1$ are required, then

$$\phi_l \leq 1 - \frac{(r(1 - \alpha) - 2\beta)^2 - \alpha^2}{4\delta r^2(1 - \alpha)} \equiv \Phi_l. \quad \text{(B.3)}$$

$$\phi_c \leq 1 - \frac{(r(1 - \alpha) + \alpha)^2 - 4\beta^2}{4\delta r^2(1 - \alpha)} \equiv \Phi_c. \quad \text{(B.4)}$$

correspond to regularity conditions over $\phi_c$ and $\phi_l$. It may be that for certain values of the parameters, $\Phi_l$ and $\Phi_c$ are higher than one. In that case, there is no need for a lower bound, since there are no $\phi_i$ values that may cause a corner solution.

2.C Heterogeneous office-holding motives

In order to simplify the analysis, throughout the paper it has been assumed that both incumbents have the same office-holding motive equal to $r$.

The cost of this assumption in terms of realism is not very high since what really matters for the model is that any incumbent has a sufficiently strong motive to hold the office, regardless of the precise magnitude of this motive. However, since it is not unreasonable to think that being in charge at the national level and being in charge at the local level is not the same in terms of benefit, in this section this assumption is relaxed.$^{18}$

Let then $r$ be the payoff premium of the local incumbent and let $R \in [0, 1]$ be the payoff premium of the central incumbent.

Since at the local level nothing has changed, the reaction functions are the same:

$$g(t) = \frac{b + t}{2} + \frac{\alpha}{2\delta(1 - \alpha)r}, \quad \text{(C.1)}$$

and

$$m(t) = \frac{b + t}{2} - \frac{\alpha}{2\delta(1 - \alpha)r}. \quad \text{(C.2)}$$

On the other hand however, the utility function of the central incumbent is now:

$$U_c = \beta G + R(\phi_c + (1 - \sigma)(1 - \alpha)g). \quad \text{(C.3)}$$

$^{18}$For the sake of brevity assume that $\alpha$ and $\beta$ are such that an interior solution always exists.
The central incumbent then solves his own maximisation problem, from which the optimal transfer \( \tilde{t} \) is obtained:

\[
\tilde{t} = \frac{1}{\delta} - b - \frac{2\beta}{\delta R(1 - \alpha)}.
\]  

(C.4)

Therefore, when choosing its optimal expenditure, each branch of government cares only about its own interest of being reelected, and does not take into account the one of the other.

Thus the optimal level of the local public good provided is equal to:

\[
\tilde{\theta} = \frac{1 - \alpha}{2\delta} + \frac{\alpha}{2\delta r} - \frac{2\beta}{2\delta R}.
\]  

(C.5)

Differently from the simplified case, it is clear the positive effect of \( R \) and the negative effect of \( r \) on the final expenditure in the local public good.

Comparing this result with the one derived in the paper, it follows that \( \tilde{\theta} > \theta^* \) implies:

\[
\frac{1 - \alpha}{2\delta} + \frac{\alpha}{2\delta r} - \frac{2\beta}{2\delta R} > \frac{1 - \alpha}{2\delta} + \frac{\alpha}{2\delta r} - \frac{2\beta}{2\delta r}
\]  

which is true if and only if \( R > r \).

Therefore, if, *ceteris paribus*, the central incumbent has a stronger office holding motive than the local one, the provision of the public good increases, and the other way around. As discussed in the paper, the reason for this result is that the office holding motive impacts in an opposite way on the behaviours of incumbents: it has a positive effect on the willingness to transfer resources for the central incumbent, and a negative impact on the willingness to spend for the local public good for the local one.

### 2.D Partially exogenous voters’ belief

Equation (2.8) states that voters’ belief entirely depends on the expenditure in \( \text{m} \). As a consequence if \( \text{m} \) is zero, the provision of the public good does not give any electoral credit. This is clearly a simplification that can be relaxed by assuming an exogenous part in the definition of \( \sigma(c) \).

\[
\sigma(m) = a + \delta m,
\]  

(D.1)

where \( a \in [0, 1] \) represents voter’s baseline beliefs about the relative contribution of the local government to the public good expenditure.

The case analysed in the paper corresponds to the case \( a = 0 \), meaning that without an investment in the side good, voters do not assign any credit to the local incumbent. The opposite extreme, \( a = 1 \) corresponds to the case in which voters already believe that the local public good is entirely provided by the local government, hence there is no need to invest in the side good since its impact would be zero.

Given this assumption, and without taking into account the baseline probability parameters, it then follows that the objective functions of the incumbents take respectively the following form:

\[
U_i = \alpha g + r(a + \delta m)(1 - \alpha)g
\]  

(D.2)
for the local incumbent and
\[ U_c = \beta G + r(1 - a - \delta m)(1 - \alpha)g, \]
for the central one.

From equation (D.2), it follows that the reaction functions of the local incumbent, in case of an internal solution, are:
\[ g(t) = \frac{b + t}{2} + \frac{a}{2\delta} + \frac{\alpha}{2\delta(1 - \alpha)r}, \]
and
\[ m(t) = \frac{b + t}{2} - \frac{a}{2\delta} - \frac{\alpha}{2\delta(1 - \alpha)r}. \]

The first result is that in \( a > 0 \), the amount allocated for the provision of the local public good increases with respect to the previous case, which is obvious since the higher \( a \) the lower is the need to influence voters’ belief.

Given equation (D.3) and the reaction functions of the local incumbent, the optimal transfer is equal to:
\[ t^* = \frac{1 - a}{\delta} - b - \frac{2\beta}{\delta r(1 - \alpha)}. \]

The second result is that the central incumbent takes into account the effect of \( a \) on the expenditure in the local public good provision. Therefore, the higher \( a \) the lower the electoral reward that the central incumbent gets from the provision of the public good, the lower the amount of resources transferred to the local incumbent.

Moreover, given the linearity of the model, the central incumbent is able to fully take into account the reaction of \( g \) to \( a \). In fact, optimal values of \( g^* \) and \( m^* \) have the following forms:
\[ g^* = \frac{1}{2\delta} - \frac{2\beta - \alpha}{2\delta r(1 - \alpha)}. \]

The level of public expenditure in the local public good does not depend on the value of \( a \) and is exactly equal to the case analysed in the paper. The effect of an increase in \( a \) impacts totally on \( m \).

\[ c^* = \frac{1 - 2a}{2\delta} - \frac{2\beta + \alpha}{2\delta r(1 - \alpha)}. \]

Thus, what follows from this analysis is that, taking into account the internal solution, an increase in \( a \) does not affect the level of the public good provided, even though it reduces the spending in the side good, which one may argue is an inefficiency.

However, it is immediate to spot how for values of \( a \) large enough the local incumbent does not have any incentive to spend in the provision of the side good, while at the same time it may be the case that the central incumbent does not have any incentive to transfer any resource to the local incumbent. This may be worth being explored more, however, it is beyond the purpose of this analysis.
2.E Non discretionary transfer

In this model, we assume that the central government does not have any sort of control over the transfer it provides to the region. So in principle, the local government could spend the entire transfer in advertisement and not for the production of $\theta$. Another equally reasonable possibility is that the transfer cannot be used for purposes different than the production of $\theta$. In a way, this would mean that, even though the central government is not directly involved in the provision of $\theta$, it still maintains a certain degree of control over its money. In a formal way that could be done by assuming that instead of the transfer entering into the budget constraint of the local government, both the branches concur directly to the provision of $\theta$. It is easy to show that if we assume an additive linear production function,$^{19}$ the results are invariant with respect of the model.

\[ \hat{g} = x + t, \]

where $\hat{g}$ is the amount of total revenues devoted to the provision of $\theta$, $t$ is the quota of the central government, while $x$ is the quota of the local one that it has to be taken only out of its own budget $b$.

So the optimisation problem of the local government is now:

\[
\begin{align*}
\max_x & : \alpha(x + t) + r[\phi_l + \delta m(1 - \alpha)(x + t)] \\
\text{s.t.} & : b = x + m
\end{align*}
\]

(E.1)

from which the reaction functions obtained are:

\[ x^*(t) = \frac{b - t}{2} + \frac{\alpha}{2\delta r(1 - \alpha)} \]

(E.2)

and

\[ m^*(t) = \frac{b - t}{2} - \frac{\alpha}{2\delta r(1 - \alpha)} \]

(E.3)

As one should expect, the contribution of the regional government is decreasing with respect to the contribution of the national government.

However, if the total share of revenues devoted to the provision of $\theta$ is computed, the result obtained is:

\[ \hat{g}^*(t) = \frac{b + t}{2} + \frac{\alpha}{2\delta r(1 - \alpha)} \equiv g^*(t). \]

(E.4)

The total amount of resources devoted to the provision of $\theta$ is exactly the same of the model, and, since the optimisation problem for the central government is unchanged, it then leads to the same optimal transfer

\[ t^* = \frac{1}{\delta} - b - \frac{2\beta}{\delta r(1 - \alpha)}. \]

(E.5)

It then follows that as long as internal solutions are considered, direct contribution from the central government to the local public good expenditure does not provide any different result to the analysis.

\[ ^{19} \text{which is the most logical assumption since we are dealing with the combination of budget shares.} \]
Chapter 3

Environmental externalities in a Cournot competition model: innovation vs lobbying

3.1 Introduction

Environmental policy has always been an important political issue, characterised by potentially enormous economic consequences. Indeed, the analysis of the economic impact of pollution and of the measure to correct it dates back to Pigou.

In the last decades, the growing awareness of the damage caused by pollution has increased the support for increasingly stricter regulations. However, the industries that would be more affected by these reforms have reacted by investing increasing amounts of money in an effort to undermine them.

The Center for Responsive Politics, a nonprofit, non-partisan research group based in Washington D.C., tracks money spent on lobbying and campaign contributions in the U.S. political system. Consider, for example, the expenditure in lobbying of the firms belonging to the Oil&Gas and Transportation sectors, two of the industries most affected by environmental regulations.\(^1\) In particular, Figure 3.1 reports the total amount of money spent in lobbying by these two interest groups in the period 1998 - 2018.

![Figure 3.1: Total Lobbying 1998-2018](image)

\(^1\)Data and reports are available on the website of the Center for Responsive Politics, [https://www.opensecrets.org](https://www.opensecrets.org).
It is clear from the figure that both industrial groups have increased their spending on lobbying at the federal level. In particular, the transport sector spent more than $240 million annually during President Obama’s first term of office. Nevertheless, aggregate expenditure, however impressive it may be, does not make it possible to isolate the political preference of these groups.

Therefore, Figure 3.2 shows, for each election year in the period 1990-2018, the electoral contributions made by these two groups to the Democratic Party and the Republican Party respectively.

Figure 3.2: Campaign contribution: Party split in election years, 1990 - 2018

The situation emerging from the figures shows how these two industrial sectors have greatly increased their spending on lobbying over time, and in both cases with a clear preference for the Republican party, traditionally very opposed to environmental regulation. Particularly interesting is the case of the contribution from the Oil&Gas sector, which presents a pike since the election of President Obama, whose program included vigorous measures to fight climate change.

This case shows how in a context where the firms involved can form lobbies trying to influence the political outcome, the process of defining an environmental regulation can be arduous and thus it may deserve a political economy analysis. In literature, however, only a few contributions have dealt with this aspect. Polk, Schmutzler, and Müller (2014), for example, study a lobbying game between government and a monopolist involving the level of environmental regulation and the choice of a firm to install a plant in the country of reference, aiming to discuss whether a multinational firm would be able than a domestic firm to influence the political outcome. More closely related to this work, Fredriksson and Wollscheid (2008) analyse the relationship between lobbying and the choice of a monopolist to invest in the abatement of technology in presence of political instability.

In both cases, however, the market structure is taken as given. The authors study a monopoly, where they overlook the impact of the number of firms interested in the regulation present into the market and the possible effects on the final outcome.

Aiming to fill this void, this paper develops an oligopoly model to analyse the incentive for firms to adopt a technological innovation that reduces the environmental externality in a situation in which they can decide to exert pressure on the government.

The game has four stages. In the first stage, each firm has to decide whether to innovate, increasing its marginal cost but internalising the environmental damage, or not to innovate and join a lobby. In the second stage, the polluting lobby pays
the political contribution while in the third stage the government, given its policy objective and the contribution, decides the tax level. Finally, in the last stage of the game firms compete in a Cournot oligopoly.

In the paper three possible policy objectives are analysed and each case provides a comparison with a benchmark case in which the government is not interested in the political contribution. For each case, given the number of firms active in the market, a complete characterisation of the optimal tax rate and share of firms that decide to innovate is provided.

The main result of the paper is that, regardless of the policy objective, a positive share of polluting firms is always present in the market, for n sufficiently low. In addition, if the policy objective corresponds to the consumer Surplus standard, then for n small enough all firms decide not to innovate while for n big enough all firms decide to innovate. Therefore, markets characterised by small oligoplies are at risk of a significant weakening of environmental regulation. At a time when environmental degradation is considered, among scientists, one of the main emergencies of our century, one of the main policy implication of this paper is that breaking those oligoplies could be an efficient way to reduce pollution.

Indeed, this model could provide an interpretation, for example, for the recent scandal in the automotive sector, the so-called “Dieselgate”. Begun in 2014, this scandal was disclosed by a report from the International Council on Clean Trasportation (ICCC), revealing that diesel cars manufactured by major companies emitted far higher emissions than reported, despite the increasingly strict European regulations. On the one hand, this affair exposed how the major companies formed a cartel against environmental policies, and, on the other hand, how national governments actively engaged in covering up the violations of these companies.

The paper is organised as follows: after the survey of the related literature in section 3.2, section 3.3 presents the model in its general set up. Section 3.4 analyses the final stage of the game, while sections 3.5, 3.6 and 3.7 solve the game in three different setups. Finally section 3.8 concludes the paper.

### 3.2 Related literature

The environmental regulation of oligopolistic markets is a topic that has been extensively studied in recent decades, starting with the first critiques of the Pigouvian taxation in a monopolistic market by Buchanan (1969) and Barnett (1980), who point out that, while under perfect competition the Pigouvian approach may be optimal, if the firms have market power, then there is a trade-off between the externality reduction and the underproduction of the goods, given that oligopolists will react to taxation by a further reduction of production, possibly leading to a reduction of aggregate welfare.

Focusing on more recent contributions to the analysis of optimal tax level, Yin (2003) enriches the standard argument of Barnett by taking into account the presence of inter-firm externalities, where each firm benefits from the reduction of emissions by other firms. Antelo and Loureiro (2009) focus on the role of asymmetric

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2Or similarly in which firms are not able to set up a lobby.
information in reducing the taxation level, while Fujiwara (2009) on the effect on optimal emission tax/subsidy scheme in presence of product differentiation and free entry.\footnote{For an extensive review of the environmental policy in cases of imperfect competition see Requate (2006).}

Besides the optimal determination of a Pigouvian taxation, different strands of literature have focused on different aspects.

The first and more natural way to tackle the critiques to the Pigouvian solution is to try to reduce this underprovision incentive by combining the environmental taxation with a (indirect) production subsidy.\footnote{The focus has been moved to indirect subsidies, as direct subsidies on output are, in general, not allowed or not politically feasible. For example, this is particularly true within the European Union.} This is the so called \textit{emission taxes refunding scheme} where the government uses part of the revenue coming from the Pigouvian tax to subsidy firms, leading to a partial (or even total) refund scheme of the taxation; this refunding can be designed according to different rules. For example, Gersbach and Requate (2004) study, in a Cournot oligopoly the case of refunding emission taxes depending on market shares as it happens in Sweden. They show that this refunding scheme can provide the social optimum if firms cannot pre-invest in cleaner technology; in that case, the refunding scheme is able to provide the first best only if the government is able to allocate the refund according to both market share and investment share. Li, Fan, and Zhu (2016) analyse a refunding mechanism based on output in presence of a given abatement target. They are able to show that if the environmental damage is proportional to the increase of flow, then this scheme is able to achieve the first best, whereas if the environmental damage mainly depends on the emission stock, then the refunding mechanism is only able to achieve the second best.

The second way to deal with the issue is the study of different instruments rather than only taxation to correct externality.\footnote{For an exhaustive list of possible instruments see Goulder and Parry (2008).} For example, Requate (1993) studies the regulation of an asymmetric Cournot duopoly when the government can either impose a Pigouvian taxation or create a market for tradable permits; he shows that neither instruments are able to reach the first best and that in general, no instrument is superior to the other. Moraga-González and Padrón-Fumero (2002) study how different instruments, emission tax, technology subsidies and product charges perform in a Cournot oligopoly where firms can choose to sell either a cleaner or a dirtier variant when consumers have a higher willingness to pay for cleaner products. David (2005) compares an emission tax with a voluntary agreement of pollution reduction through an \textit{end-of-the-pipe} technology, on the model of what introduced in Denmark in 1996. She is able to prove that under some conditions (cheap abatement technology or very concentrated market) voluntary agreement is more efficient than emission taxation. However, these instruments do not come without problems: for example, as pointed out by Heyes (2000) environmental regulation is connected with major enforcement and compliance issues, often overlooked in the literature, while Sartzetakis (2004) proved that a market for emission permits fails to ensure efficiency if the product market is oligopolistic and that in some cases it can turn out to be worse, in term of welfare, than a bureaucratic allocation mechanism. Re-
cently there have been some attempts to explicitly take into account a policy mix of these instruments; for example, Leal, Garcia, and Lee (2018) study the policy mix of regulation through tradable polluting permits and emissions taxes in the specific case of a duopoly with a consumer-friendly firm.

A different strand of literature studies the relationship between environmental regulation and emission-reducing R&D in markets characterised by imperfect competition. Poyago-Theotoky (2007), presents a framework to study the effect of an emission tax on the effort in R&D undertook by firms in a duopoly analysing the case of independent effort and an environmental R&D cartel. Ouchida and Goto (2014) extend this framework showing that optimal taxation can be negative (i.e. a subsidy) when the environmental damage parameter is sufficiently small. And nevertheless, emission might be lower even in presence of a subsidy. Moreover, social welfare is always higher with taxation or subsidy, even in cases in which emissions increase with respect to laissez-faire.

Another large stream of literature considers the effect of environmental taxation on the firm’s location choice, and its subsequent effect on the willingness of the government to implement an environmental regulation at the risk to lose investments. The early contribution in this literature focused on the negative impact of taxation over the location choice, leading to the formulation of the so-called Pollution Heaven Hypothesis (PHP) according to which, in the long run, polluting firms will locate in countries with looser environmental regulation (see for example Rauscher, 1995). However, in the last decade, following the inconclusive empirical evidence on the PHP, a new interest has emerged for the subject with new papers providing theoretical frameworks showing that under certain conditions the polluting capital can move to more regulating countries rather than the opposite. For example, Dijkstra, Mathew, and Mukherjee (2011) study the problem of the location choice in a Cournot duopoly characterised by a domestic and a foreign firm with the environmental regulation being stricter in the domestic country. They show how, under certain circumstances, the foreign firm could be willing to relocate to the domestic country, instead of exporting, because, given the subsequent higher taxation, its increase in cost would be overcompensated by the increase in the cost of its rival. Elliott and Zhou (2013) relax the assumption of a given market structure, and, through an entry game, they are able to show that an already established domestic firm may be willing to invest in the foreign country, where the environmental regulation is stricter, in order to prevent a local firm to decide to enter into the market.

A particular field is the study of Corporate Social Responsibility where (in addition to the already mentioned Leal et al., 2018) some papers study the impact of a direct government intervention in reaction to environmental externality either through taxation (Lambertini & Tampieri, 2015) or through different instruments (for example Manasakis, Mitrokostas, & Petrakis, 2013, study the set up of public CSR certifications).

Despite all the differences, all the strands of literature have in common the use of oligopolistic games to take into account different aspects of the same big picture.

The other main literature to which this paper contributes is the literature on

\[6\]See Requate (2005) for a thorough review of this literature.
lobbying. In particular, this paper follows the strand of literature originated by the seminal paper of Grossman and Helpman (1994), modelling the lobbying activity as an agency problem, assuming that the government is interested in receiving a political contribution, possibly to build campaign funds for a future electoral competition but without explicitly taking that into the model. In addition, this paper is connected to the subsequent extension of Grossman and Helpman (1994) provided by Mitra (1999), and subsequent developments (see for example Laussel, 2006; Bombardini, 2008), who endogenises the lobby formation process in order to discuss the incentives for firms to form a lobby in the first place.

3.3 The model

Consider a sequential game of environmental regulation for a market into which a new regulation is set up. Subsection 3.3.1, presents the general framework of the model, while subsection 3.3.2 discusses in detail the various stages of the game.

3.3.1 The environment

This subsection presents the environment of the game, including players and timeline.

The players of the game are the n firms active into the market, the lobby formed by the firms that decide not to innovate and finally the government.

Firms

Consider an oligopolistic market with n firms. The baseline production technology, that is common to all firms, is characterised by the production of pollution and by a constant marginal cost that, for simplicity, is normalised to zero.

At the beginning of the game, an innovation of the production technology allows firms to abate pollution within the production process at a constant marginal cost $c$. It is reasonable to think that the cost of the innovation will be larger the more polluting is the production process in the first place. If the baseline production process is not very polluting, then it is very likely that implementing a technology that reduces emissions would not be very costly for the firm. If on the contrary, the baseline production is very polluting, then the increase in the cost for the implementation of the new technology will be much higher.

Therefore, $c$ can be considered as a measure of how much polluting a certain production process is. A relatively green production will be characterised by a lower $c$, whereas a very polluting process will be characterised by a high value of $c$.

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7 See Grossman and Helpman (2001, 2002) for an exhaustive discussion on this approach.
8 The concept of “within the production process” can be interpreted in a very broad sense. In the case of the automotive industry, for example, the abatement of pollution can derive from an innovation that reduces the amount of emission during the utilisation of a car; however, the cost of such innovation would still be sustained by the firm.

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Each firm then has to decide whether or not to implement the new technology. If the firm decides to innovate, it internalises the cost of pollution and pays a constant unitary cost $c$. If the firm decides not to innovate, it treats pollution like an externality, hence the production cost remains equal to 0. However, it has to pay a linear Pigouvian tax $t$ on each unit produced.

From now on firms that decide to innovate will be labelled as “green” (indexed by the subscript $g$) while firms that decide not to innovate will be labelled as “polluting” (indexed by the subscript $p$).

Summing up the cost structure of the market is the following:

$$\begin{cases} 
\epsilon_p \equiv t \\
\epsilon_g \equiv c
\end{cases} \quad (3.1)$$

Since the objective of this analysis is the evaluation of the externality produced by firms that choose not to innovate, two simplifying assumptions are introduced.

The first assumption is that, once they have innovated, green firms do not produce pollution at all. This is a convenient normalisation that can be justified if one thinks that the initial innovation brings the production technology to the frontier, therefore there would be no other possible improvement from an environmental point of view.

The second assumption is to measure the level of externality through the cost that the firm does not internalise. In other words, the gross unitary cost of externality produced by a polluting firm is equal to $c$. This is a simplified version of Lambertini and Tampieri (2015) who assume a linear environmental externality function, with $g > 0$ as the marginal polluting intensity of output. However, given that in this setup $c$ measures the degree of pollution associated with a certain production process, the higher $c$, the higher the externality. Therefore, to assume that $g = c$ greatly simplifies the analysis without a great loss in generality.

However, the presence of the Pigouvian taxation reduces the amount of unitary cost that polluting firms are able to externalise by $t$. This would imply either that the government redistribute the total revenues to citizens, or that it uses the total revenue to finance a technology that reduces the environmental damage.

Summing up, polluting firms produce a net unitary level of externality $d$ equal to:

$$d = (c - t), \quad (3.2)$$

that hereafter will be defined as “environmental damage”.

The Lobby

The firms that choose not to innovate form a lobby in order to exert pressure over the government aiming to obtain a tax reduction.

However, it is well known how both the formation and the internal mechanism of a lobby present several issues from a microeconomic point of view since in general, it involves a problem of free riding and coordination.

The first problem arises because the linear taxation is applied to every polluting firm, without the possibility to discriminate between the ones that have joined the
lobby and those who have not. Thus the lobbying activity aimed to tax reduction is a public good from the point of view of the firms (see Polk & Schmutzler, 2005, for a related example).

Once the lobby is set up, a coordination problem could be present in the decision making process, since different firms within the lobby may have different objectives, and different decision rules could be possible, each of them associated to coordination costs, and with the possibility of dropouts.

While the second problem is less important in this set up because all firms are symmetric, the free riding problem in the formation of the lobby still remains.

Following the literature (see for example Aidt, 1998), in this paper, it is assumed that, for some reasons external to the model, polluting firms are able to overcome these problems, and therefore form a lobby, who interacts on their behalf with the government.

The lobby operates through a political contribution $L$ to the government, which is a sum of money that the lobby pays to the government, or to the official in charge, in exchange for a better treatment in terms of environmental taxation.

The objective of the lobby, $V$, is to choose $L$ in order to maximise the profits of its members, that in this case coincide with the aggregate profit of all polluting firms.

$$V = \alpha n \Pi_p - L,$$  \hspace{1cm} (3.3)

However, as extensively discussed by Mitra (1999), forming a lobby presents additional costs, other than the mere political contribution. The simplest case (see for example Laussel, 2006) is the fixed cost of hiring lobbyists or renting offices at government offices.

Thus the members of the lobby have to transfer enough money to cover both the political contribution and the fixed cost.

$$\sum_p l_p = L + F$$  \hspace{1cm} (3.4)

Where $F \geq 0$, is the fixed cost necessary to set up the lobby.

**Government**

Consider a government who is in charge of the environmental policy for this market, which, as already stated, consists of a linear taxation over production, to be applied to firms that have decided not to innovate.\(^9\)

The government is moved by a mixed motive. On the one hand, it has a policy objective which, at least to a certain degree, includes an environmental concern, that is to prevent firms from externalising the environmental cost. On the other hand, however, the government is also inclined to accept political contributions from the lobby group in exchange for a more accommodating environmental policy.

Formally, in setting up the values of taxation, the government maximises its objective function $G$, equal to the sum of its “policy objective”, $W$, and political

---

\(^9\) The term “government” should be understood in a broad sense. This definition may include any public authority with the power to impose a tax in the market and which responds to political incentives.
contributions from the lobby, \( L \).

\[
G = W + L.
\]  

(3.5)

The specific choice of the policy goal of the government is purely arbitrary since it depends on many factors which are outside the aim of this paper. For this reason, three possibilities will be considered.

In the first case, the policy goal is purely the pollution abatement (from now on purely Pigouvian government), that would correspond to a situation in which the government is an environmental agency whose only goal is to tackle pollution with no regard of the economic consequences. In the second case, the government is still interested in the reduction of the environmental externality, but it is also interested in increasing the quantity produced. Finally in the third case the government adopts the consumes standard (see Neven & Röller, 2005), meaning that it maximises consumers’ Surplus net of the environmental externality.

One may ask why not to assume, as it is common in partial equilibrium literature, that the government maximises the standard Social Welfare function, which usually would include also aggregate profits. Even without taking into account the fact that such a choice is questionable itself, there are three main reasons for not including aggregate profits into the policy objective of the government.

The first reason is that, in this framework, the inclusion of aggregate profits into the Social Welfare function would create an additional problem in terms of conflict of interest. In fact there are two types of firms whose profits are affected by the tax in the opposite way; therefore, it is not clear why the government should be interested in the total profit obtained, or why it should assign the same (or a different) weight to the profit of each group.

The second reason is that in this paper, the government is not a Social Planner, but a political actor which is (at least partially) interested in its reelection. This eventually depends on the decision of voters, which in this partial equilibrium approach correspond to consumers. Therefore it seems totally reasonable to assume that since the government responds directly to voters, it takes into account only the variables that directly affect them. On the contrary, the business sector is able to influence its behaviour only through the lobbying channel.

The third reason is that this approach is rather common in the literature when the aim is to address firms’ strategic reaction to environmental regulation (see for example Markusen, Morey, & Olewiler, 1995; Rauscher, 1995; Polk et al., 2014).

Consider now the possible restrictions over the value of \( t \).

Given that the tax rate is paid by the polluting firms to compensate for the environmental externality, it follows that the best case scenario in terms of revenues would be the one in which the resources collected are exactly equal to the total cost of the externality produced by polluting firms.

In other words, one may think that since the legal basis to impose this taxation is the reduction of the environmental externality, which by definition cannot be negative, then the government cannot collect more resources than those needed to compensate for the environmental damage. Therefore, the best case scenario is the one in which the environmental externality is equal to zero, either because no firm chooses to pollute or because the government obliges them to internalise the entire
cost. If this is the case, it follows that $c$ is the upper bound for the tax rate, i.e. $t \leq c$.

With regard to the lower bound, however, no restriction is introduced, meaning that $t$ may assume negative values. In that case, the tax would turn into a subsidy that the government would give to polluting companies. This assumption is not unreasonable, given that the presence of incentives of various kinds in favour of polluting business is still a reality in many economic systems (for example see Coady, Parry, Sears, & Shang, 2017, for an analysis of the subsidy to polluting firms to the energetic sector). Moreover, it is not uncommon in the literature (see, among others, Requate, 1993; Fujiwara, 2009; Ouchida & Goto, 2014).

**Time of the Game**

The game is divided into four stages. At the beginning of the game, each firm has to decide whether to switch to the green technology or to maintain the baseline technology and join the lobby. In the following stage, the lobby of polluting firms engages the government with its lobbying activity. In the third stage, the regulator sets up the level of enforcement of the environmental regulation, and in the final stage firms compete in an oligopoly a l`a Cournot.

The timeline of the game is represented in Figure 3.3.

![Figure 3.3: Timeline](image)

The game is solved by backward induction.

### 3.3.2 Stages of the Game

In the proceeding of the paper, different cases of the game will be analysed, depending on different political objective functions of the government. For the sake of clarity, in this section, each stage of the game is discussed in general terms, in order to highlight the main point and assumptions of the analysis.

**Cournot Competition**

The market analysed is a Cournot oligopoly with $n$ firms. Given the choice made in the first stage of the game, firms will be divided into two groups, the polluting one, composed by $n_p$ firms, and the green one composed by $n_g$ firms, where $n_p + n_g = n$.

Let $\alpha$ be the share of polluting firms active into the market, such that

$$\alpha = \frac{n_p}{n}.$$
It follows that $\alpha \in [0, 1]$, however given that $n$ is the number of firms, it follows that $n \in \mathbb{N}$. Therefore either there are no polluting firms, i.e. $\alpha = 0$, or there is at least one, i.e. $\alpha = \frac{1}{n}$.

Given that it will simplify the analysis, it is convenient to assume that $\alpha \in \left[\frac{1}{n}, 1\right]$, with the particular case $\alpha = 0$ discussed aside.

The equilibrium of this stage of the game will define the total quantity $Q$ produced in the market and in particular the total quantity $Q_p$ produced by the polluting firms, and the total quantity $Q_g$ produced by the green firms.

Given equation (3.2) it follows that the total amount of externality (total environmental damage) produced by the polluting firms, $D$, is equal to:

$$D = (c - t)Q_p. \quad (3.6)$$

Moreover gross profits of each firm will be defined, where $\Pi_p$ and $\Pi_g$ are respectively the gross profits of a polluting firm and a green firm.

### Lobbying Game

Prior to the actual Cournot competition, the government has to set up the level of taxation $t$.

Given the government’s objective function defined in equation (3.5), the first step is to analyse the benchmark case in which there is no lobbying activity.

If $L = 0$ it follows that the government chooses $t$ in order to maximise its political objective $W$.

Defining $t_0$ as the tax rate that the government sets up if there is no intervention from the lobby, it follows that

$$t_0 = \arg \max_t W. \quad (3.7)$$

Therefore, $W_0 \equiv W(t_0)$ represents the fall back utility for the government (from now on $G_0$).

Following Grossman and Helpman (1994) in this paper it is assumed that the lobby offers a contribution schedule to the government, which is a map that assigns for every possible level $t$ chosen by the government the associated amount of contribution, since the lobby is aware that in choosing the tax rate, the government maximises its own objective function.

More specifically, since there is only one lobby, given the tax rate $t < t_0$, the minimum required contribution is the one that compensates exactly the government for the loss in utility, i.e. the one that guarantees the government $G = G_0$.\(^{10}\) Thus, for any tax rate $t < t_0$, the contribution schedule $L(t)$ takes the following form:

$$L(t) = W_0 - W(t). \quad (3.8)$$

Given equation (3.8), a well established result in the lobbying literature (Polk et al., 2014) is that the problem can be analysed directly assuming that it is the lobby that actually sets up the tax rate $t$ by defining the contribution schedule $L$.

\(^{10}\)The implicit assumption is that when indifferent, the government chooses the tax level preferred by the lobby group.
Therefore, given equation (3.3), it follows that the equilibrium tax rate $t^*$ will set up such that:

$$\alpha n \frac{\partial \Pi_p}{\partial t} = \frac{\partial L}{\partial t}.$$  

(3.9)

**Equilibrium share of polluting firms**

In the first stage of the game, each firm has to choose whether to implement the innovation, becoming a green firm or to join the lobby becoming a polluting one.

Since the aim of the paper is to determine how many firms will choose to innovate and how many will join the lobby and remain polluting, the choice can be treated as a modified version of an entry game.

In such a game, one firm at the time is called to decide, independently from all the others, whether or not to implement the innovation. If it chooses to innovate, it becomes a green firm and moves to the final stage of the game, whereas if it chooses not to innovate then it joins the lobby, pays the contribution and then moves to the final stage of the game.

Looking at the problem from the other way round, the choice for each firm $i$ is then whether or not to enter the lobby. In the first case, it pays a sunk set up cost for the lobby and then obtains the profit of a polluting firm. In the second case, it obtains the fall back profit, that in this case, differently from a standard entry game, is not zero, but it is equal to the profit realised by a green firm.

Therefore the equilibrium fraction of polluting firms over the total will be determined by imposing that the last firm choosing one technology is indifferent between the two.

In other words, since each firm chooses the technology that guarantees the higher profit, the equilibrium value of $\alpha$ is the one that equals the profits obtained with the different technologies.

A green firm does not sustain any extra cost out of the cost of production, therefore its net profit is exactly its gross profits obtained in the final stage of the game.

$$\pi_g = \Pi_g.$$  

(3.10)

On the other hand, each polluting firm has to pay its share to the lobby, therefore its net profits are lower than the gross profit:

$$\pi_p = \Pi_p - l_p.$$  

(3.11)

Given equation (3.4), and assuming that all the expenses are divided equally among the members of the lobby, the amount of money that each firm has to transfer to the lobby is equal to:

$$l_p = \frac{L + F}{\alpha n}.$$  

(3.12)

The equilibrium condition will be determined by equating the net profit of choosing to be polluting ($\pi_p$) and the neat profit of being green ($\pi_g$).

$$\alpha^*: \quad \pi_p = \pi_g.$$  

(3.13)
3.4 Cournot competition

In the last stage firms, given their choice in terms of technology and the tax rate $t$, compete in a Cournot oligopoly. Demand is equal to

$$ p = a - Q, $$

where $a$ represents the dimension of the market.

The following Lemma provides the characterisation of the final equilibrium in the market.

**Lemma 3.1.** Given the Cournot competition in the last stage of the game, let $\alpha$ be the share of polluting firms into the market, and let $t \geq t$.

The equilibrium quantities produced by the single polluting and green firms are respectively:

$$ q_p = \frac{a - t + (1 - \alpha)n(c - t)}{1 + n}, $$

$$ q_g = \frac{a - c - \alpha n(c - t)}{1 + n}, $$

(3.15)

The total quantity produced into this market is equal to:

$$ Q = \frac{n}{n + 1} (a - \alpha t - (1 - \alpha)c) $$

(3.16)

and the profits of a single green and polluting firms are equal to:

$$ \Pi_p = \frac{(a - t + (1 - \alpha)n(c - t))^2}{(n + 1)^2}, $$

$$ \Pi_g = \frac{(a - c - \alpha n(c - t))^2}{(n + 1)^2}. $$

(3.17)

Lemma 3.1 states that if the tax rate is sufficiently high, the cost advantage of the polluting firms is not enough to push green firms out of the market, and the outcome of the market is a Cournot equilibrium where neither the total number of firms present into the market, nor the relative size of the two groups are predetermined.

However, if the tax rate is too low, i.e. if $t < c - \frac{a-c}{\alpha n}$, the cost advantage of the polluting firms is sufficient to push green firms out of the market, the outcome is then a Cournot equilibrium with $\alpha n$ symmetric polluting firms.

Given these results, it is now convenient to define some additional functions that will be needed in the proceeding of the paper.

Firstly, given equation (3.15), it is possible to define total quantities produced by each group by aggregation:

$$ Q_p = \sum q_p \equiv an \frac{a - t + (1 - \alpha)n(c - t)}{1 + n}, $$

$$ Q_g = \sum q_g \equiv (1 - \alpha)n \frac{a - c - \alpha n(c - t)}{1 + n}. $$

(3.18)

---

11 All proofs are provided in Appendix 3.A.
3.5 Purely Pigouvian Government

Suppose that the government is moved only by a Pigouvian motive, that is it only cares about minimising the level of externality produced by firms.

Its policy objective $W$ will be exactly equal to the total environmental damage $D$:

$$W = -D.$$  \hfill (3.19)

Therefore, the political objective of the government is to minimise pollution.

3.5.1 Lobbying Game

It follows that without any intervention of the lobby, the government would set the highest possible tax rate, $t_0 = c$, associated with $G_0 = 0$.

It then follows from previous equation:

$$L = D,$$  \hfill (3.20)

from which it follows

**Proposition 3.1.** If the government’s policy objective is purely Pigouvian, then, in presence of the lobby, the equilibrium tax rate $t^*$ assumes the following form:

(i) If $\alpha \leq \bar{\alpha}$, the equilibrium tax rate is equal to:

$$t^* = c - \frac{a - c}{2\alpha} \cdot \frac{n + 1 - 2\alpha n}{n + 1 - \alpha n},$$  \hfill (3.21)

which is higher the higher the share of polluting firms, the higher the number of firms into the market and the higher the unitary cost $c$.

(ii) If $\alpha > \bar{\alpha}$, the equilibrium tax rate is equal to $c$.

Finally the tax rate $t^*$ is always sufficiently high to allow green firms to stay into the market.

The main result of Proposition 3.1 is twofold. On the one hand, the polluting lobby is able to obtain a more favourable treatment in terms of taxation. On the other hand, however, this ability is not unbounded.

In fact, despite the willingness to reduce the tax rate in exchange for a political contribution, the government will increase the taxation the higher the number of polluting firms into the market. Eventually, if the share of polluting firms into the market is too high, the government will set up the highest possible tax rate, $t^* = c$, regardless of the contribution.\footnote{As a matter of fact, if $\alpha > \bar{\alpha}$, it follows that $t^* > c$ and $L^* < 0$. In other words, the optimal behaviour of the government would be to set up a tax rate higher than the one needed to ensure no environmental damage and then compensate the lobby with a monetary transfer. This is obviously meaningless, therefore it can be ruled out by assuming that whenever $\alpha > \bar{\alpha}$, the government will set the maximum tax rate without any lobbying activity from the polluting firms.}

It is worth noticing that the tax rate may assume negative values. This would happen for values of $\alpha$ sufficiently low. The intuition is simple: when the share of polluting firms is too high, the government will set the maximum tax rate without any lobbying activity from the polluting firms.\footnote{As a matter of fact, if $\alpha > \bar{\alpha}$, it follows that $t^* > c$ and $L^* < 0$. In other words, the optimal behaviour of the government would be to set up a tax rate higher than the one needed to ensure no environmental damage and then compensate the lobby with a monetary transfer. This is obviously meaningless, therefore it can be ruled out by assuming that whenever $\alpha > \bar{\alpha}$, the government will set the maximum tax rate without any lobbying activity from the polluting firms.}
polluting firms is very small there are two effects. On the one hand, the environmental damage that they cause will be almost negligible, while, on the other hand, these firms will face a weaker competition into the market given that the vast majority of the firms are green. Both these effects work in favour of the Lobby, whose action is therefore very effective. In particular, if profits are large enough, the tax rate will be negative.

A somewhat related result can be found in Ouchida and Goto (2014) who, even with a different setup, show that a linear tax can turn into a subsidy if emissions are small enough.\footnote{To be precise Ouchida and Goto (2014) consider a model of emission regulation and emission reducing R\&D, (building on Poyago-Theotoky, 2007). Nevertheless, despite the difference in the subject of the analysis between this work and their work, the decision of the government about the tax rate is analogous.}

For example, consider the case in which only one polluting firm is present. If $\alpha = \frac{1}{n}$, the optimal tax rate is equal to

$$t^* = c - \frac{a - c}{2} \cdot \frac{n - 1}{n},$$

(3.22)

which is negative if

$$c < \frac{n - 1}{3n - 1} a,$$

(3.23)

Figure 3.4 depicts the optimal tax rate

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.4}
\caption{Optimal tax rate $t^*$ \hfill ($a = 100$, $c = 25$, $n = 10$)}
\end{figure}

A further result is that green firms are always able to stay in the market. This is because on the one hand if the fraction of polluting firms is relatively large, the tax is sufficient to offset their competitive advantage. On the other hand, when the tax becomes a subsidy, the fraction of polluting firms is small enough to maintain the competitiveness of green firms.

In terms of comparative statics, the impact of the number of firms into the market is fairly clear. The higher the number of firms into the market, the lower
the profits, therefore the lower the possibility for the lobby to pay a contribution sufficient to influence the policy outcome.

A similar reasoning applies for the cost $c$. Given that $c$ represents the value of the externality that polluting firms impose on society, if $c$ increases, there are two effects. On the one hand, the higher $c$ the higher firms gain to externalise the cost. On the other hand the higher $c$, the higher the willingness of the government to increase taxation, therefore the higher the cost of the political contribution from the lobby.

From the result of Proposition 3.1 it follows that this second effect is overall stronger than the first one, meaning that ceteris paribus the higher the unitary environmental damage, the higher the tax rate.

### 3.5.2 Equilibrium share of polluting firms

Consider now the first stage of the game, in which each firm has to decide whether or not to enter into the lobby. Given Proposition 3.1, the following proposition characterises the equilibrium of the game:

**Proposition 3.2.** If the government’s policy objective is purely Pigouvian, then in presence of a lobby, the equilibrium share of polluting firms is equal to:

(i) If $n \leq \overline{\pi}$

$$\alpha^* = \frac{n + 1}{n} - \frac{a - c}{a} \frac{\sqrt{(a - c)^2 - 4F(n + 1)}}{4Fn},$$

with $\alpha^*$ decreasing in $n$.

(ii) If $n > \overline{\pi}$, all the firms choose to be green.

Figure 3.5 depicts the result.

![Figure 3.5: Equilibrium share of polluting firms $\alpha^*$](image)

Given Proposition 3.1, it is already known that $\alpha$ will have an upper bound. In fact, if $\alpha = \frac{1+n}{2n}$, the tax rate is equal to $c$. Therefore there is no incentive for any
firm to join the lobby, to pay its share of set up costs, and then to obtain the same gross profit of a firm that has chosen to be green.\textsuperscript{14}

Proposition 3.2 states that the share of polluting firms into the market is a decreasing function of the number of firms and in particular if \( n \) is large enough, no firm will choose to enter the lobby. This result is largely expected because, as the number of firms into market increases, profits decrease. Therefore the possibility for firms to pay the extra costs related to the lobbying activity decreases. Eventually, profits will no longer be sufficient to cover those costs, and all firms will choose to internalise the environmental externality. Moreover, even if \( n \) is sufficiently low to support a positive number of polluting firms, green firms will remain active.\textsuperscript{15}

3.6 Government cares about total quantity

Assume now that the government is interested not only in the reduction of the environmental externality but also in the total quantity produced. The reason for which the government may care about the quantity could be twofold.

On the one hand, the higher the quantity, the lower the price, therefore consumers are better off.\textsuperscript{16}

On the other hand, assuming that the government cares about the level of occupation, the higher the quantity produced, the higher the number of production factors used, therefore, in general, the higher the occupation.\textsuperscript{17}

If this is the case, the government’s policy objective function \( W \) is equal to

\[
W = Q - D. \tag{3.26}
\]

This section is organised just like the previous one. Moreover, given that many economic intuitions and many results, in particular comparative statics, are analogous to the previous case, they are, for the sake of brevity, omitted.

Finally, in order to distinguish the results of this section with the other one, they will be labelled with the subscript \( q \).

3.6.1 Lobbying Game

First of all, Lemma 3.2, characterises the benchmark situation.

\textbf{Lemma 3.2.} Given \( c \in (0, a - 1) \) and absent any pressure from the polluting lobby, the government sets up the highest possible tax rate.

\textsuperscript{14}As a matter of fact one may notice that

\[
\lim_{F \to 0} \alpha^* = \frac{1 + n}{2n}, \tag{3.25}
\]

meaning that if there is no cost to set up the lobby, the share of polluting firms would be such that \( t^* = c \) and \( L = 0 \), with polluting and green firms obtaining the exact same net profit.

\textsuperscript{15}It is interesting to notice that in case of a monopoly, the government would set \( t = c \) and consequently the firms would choose to be green.

\textsuperscript{16}In fact the higher the quantity, the higher the consumers’ Surplus.

\textsuperscript{17}Of course the underlying assumption is that the economy under analysis is a closed one.
The total quantity produced, $Q_0$, is equal to the one obtained in a market with $n$ green firms, which is the lowest possible in this market.

A direct result of Lemma 3.2, is that

$$L = D - (Q - Q_0).$$

(3.27)

Given that the quantity produced is a decreasing function of the tax rate, the amount that the lobby has to pay for a given tax rate will be, ceteris paribus, lower with respect to the Pigouvian case.

**Proposition 3.3.** If the government’s policy objective includes the total quantity produced, then, in presence of the lobby, the equilibrium tax rate $t_q^*$ assumes the following form:

(i) If $\alpha \leq \overline{\alpha}_q$, the equilibrium tax rate is equal to:

$$t_q^* = c - \frac{a - c}{2\alpha n} \cdot \frac{n + 1 - 2\alpha n}{n + 1 - \alpha n} - \frac{n + 1}{2\alpha n(n + 1 - \alpha n)},$$

(3.28)

which is always lower than $t^*$.

(ii) If $\alpha > \overline{\alpha}_q$, the equilibrium tax rate is equal to $c$.

Finally the tax rate $t_q^*$ is always sufficiently high to allow green firms to stay into the market.

From a qualitative point of view, Proposition 3.3 is completely analogous to Proposition 3.1. The tax rate is a decreasing function of the share of polluting firms and eventually for $\alpha$ too big the government chooses the highest possible tax rate.

From a quantitative point of view, however, this case is more favourable to the lobby because of the tradeoff faced by the government. On the one hand, by raising the tax rate the government decreases the environmental externality but reduces the quantity produced, and the other way around, if it decreases the tax rate. Overall, this tradeoff increases the willingness of the government to accept a contribution from the lobby, therefore, in the space of the interior solution, the tax rate will be, ceteris paribus, lower than in the previous case.

It then follows that the tax rate will become a subsidy for a larger combination of parameters.

Sticking to the example of just one polluting firm, it follows

$$t_q^* = c - \frac{a - c}{2} \cdot \frac{n - 1}{n} - \frac{n + 1}{2n},$$

(3.29)

that is negative if

$$c < \frac{n - 1}{3n - 1} a + \frac{1 + n}{3n - 1},$$

(3.30)

which is clearly a less demanding condition with respect to the previous case.

Finally, Proposition 3.3 states that, interestingly enough, despite the reduction of the tax rate (or of the increase of the subsidy) green firms are still able to remain into the market. The share of green firms is going to be lower than the previous case, nevertheless, it will still be positive.

Figure 3.6 depicts the optimal tax rate.
3.6.2 Equilibrium share of polluting firms

Consider now the firms’ choice whether to join the lobby or innovate. Given Proposition 3.3, the equilibrium of the game can be characterised as follows:

**Proposition 3.4.** The equilibrium share of polluting firms is equal to:

(i) if \( n \leq \bar{n}_q \)

\[
\alpha_q^* = \frac{n + 1}{4n} \frac{(a - c + 1)^2 - 4F(n + 1) - (a - c - 1)\sqrt{(a - c + 1)^2 - 4F(n + 1)}}{a - c - F(n + 1)}.
\]

(ii) if \( n > \bar{n}_q \), all the firms choose to be green.

Figure 3.7 depicts the result.
The upper bound is consistent with Proposition 3.3, as it was for Proposition 3.2, and in fact the result is completely the same than the one obtained in the previous section.

3.7 Government implements Consumer Standard

This section investigates the case in which the government, when it has to set up the tax rate $t$, takes also into account the consumers’ Surplus, the so called consumers standard. It then follows:

$$W = S - D. \quad (3.32)$$

Analogously to section 3.6, the structure is unchanged and results that have already been discussed will be omitted. Moreover, the subscript $s$ will be used to label the results so as to distinguish them from those of other sections.

3.7.1 Lobbying Game

**Lemma 3.3.** Absent any pressure from the polluting lobby, the government sets up the highest possible tax rate.

Consumer Surplus, $S_0$, is equal to the one obtained in a market with $n$ green firms, which is the lowest possible in this market.

Therefore, exactly like a fully Pigouvian government, any political contribution being absent, the government sets up $t_0 = c$. In this case, however, the baseline utility is different: $G_0 = S_0$.

It then follows from equation (3.8):

$$L = W_0 - W = D - \Delta S. \quad (3.33)$$

Considering equation (3.33) and Lemma 3.3, it follows that compared to the previous case, the amount of the contribution, in this case, is lower. In fact consumers’ Surplus increases if the tax rate decreases, therefore the higher Surplus partially offsets the higher level of externality, reducing the utility compensation that has to be provided by the lobby.

This reduction in the required political contribution has a dramatic impact on the optimal tax rate, as stated by the following proposition.

**Proposition 3.5.** If the government’s policy objective takes into account the environmental externality and consumers’ Surplus, then, in presence of the lobby, the equilibrium tax rate $t^*$ assumes the following form:

$$t^*_s = c - \frac{a - c}{an}, \quad (3.34)$$

such that only polluting firms produce a positive quantity.

Proposition 3.5 states that if the government is also interested in the consumers’ Surplus, the action of the polluting lobby is very effective. The government moves from the maximum tax rate to the rate that pushes the green firms out of the market.
In fact, it is worth noticing that, even if the tax rate increases when the share of polluting firms into the market does so, it never reaches the maximum level $c$.\footnote{Taking the limit with respect to the number of firms}

Consider the case in which there is only one polluting firm into the market. If $\alpha = \frac{1}{n}$, the optimal tax rate is equal to

$$t_s^* = 2c - a,$$

which assumes the form of a subsidy under the not very demanding condition

$$c < \frac{a}{2}.$$\hspace{1cm} (3.37)

The comparative statics is analogous to the one provided by Proposition 3.1. The tax rate is an increasing function of the marginal cost $c$ and the share of polluting firms $\alpha$, while it is a decreasing function of the size of the market $a$.

Figure 3.8 depicts the optimal tax rate.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure3.8}
  \caption{Optimal tax rate $t_s^*$}
  \label{fig:optimal_tax_rate}
\end{figure}

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure3.8}
  \caption{Optimal tax rate $t_s^*$}
  \label{fig:optimal_tax_rate}
\end{figure}

\section{Equilibrium share of polluting firms}

As one may expect, the extremely favourable outcome for polluting firms of Proposition 3.5 has a dramatic impact over the choice of the single firm in the first stage of the game.

\textbf{Proposition 3.6.} If the government’s policy objective takes into account pollution and surplus, then, in presence of the lobby

(i) If $n \leq n_s$, all firms choose to be polluting

\footnote{Therefore only if the market converges to perfect completion, the tax rate converges to the benchmark level $c$.}

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(ii) If \( n > \pi_s \), all firms choose to be green.

The result of Proposition 3.6 is extremely net.\textsuperscript{19} If the competition is not strong enough, all firms will choose not to innovate and join the lobby. Only when, due to the high number of firms, the profits are too low to sustain the expenditure to set up the lobby, all firms decide to innovate.

### 3.8 Final Remarks

This paper studies the incentive for firms in an oligopoly to adopt a new cleaner technology, promoted by a government through environmental taxation when they have the possibility to form a lobby trying to resist to the enforcement of this tax.

Firms are assumed to compete in a Cournot oligopoly, while the lobbying game is modelled as an agency problem. Each firm at the beginning of the game can choose to become a green firm, internalising the environmental damage though increasing its marginal cost, or to become a polluting firm, paying a linear tax and joining the polluting lobby.

The government chooses the tax rate in order to maximise the sum of its political objective function and the contribution that it may receive from the polluting lobby.

In order to offer a larger analysis, three different policy objectives are analysed: simple environmental concern, interest in quantity produced minus environmental damage and, finally, consumers standard.

In each of these cases, without the action of the lobby the government would enforce the strictest regulation (i.e. the highest tax rate), obliging all firms to choose to innovate.

If polluting firms can form a lobby to act against the regulation, the effectiveness of this action will depend on the specific policy objective of the government and on the number of firms active into the market.

Under the first two policy objectives the result is that, in equilibrium, if the number of active firms is small enough, both green and polluting firms coexist into the market; whereas, if the number of firms is too large, all firms will choose to become green.

Under the consumers standard, however, the result is more neat. If the number of firms is sufficiently small, all firms choose to be polluting, and the other way around when the number of firms is big enough.

In conclusion, the aim of the paper was to study the effect of the number of firms over the enforcement of the environmental regulation. The analysis has shown how the larger the number of firms, the lower the ability of firms to undermine the enforcement of the regulation. However, this raises the question of firms’ entry into the market. In particular, future research could extend this model to study the possibility of already active firms preventing the entry of new competitors. Indeed, the increased effectiveness of lobbying in a small oligopoly would provide an additional incentive for such a deterrent action.

\textsuperscript{19}One may argue that the reason for this strong result is the assumption that there is a unique fixed set up cost \( F \), divided among all the polluting firms. Appendix 3.B analyses a different scenario in which each polluting firm pays a fixed fee to join the lobby.

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Appendix

3.A Proofs

Proof of Lemma 3.1.

The first step of the proof is to derive the Cournot equilibrium for the entire market.

Given equation (3.14), the quantity produced by firm \( i \) in a Cournot oligopoly with \( n \) firms is equal to:

\[
q_i^n = \frac{a - c_i}{(n + 1)} + \frac{n(\bar{c} - c_i)}{(n + 1)},
\]

(3.A.1)

where \( \bar{c} \) is the average marginal cost.

At this stage of the game the \( n \) firms are divided into two groups: the green ones, and the polluting ones; define \( n_g \) and \( n_p \) the number of firms of each type. It then follows that the average marginal cost is equal to

\[
\bar{c} = \frac{n_p t + n_g c}{n},
\]

(3.A.2)

defining \( \alpha \) as the share of polluting firm into the market, \( \frac{n_p}{n} \), the average cost can be written as:

\[
\bar{c} = \alpha t + (1 - \alpha)c.
\]

(3.A.3)

substituting equation (3.A.3) in equation (3.A.1) and recalling that by equation (3.1)

\[
c_i = \begin{cases} c & \text{if } i = p \\ t & \text{if } i = g \end{cases},
\]

(3.A.4)

equation (3.15) can be derived.

\[
q_p = \frac{a - t + (1 - \alpha)n(c - t)}{1 + n},
\]

\[
q_g = \frac{a - c - \alpha n(c - t)}{1 + n}.
\]

(3.A.5)

In order to derive equation (3.A.10), first notice that by (3.16) it follows that

\[
p = \frac{a + n(t\alpha + (1 - \alpha)c)}{(1 + n)},
\]

(3.A.6)

given the equation of price, equation (3.A.10) follows immediately.
The second step of the proof is to analyse the corner solution that originates if the cost advantage of the polluting firms is too high.

Green firms stay in the market only if they have the incentive to produce a non-negative quantity, i.e. only if $q_g \geq 0$, which implies

$$t \geq c - \frac{a - c}{\alpha n} \equiv t_c.$$  \hspace{1cm} (3.A.7)

Therefore if $t < t_c$, green firms prefer not to produce, then the market will be composed only by the $\alpha n$ polluting firms.

$$q = \frac{a - t}{\alpha n + 1}. \hspace{1cm} (3.A.8)$$

The total quantity produced in this market is equal to:

$$Q = \frac{\alpha n}{\alpha n + 1}(a - t). \hspace{1cm} (3.A.9)$$

and the profits of a single green and polluting firms are equal to:

$$\pi = \left(\frac{a - t}{\alpha n + 1}\right)^2. \hspace{1cm} (3.A.10)$$

**Proof of Proposition 3.1.**

Firstly, given equations (3.6) and (3.20) it follows that:

$$L = (c - t)Q_p. \hspace{1cm} (3.A.11)$$

Therefore the optimal tax rate $t^*$ of equation (3.21) is obtained by substituting equation (3.A.11) into the FOC, of equation (3.9).

The partial derivatives can be derived fairly easily. The partial derivative of $t^*$ with respect to $\alpha$ and $c$ are equal respectively to:

$$\frac{\partial t^*}{\partial \alpha} = \frac{a - c}{2\alpha^2n} + \frac{2n(a - c)}{(2n - 2\alpha n + 2)^2},$$ \hspace{1cm} (3.A.12)

and

$$\frac{\partial t^*}{\partial c} = 1 + 1 + \frac{1}{2an} - \frac{1}{2(n + 1 - \alpha n)},$$ \hspace{1cm} (3.A.13)

and it is immediate to recognise that both these derivatives are always positive in the space of the parameters of this model.

Finally, consider the partial derivative of $t^*$ with respect to the number of firms into the market, $n$:

$$\frac{\partial t^*}{\partial n} = \frac{a - c}{2} \left(\frac{1}{\alpha n^2} - \frac{1 - \alpha}{(1 + n - \alpha n)^2}\right),$$ \hspace{1cm} (3.A.14)

which is also strictly positive for $\alpha < \overline{\alpha}$.

The corner solution follows directly from the constraint $t^* \leq c$, which implies that

$$\alpha \leq \frac{1 + n}{2n} \equiv \overline{\alpha},$$ \hspace{1cm} (3.A.15)
therefore, if \( \alpha > \pi \), the consequence would be that \( t^* > c \) and therefore \( L^* < 0 \).

The final result to be proved is that green firms are always able to remain in the market.

In order to prove this result, recall that by Lemma 3.1, green firms stay into the market as long as \( t \geq \frac{1}{2} \). Given the optimal tax rate this condition implies:

\[
c - \frac{a - c}{2an} \left( \frac{n + 1 - 2\alpha n}{n + 1 - \alpha n} \right) > c - \frac{a - c}{\alpha n} \Rightarrow \frac{n + 1 - 2\alpha n}{n + 1 - \alpha n} < 2, \tag{3.A.16}
\]

which is always true, meaning that, for any level of the optimal tax, the condition \( t > \frac{1}{2} \) is always verified.

Thus, the firms that at the beginning of the game have chosen to be green, are always able to stay into the market in the last stage of the game.

**Proof of Proposition 3.2.**

Given equation (3.21), it follows that

\[
\Pi_p = \frac{(a - c)^2}{4\alpha^2n^2}, \tag{3.A.17}
\]

while

\[
\Pi_g = \frac{(a - c)^2}{4(1 + n - \alpha n)^2}. \tag{3.A.18}
\]

Moreover, given that

\[
L = (a - c)^2 \frac{1 + n(1 - 2\alpha)}{4\alpha n(1 + n - \alpha n)} \tag{3.A.19}
\]

it follows that the equilibrium condition stated in equation (3.13) can be rewritten as:

\[
(a - c)^2 \frac{1 - n(1 - 2\alpha)}{4\alpha n(1 + n - \alpha n)^2} = \frac{F}{\alpha n}, \tag{3.A.20}
\]

and solving for \( \alpha \) gives equation (3.24).

However, one can immediately recognise that such optimal value \( \alpha^* \) exists as long as the \( \Delta \) is non-negative. This implies that \( (a - c)^2 - 4F(n + 1) \geq 0 \), from which it follows

\[
n \leq \frac{(a - c)^2}{4F} - 1 \equiv \pi \tag{3.A.21}
\]

If \( n > \pi \), the profit that a firm obtains by choosing not to innovate is always lower than the profit obtained by the one choosing to innovate, therefore all firms choose to be green.

The partial derivative with respect to the number of firms takes the following form

\[
\frac{\partial \alpha^*}{\partial n} = \frac{(a - c)^2 - 4F}{4Fn^2} - \frac{(a - c)}{4Fn^2} \cdot \frac{(a - c)^2 - 2F(n + 2)}{\sqrt{(a - c)^2 - 4F(n + 1)}}, \tag{3.A.22}
\]

which in the space of parameters such that \( \Delta > 0 \) is always negative.
Proof of Lemma 3.2.

The benchmark case corresponds to the one in which $L = 0$. In this case, the government objective function is equal to $W$:

$$G = Q - D.$$  \hfill (3.A.23)

Substituting equation (3.16) and computing the FOC, it follows that the optimal tax rate in the benchmark case is equal to:

$$t_0 = c + \frac{a - c - 1}{2 + 2(1 - a)n}.$$  \hfill (3.A.24)

The second term of this expression is positive as long as $a - c$ is greater than 1. Therefore, under this assumption the optimal tax rate is above the upper bound $c$ from which it follows $t_0 = c$. The first part of the Lemma is then proved.

The second part of the Lemma follows immediately by substituting $t = c$ in equation (3.16).

$$Q(c) = \frac{n}{n + 1}(a - c) \equiv Q_0,$$  \hfill (3.A.25)

which obviously corresponds to the case in which all firms choose to be green.

Proof of Proposition 3.3.

Firstly, given equations (3.6) and (3.27) it follows that:

$$L = (c - t)Q_p - (Q - Q_0).$$  \hfill (3.A.26)

Given equation (3.A.26), equation (3.28) follows directly by the FOC, equation (3.9).

Given that the term

$$\frac{n + 1}{2an(n + 1 - \alpha \eta)};$$

is always positive, it immediately follows that $t_q^* < t^*$.

The corner solution follows directly from the constraint $t_q^* \leq c$, which implies that

$$\alpha \leq \frac{n + 1}{2n} \cdot \frac{a - c + 1}{a - c} \equiv \alpha_q$$  \hfill (3.A.27)

therefore, if $\alpha > \alpha_q$, the consequence would be that $t^* > c$ and therefore $L^* < 0$.

Moreover, it is immediate to recognise that $\alpha_q > \alpha$, meaning that $t_q^*$ reaches the upper bound $c$ for higher values of $\alpha$ than $t^*$. This complete the proof that $t_q^* < t^*$ for any value of $\alpha$ lower than $\alpha_q$.

Finally, it has to be proved that green firms are always able to remain in the market.

In order to prove this result, recall that by Lemma 3.1, green firms stay into the market as long as $t \geq \bar{t}$. Given the optimal tax rate this condition implies:

$$c - \frac{a - c}{2an} \cdot \frac{n + 1 - 2\alpha}{n + 1 - \alpha} - \frac{n + 1}{2an(n + 1 - \alpha)} > c - \frac{a - c}{\alpha n}.$$  \hfill (3.A.28)

By rearranging this expression, the following condition can be obtained

$$\frac{\alpha n}{1 + n - \alpha n} > -1$$  \hfill (3.A.29)

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which in the parameter space of the model is always verified. Therefore, for any level of the optimal tax, the condition \( t > t^* \) holds.

Finally, the partial derivatives are computed.
The partial derivative of \( t^* \) with respect to \( \alpha \) and \( c \) are equal respectively to:

\[
\frac{\partial t^*_q}{\partial \alpha} = \frac{a - c + 1}{2\alpha^2n} + \frac{2n(a - c - 1)}{(2(1 - \alpha)n + 2)^2},
\]

and

\[
\frac{\partial t^*_q}{\partial c} = 1 + \frac{1}{2\alpha n} - \frac{1}{2(n + 1 - \alpha n)}.
\]

It is immediate to recognise that both these derivatives are always positive in the space of the parameters of this model.

Finally, consider the partial derivative of \( t^* \) with respect to the number of firms into the market, \( n \):

\[
\frac{\partial t^*_q}{\partial n} = \frac{a - c + 1}{2\alpha n^2} - \frac{2(1 - \alpha)(a - c - 1)}{(2(1 - \alpha)n + 2)^2},
\]

which it is also strictly positive for \( \alpha < \pi_q \).

**Proof of Proposition 3.4.**

Given equation (3.28), it follows that

\[
\Pi_p = \frac{(a - c + 1)^2}{4\alpha^2n^2},
\]

while

\[
\Pi_g = \frac{(a - c - 1)^2}{4(1 + n - \alpha n)^2}.
\]

therefore equation (3.13) takes the following form:

\[
\frac{4\alpha^2n^2(a - c) - 2\alpha n(n + 1)(a - c + 1)^2 + (n + 1)^2(a - c + 1)^2}{4\alpha n(n + 1)(1 + n - \alpha n)^2} = \frac{F}{\alpha n}
\]

where equation (3.31) is derived by solving with respect to \( \alpha \).

The non-negativity condition over \( \Delta \) provides the upper bound to the number of firms into the market, \( \pi_q \). This implies that \( (a - c + 1)^2 - 4F(n + 1) \geq 0 \), from which it follows

\[
n \leq \frac{(a - c + 1)^2}{4F} - 1 = \pi_q
\]

If the number of firms present into the market is above this threshold, the profit of a polluting firm is always lower than the profit of a green one, therefore all firms will choose to be green, i.e. \( \alpha^*_q = 0 \).

**Proof of Lemma 3.3.**

The benchmark case corresponds to the one in which \( L = 0 \). This case then corresponds to the case in which the government maximises

\[
G = S - D,
\]

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that is a concave function.

In a Cournot oligopoly with linear demand and linear cost function, consumers’ Surplus is equal

\[ S = \frac{Q^2}{2}. \quad (3.\text{A}.38) \]

Therefore from the FOC, it follows that the optimal tax rate is equal to

\[ t_0 = c + \frac{a - c}{2(n + 1)^2 - \alpha n(2n + 3)} \quad (3.\text{A}.39) \]

For \( \alpha \in [0, 1] \) the denominator is positive, therefore the optimal tax rate is greater than \( c \).

Therefore the desired level of taxation is out of the set of available choices and the government will choose the highest possible tax rate, i.e. \( t_0 = c \).

From here the second part of the Lemma follows immediately. First of all by imposing \( t = c \) in equation (3.16) and substituting in equation (3.\text{A}.38) it follows

\[ S(c) = \frac{n^2}{(n + 1)^2} \frac{(a - c)^2}{2}, \quad (3.\text{A}.40) \]

and it is straightforward to recognise this as the same equation of a Cournot oligopoly with \( n \) equal green firms. It is also immediate to recognise how this is the minimum Surplus possible since \( Q \) is a decreasing functions of \( t \).

**Proof of Proposition 3.5.**

Firstly, given equations (3.6) and (3.33) it follows that:

\[ L = (c - t)Q_p - \frac{Q^2 - Q_0^2}{2}. \quad (3.\text{A}.41) \]

Therefore the optimal tax rate \( t^* \) is obtained by substituting equation (3.\text{A}.41) into the FOC, of equation (3.9).

It is immediate to recognise that the superior constraint is never reached, since \( t > c \) requires \( a < c \).

For the sake of completeness partial derivatives are also provided. The partial derivatives of \( t^* \) with respect to \( \alpha \) \( c \) and \( n \) are respectively:

\[ \frac{\partial t^*}{\partial \alpha} = \frac{a - c}{\alpha^2 n}, \quad (3.\text{A}.42) \]

\[ \frac{\partial t^*}{\partial c} = \frac{1 + \alpha n}{\alpha n}, \quad (3.\text{A}.43) \]

and

\[ \frac{\partial t^*}{\partial n} = \frac{a - c}{\alpha n^2}, \quad (3.\text{A}.44) \]

and it is immediate to recognise that all these derivatives are always positive.

\(^{20}\)In fact the denominator can be written as \( 2n^2(1 - \alpha) + n(4 - 3\alpha) + 2 \), and it is immediate to recognise that for \( \alpha \in [0, 1] \) this expression is always positive.
Proof of Proposition 3.6.

Given equation (3.34), it follows that
\[ \Pi_p = \left( \frac{a-c}{an} \right)^2, \]  
(3.A.45)
while \( \Pi_g = 0 \).

Taking into account equation (3.11), it follows that the net profits of a polluting firm are equal to:
\[ \pi_p = (a-c)^2 \frac{(2n+1)}{2an(n+1)^2} - \frac{F}{an}, \]  
(3.A.46)
while obviously \( \pi_g = 0 \).

It follows that the equilibrium condition stated in equation (3.13) can be rewritten as:
\[ \frac{(a-c)^2(2n+1)}{2an(n+1)^2} = \frac{F}{an}, \]  
(3.A.47)
and it is immediate to verify that there is no \( \alpha^* \) satisfying this condition. In fact, either \( \pi_p > \pi_g \) for every \( \alpha \) or the other way around.\(^{21}\)

Condition \( \pi_p > \pi_g \) holds if
\[ n \leq \frac{(a-c)\left( \sqrt{(a-c)^2 - 2F + a-c} \right)}{2F} - 1 \equiv \pi_s, \]  
(3.A.48)
If it is the case, then it is always convenient to be polluting rather than green, therefore all firms will take this decision and \( \alpha^* = 1 \).

On the other hand, if \( n > \pi \) the inequality holds in the opposite direction, \( \pi_p < \pi_g \), therefore all firms choose the innovate and \( \alpha^* = 0 \).

3.B Fixed cost as enrolment fee

The very neat result in proposition 3.5 may also depend on the assumption that there is a single fixed cost \( F \) to set up the Lobby, which is divided among all the firms that choose to enter.

The interpretation of this assumption would be that, in a way, coordination costs within the lobby are absent, and therefore the higher the number of firms in the lobby, the lower the setup cost per capita.

However, one may argue that, in general, increasing the number of firms within a lobby has two effects. On the one hand, it would reduce the setup cost per capita, but, on the other hand, it would increase the coordination costs within the lobby since there are more members that have to agree on the strategy to be implemented. These two effects may have different magnitudes, and, in principle, there is no reason to believe that an increase in the number of firms in the lobby is always positive in terms of costs for the members.

\(^{21}\)Actually for \( n = \pi \) it follows that \( \pi_p = \pi_g \) for every \( \alpha \). In this case assume that firms prefer to be polluting.

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This section analyses the case in which each firm has to pay a fixed enrolment cost \( f \in (0, a - c) \), which would correspond to the case where the two opposite effects have the same magnitude in terms of costs for each participant.

If this is the case, the equilibrium condition stated in equation (3.13) can be rewritten as:

\[
\frac{(a - c)^2(2n + 1)}{2an(n + 1)^2} = f.
\]

This equation has a unique solution which is equal to

\[
\alpha^* = \frac{(a - c)^2(2n + 1)}{2fn(n + 1)^2}.
\]

The first thing to notice is that \( \alpha^* \) is a decreasing function of \( n \) and despite being always positive

\[
\lim_{n \to \infty} \alpha^* = 0.
\]

Therefore this result is coherent with the main results of the paper.

Moreover, it may be the case that \( \alpha^* > 1 \). This happens if

\[
\frac{n(n + 1)^2}{(2n + 1)} < \frac{(a - c)^2}{2f}.
\]

Therefore if the active number of firms into the market is small enough, they will all choose to join the lobby instead of innovating the production process.

It is not possible to obtain a closed form for the number of however it can be proved that such \( n \) is unique.

**Proposition 3.7.** \( \exists! \ n^* > 0 \ such \ that \ equation \ (3.4) \ holds \ \forall \ n < n^* \).

**Proof.** Starting from (3.4), let

\[
\begin{cases}
  g_1 = 2fn(n + 1)^2 \\
g_2 = (a - c)^2(2n + 1)
\end{cases}
\]

where both \( g_1 \) and \( g_2 \) being monotonically strictly increasing in \( n \) and

\[
\lim_{n \to \infty} \frac{g_1}{g_2} = +\infty
\]

it follows that \( \exists! \ n^* \) intersection point if and only if

\[
\lim_{n \to 0^+} g_2 > \lim_{n \to 0^+} g_1 \iff (a - c)^2 > 0
\]

which is always true.

**Proposition 3.7** states that for \( n \) positive, there is only one intersection with the horizontal line 1. However it might be the case that \( n^* \) lays in the range \([0, 2)\) which is assumed to be out of the valid range of this model. The following corollary restricts the proposition to the allowed set of values.

**Corollary 3.1.** If \( \frac{(a - c)^2}{f} > \frac{36}{5} \), then \( n^* > 2 \)
Proof. \( \exists \ n^* > 2 \) intersection point if and only if
\[
\lim_{n \to 2^+} g_2 > \lim_{n \to 2^+} g_1 \iff 5(a - c)^2 > 36f
\]
Recalling that, by assumption \( a > c + f \) and \( a - c > 1 \), the following condition holds
\[
\frac{(a - c)^2}{f} > \frac{36}{5}, \tag{3.B.7}
\]
which completes the proof. \( \blacksquare \)

Figures 3.B.1 presents three different scenarios for \( \alpha^*(n) \), highlighting for each of them the cut off \( n^* \), above which the curve is dashed since a corner solution is present.

Figure 3.B.1: Different scenarios for \( n^* \)
The curve in blue has a parameter set \( (a = 10, c = 1, f = 2) \), the red curve has a parameter set \( (a = 50, c = 7, f = 15) \) and finally the black curve corresponds to the parameter set \( (a = 100, c = 15, f = 40) \)
Bibliography


