The Strategic Foundations and Evolutionary Dynamics of Poverty Traps

by

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Dissertation submitted to the Department of Economics

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

in the subject of

Economics

University of Siena, Italy.

[May] [2010]

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ACKNOWLEDGMENTS

To Elvio Accinelli and Lionello Punzo since they showed me a taste for economic theory and the theory of games as a tool to understand the strategic foundations of economic growth and economic development, which have motivated my life as economist scientist.

To Costas Azariadis, Samuel Bowles and Herbert Gintis for their helpful feedback to improve my research.

To the Faculty members of the Department of Economics at The University of Siena.

Como fuente de inspiración para el ejemplar amor de

Yolanda Carrera Rdz y Humberto Perez Galvan$^{(+)}$

Abstract

This thesis shows that individual behavior driven by imitation can lead an economy either into a low level equilibrium - a poverty trap - or a high level equilibrium, both of which are evolutionarily stable. To overcome the poverty trap there exists a threshold level of high-profile economic agents. The threshold level depends on education costs, skill premia (or bonuses) and income taxes.

We consider three models: a baseline model, and two models in which we apply two different imitation rules. The baseline model studies an evolutionary game with two asymmetric populations where players from each population are paired with members of the other population. We consider imitation as the facilitator of evolutionary dynamics, resulting in evolutionarily stable strategies. We present two imitation models. In the first model dissatisfaction drives imitation. In the second model agents imitate the successful. In the first model we use a simple reviewing rule, while in the second model we use a proportional imitation rule where switching depends on agents comparing their payoffs to others' payoffs. We show that such imitation is approximated by a replicator dynamic system following which we apply the well-known relationships between stability of stationary states, evolutionarily stable strategies (ESS), and Nash equilibria. We characterize the evolutionarily stable strategies of our two asymmetric populations and we show that a mixed strategy profile distribution of inhabitants can be an ESS if it is the strict Nash equilibrium. We offer one clear conclusion: whom an agent imitates is more important than how an agent imitates. The rest of the thesis applies evolutionary game theory on three models concerning the notions of economic agents and poverty traps.

First, a coordination game interpreted as a game between economic agents: namely a leader and a follower. The leader must hire the follower and each player can be either a high or a low profile economic agent. The follower can decide to be of the high profile type by incurring some training cost. Both players must also pay some income tax. Play that is stuck in the low-level or inefficient equilibrium is interpreted as a poverty trap. Self-confirming equilibria are analyzed before looking at the replicator dynamics driven by imitation. We show that the economy can be located in a low-level or high-level equilibrium, both of which are evolutionarily stable strategies against the field. Furthermore, we show that to overcome the poverty trap there exists a threshold number of high-profile economic agents which depends mainly on training costs (or education costs), skill premia or bonuses to skillful agents, and income taxes.

Second, an economy with two types of firms (innovative and non-innovative) and two types of workers (high-skilled and low-skilled) where the workers' decisions are driven by imitative behavior, while firms' decisions depend on the number of high-skilled workers. We show that such an economy's evolution depends on the initial distribution of the firms. The multiple equilibria

of this model can be characterized either by a continuum of high level steady states or only one low level steady state for which there exists a threshold level to be overcome to attain the higher level equilibria. We show that in each high level equilibrium there coexist a percentage of innovative firms with a percentage of non-innovative firms, and a set of high-skilled workers coexisting with a set of low-skilled workers. But if the initial percentage of innovative firms is lower than the threshold value, then the economy devolves to a low level equilibrium wholly composed of non-innovative firms and low-skilled workers.

Third, we consider the decisions of both firms and workers are driven by imitating successful strategies. Firms and workers should decide whether to be high-profile or low-profile agents by following an imitative behavioral rule given the current state of the economy, characterized by the distribution of strategies. We apply evolutionary game theory to find the system of replicator dynamics, and characterize the low-level and high-level equilibria as evolutionarily stable strategies. We find that a poverty trap is an ESS and that a threshold level of high-skilled workers and innovative firms exists to overcome it. Moreover, when the current state of the economy is in the basin of attraction of the poverty trap, players should "play against the field" if they want "to change the *status quo*". We show that the threshold level can be lowered if there is an appropiate policy using income taxes, education costs and skill premia.

Keywords: Evolutionary games, imitation rule, poverty traps, replicator dynamics, signaling games, strategic complementarities, threshold value.

JEL classification: C70, C72, C73, D83, I30, O10, O12, O40.

This thesis makes partial reference to the following papers:

1. Accinelli E., G. Brida and Carrera Edgar JS (2009), "Imitative Behavior in a Two Population Model" *Annals of the International Society of Dynamic Games* XI.
2. Carrera Edgar JS (2009), "The Strategic Foundations and Evolutionary Dynamics of Poverty Traps" Status: *submitted.*
3. Accinelli E., S. London, Carrera Edgar JS, and L. Punzo (2009), "Dynamic Complementarity, Efficiency and Nash Equilibria in the Populations of Firms and Workers" *Dynamics and Games in Science*, in honour of Mauricio Peixoto and David Rand" to be published by Springer-Verlag.
4. Carrera Edgar JS (2009), "The Replicator Dynamics for Firms and Workers". Status: *submitted.*

CONTENTS

CHAPTER 1

INTRODUCTION

Economists have used the idea of economic agents to conceive of agents as decisionmakers endowed with preferences, who form expectations, and face particular constraints. The idea of the economic agent incorporates persons, firms, and other entities such as nonprofit organizations and governments. The essential characteristic of an economic agent is not its physical form but rather its status as a decisionmaker. An agent as a decisionmaker carries within it a straightforward answer to the question of how agents interact. Agents interact through their chosen actions. An action chosen by one agent may affect the actions of other agents through three channels: constraints, expectations, and preferences.

In this thesis we argue that, it is the strategic behavior of economic agents which can lead an economy to either a high or low level equilibrium. Where a low-level equilibrium is a situation where economic performance is low on most relevant counts (GDP per capita, Human Capital, R&D, Innovation, etc.). A poverty trap is a low-level equilibrium defined as any self-reinforcing mechanism which causes poverty to persist (see Azariadis and Stachurski, 2005), and in which economic agents suffer from persistent underdevelopment.

To continue the ideas and to clarify the notion of poverty trap, Matsuyama (2008) considers that the state of the economy in period t can be represented by a single variable, x_t, where a higher x means that the economy is more developed and the economic agents are characterized by high-profiles (e.g., innovative firms and human capital), and that the equilibrium path follows a deterministic one-dimensional difference equation, $x_{t+1} = F(x_t)$. Once the initial condition, x_0, is given, this law of motion can be applied iteratively to obtain the entire trajectory of the economy.

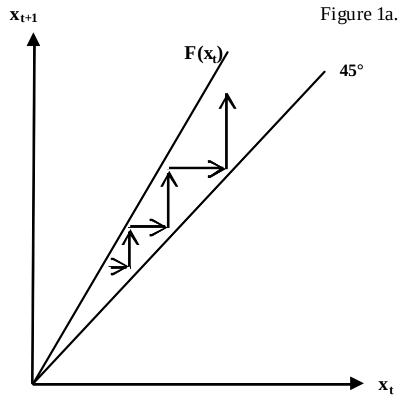

Figure 1a.

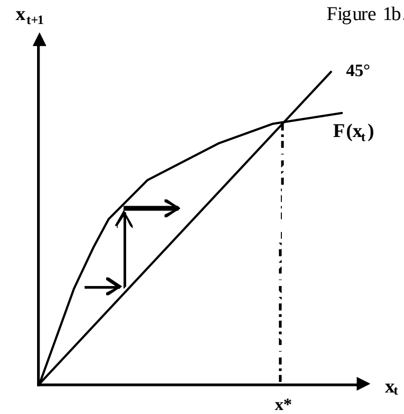

Figure 1b.

In Figure 1a, $F(x)$ stays above the $45°$ line everywhere, hence the economy grows forever (as in endogenous growth models). In Figure 1b, for any x_0, the economy converges to x^* (as in the Solow growth model). In either case, there is no poverty trap, since the long run performance of the economy is independent of the initial condition, i.e. no matter how low the number of high-profile economic agents is and how underdeveloped the economy is initially, the economy can converge to the high level equilibrium.

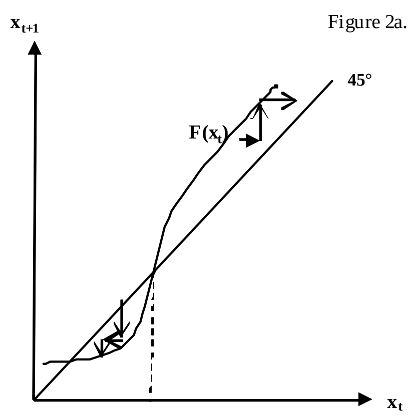

Figure 2a.

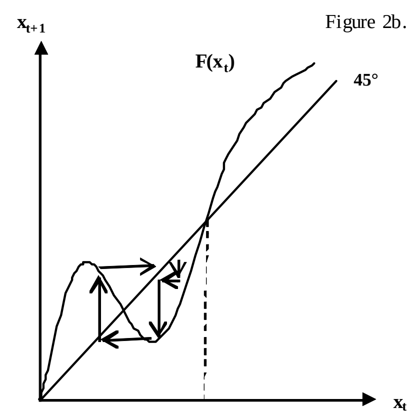

Figure 2b.

In Figures 2a and 2b, on the other hand, the long run performance depends on the initial conditions. When the economy starts above x_c, it will stay above x_c and may either grow forever or reach a higher stationary state. However, if it starts below x_c, it will be trapped forever below x_c. In this sense both figures exhibit a poverty trap in its strong form. In Figure 2a, the economy caught in the trap will converge to the low-level stationary state. In Figure 2b, it will fluctuate below x_c. In both cases, the economy will remain poor only because it began poor. Thus, persistent poverty becomes its own cause. It is this self-perpetuating nature that sets “the poverty trap” apart from “the limit to growth”. The essential message of poverty traps is that poverty tends to persist, and that it is difficult, but not necessarily impossible for economic agents to escape it.

Steven Durlauf has enriched the modeling of poverty traps by adding a spatial dimension: the idea that an agent’s socioeconomic outcomes depend upon the composition of the various groups of which she is a member over the course of her life. That is, the decision for an agent to acquire an education strongly depends on the prior existence of other educated members in a group. This interdependence of behavior induces "neighborhood effects", which generate different types of groups that have different steady states (with/without educated members). This interdependence may be intertemporal, i.e. it affects future social interactions.[1] It is this concept of neighborhood effects that for Durlauf allows for the understanding of why poverty traps exist and persist. Hence, poverty traps are defined as a community of economic agents that composed initially of poor members with low-profiles that remain in the low-level equilibrium over generations (Durlauf, 2003).

Poverty traps can therefore arise across geographical location, and within dispersed collections of agents affiliated by cultural facts and institutions. Group outcomes are then summed up progressively from the level of the agent.

In this vein, Samuel Bowles has built the seminal concept of ‘institutional poverty traps’, which emphasizes that coordination failures and poverty traps are induced by the presence of specific institutions. Bowles defines institutions as conventions in which members of a population typically act in ways that maximize payoffs given the actions taken by others such that the process supports continued adherence to the conventions (Bowles, 2006:118).

Polterovich (2008) pointed out that the formation of institutional traps due to the economic agents with low-profiles who conform to specific strategies is one of the main obstacles to improve the economic performance. He defined an institutional trap as a stable but yet inefficient equilibrium in a system where agents choose a norm of behavior (an institution) among several options. It is usually implied that multiple equilibria prevail in the system, and that an institutional trap is Pareto dominated. As with any other norm, an institutional trap’s stability means that a system absorbing a small external shock will remain in the institutional trap, having perhaps slightly changed its parameters, and will return to the inefficient equilibrium state once the source of destabilizing pressure is removed. Individuals or a small groups of people lose if they deviate from an institutional trap.

[1] In Durlauf’s (1996) model they create incentives for wealthier families to segregate themselves into economically homogeneous neighborhoods. The dynamics of these combinations explain persistent income inequality.

However, the simultaneous adoption by all agents of an alternative norm may be Pareto improving. Thus the lack of coordination is the main cause of the institutional trap's stability.[2]

Hence, the types of economic agents and their neighborhoods can explain whether a low-level or high-level equilibrium obtains for an economy. We study the idea of neighborhood effects by considering that economic agents interact with an underlying motivation to imitate others. Since, an economic agent imitates their neighbors given a particular state of the economy comprising different types of agents (high- and low-profile), firms and workers could imitate high-profiles engaging in R&D and human capital activities, because imitation is profitable and because there are strategic complementarities between the types of agents. Barret and Swallow (2006) show that when there are multiple strategies in dynamic equilibrium, it implies that poverty traps may arise. Since choosing a strategy means that one implicitly selects the equilibrium towards which one naturally moves over time, given the state of the system, such a strategy is a steady state of a dynamical system.

Therefore, we intend to study the dynamic complementarities and the evolution of an economy composed of different types of economic agents interacting within the constraints of such complementarities and the economic states that obtain.

We have chosen an evolutionary game theoretical model to represent the above phenomena. Let us continue, in order to explain the advantage of the chosen approach.

1.1 Learning by Imitation

Blackmore (1999) pointed out not only how effective a form of learning imitation is, but also the sophistication required in order to be able to imitate. To explain why agents imitate we should think of it as a kind of rational behavior. Rational imitation can be explained as follows. An agent, A, can be said to imitate the behavior of another agent, B, when observation of the behavior of B affects A in such a way that A's subsequent behavior becomes more similar to the observed behavior of B. An agent can be said to act rationally when the agent, faced with a choice between different courses of actions, chooses the course which is the best with respect to her interests, her beliefs about possible action opportunities, and the effects of these potential action opportunities (on the notion of imitation see Sanditov, 2006).

Durlauf (2001) noted that the imitative behavior may be due to:

1. psychological factors, an intrinsic desire to behave like certain others;

[2] An important class of poverty traps is due to coordination failures. Many such models are discussed in Cooper and John (1988) and Hoff (2001).

2. interdependence in the constraints that agents face, so that the costs of a given behavior depend on whether others due the same, or;
3. interdependence in information transmission, so that the behavior of others alters the information about the effects of such behaviors available to a given agent.

Each of these types of imitative behavior implies that an agent, when assessing alternative behavioral choices, will find a given behavior relatively desirable if others have behaved or are behaving in the same way.

In this thesis we deal with imitation that results in agents performing a spectrum of tasks "as others do". We assume that occasionally each agent in a finite population gets an impulse to revise her (pure) strategy choice. If these impulses arrive according to i.i.d. Poisson processes, then the probability of simultaneous impulses is zero, and the aggregate process is also a Poison process. Moreover, the intensity of the aggregate process is just the sum of the intensities of the individual processes. If the population is large, then one may approximate the aggregate process by deterministic flows given by the expected values. There are two basic elements common to these models. The first is a specification of the time rate at which agents in the population review their strategy choice. This rate may depend on the current performance of the agent's pure strategy and of other aspects of the current population state. The second element is a specification of the choice probabilities of a reviewing agent. The probability i-strategist will switch to some pure strategy j may depend on the current performance of these strategies and other aspects of the current population state.

Björnerstedt and Weibull (1996) studied a number of such models, where those agents who revise may imitate other agents in their player-population, and show that a number of payoff-positive selection dynamics, including the replicator dynamics, may be so derived. In particular, if an agent's revision rate is linearly decreasing in the expected payoff to her strategy (or to the agent's latest payoff realization), then the intensity of each pure strategy's Poisson process will be proportional to its population share, and the proportionality factor will be linearly decreasing in its expected payoff. If every revising agent selects her future strategy by imitating a randomly drawn agent in their own player population, then the resulting flow approximation is again the replicator dynamics.

Theoretical advances to understand imitation have been developed by Vega-Redondo (1997) and Schlag (1998, 1999).[3] These two previously cited

[3] Schlag (1998) analyses what imitation rules an agent should choose, when she occasionally has the opportunity to imitate another agent in the same set of types of players - which I refer to as a player-position - but is otherwise constrained by severe restrictions on information and memory. Schlag finds that if an agent wants a learning rule that leads to non-decreasing expected payoffs over time in all stationary environments, then the agent should (i) always imitate (not experiment) when changing strategy, (ii) never imitate an agent whose payoff realization was worse than her own, and (iii) imitate agents whose payoff realizations are better than her own with a probability that is proportional to this payoff difference.

works, point out how an agent who faces repeated choice problems will imitate others who obtained high payoffs. Despite their basic similarity, this two models differ, along at least two dimensions: 1) the informational structure ("whom agents imitate") and, 2) the behavioral rule ("how agents imitate"). While agents in Vega-Redondo's model observe their immediate competitors, in Schlag's model agents observe others who are just like them, but play in different groups against different opponents. Additionally, agents in Vega-Redondo's model copy the most successful action of the previous period whenever they can. In contrast, Schlag's agents only imitate in a probabilistic fashion and the probability with which they imitate is proportional to the observed difference in payoffs between their own and the most successful action. Apesteguia et al., (2007) shown that the different results between the two models occur because of the different informational assumptions rather that the different adjustment rules.

So, whom an agent imitates is more important than how an agent imitates, and in this thesis we confirm this affirmation.

Now, we proceed to explain the notion that we have in mind about high-profile economic agents and the importance of the complementarities between those agents.

1.2 High Profile Economic Agents

The notion of strategic complementarities is widely studied and well understood and thus the complementarity between R&D (innovative firms) and human capital accumulation (high-skilled workers) is widely accepted as an engine of economic growth (see the seminal paper by Lucas, 1988 or Mankiw et al., 1992; Stockey, 1991). In this thesis, therefore, we label innovative firms and high-skilled workers as "high-profile economic agents" and they are the engine driving towards a high level equilibrium.

Nelson and Phelps (1966) and Schultz (1975) pointed out the major role of education is to adapt to, and to generate, new technologies; that is, to adapt to technological changes generated by innovative firms. Nelson and Phelps (1966), modeling the idea, assert first that the major role of education is to increase the agent's capacity to innovate, and second to adapt to new technologies, thereby accelerating technological diffusion through the economy. Therefore, high-profile economic agents lead the economy to a high-level equilibrium.

Redding (1996) formalizes this idea using an R&D-based growth model developed by Aghion and Howitt (1992). He shows strategic complementarities between investment in education and firms investment in R&D, and then demonstrates the development trap when both types of investment are inactive. More recent studies develop different models to prove that high-skilled labor and high-technological firms are complements in order to obtain a high-level equilibrium (particularly see, Acemoglu, 1997; 1998). Acemoglu (1997) considers the same type of interdependence under heterogeneous individuals' human capital. Acemoglu (1998) focuses on skill-biased technological progress and inequality growth between and within groups of skilled and unskilled workers. Focusing on common situations in which workers accumulate general skills to prepare for the obsolescence of their technology (firm-)specific skills due to technological progress.

In fact, it is generally thought that new technologies reduce the demand for low-skilled workers and increase the demand for high-skilled workers, since high-skilled workers adapt more easily to technological change.[4] This is the well-known notion of "Skill-Biased Technical Change" which implies a shift in the production technology that favors high-skilled over low-skilled labor by increasing its relative productivity and, therefore, its relative demand (see Acemoglu, 2002; Aghion, 2006; Hornstein et al., 2005).

Then, a number of recent contributors have emphasized the role of skill resources as a crucial constraint on the selection of the technological profile to be implemented in underdeveloped economy. Greenwood and Yorukoglu (1997), for instance, consider that the adoption of investment specific technical change requires specific human capital in addition to physical capital, and an increase in skills labor facilitates the adoption of new technologies. Hendricks (2000) develops a model of growth through technology adoption focusing on the complementariness between technologies and skills. The workers' skills and the technological profile of firms are complementary because the level of available skills limits the profile of technologies firms can use, while the technological profile determines the rate of learning. Benhabib and Spiegel (1994), by focusing on the role of human capital in economic development, suggest that the specific role of the human is to facilitate the adoption of technology from abroad and to create domestic technology. This evidence reinforces the importance of the matching among the skills and the technological profile. In this sense, Lavezzi (2006) focused on the dynamics of human capital accumulation (by means of a Markov chain) where human capital accumulation and technology adoption are interrelated processes. According to him, matching is fundamental to isolate one of the most important aspects of the acquisition of human capital and technology. For workers the crucial issue is the type of firms they interact with, while for firms it is the type of workers they hire. In the high-skill equilibrium, for example, workers expect firms to invest in technology and then invest in human capital. Given these workers' expectations, firms find it optimal to invest, and therefore expectations are fulfilled in equilibrium.

Some empirical research for Latin American countries (LAC) is due to Cimoli et al. (2009), who pointed out that the accumulation of human capacities, technological capabilities and the specialization of the production structure shape the response and the way to overcome low level equilibria in LAC. For instance, the Latin American evidence shows a mismatch between the complementarities of innovation and human capital. The case of

[4] This assumption is akin to the Nelson and Phelps (1966) argument that greater skills allow for faster adoption of technology.

the so-called *maquiladoras* in the north of Mexico and low-skilled workers (see Kopinak, 1995),[5] where Mexican labor is inexpensive and, courtesy of NAFTA (the North American Free Trade Agreement), taxes and custom fees are almost nonexistent, which benefit the profits of corporations. 'Maquiladorized' industry paid lower wages, was non-union in orientation, classified most workers as low-skilled, and is characterized by a high proportion of women workers. An article in *Ward's Automotive Reports* (1997) indicates that the technology mix in Mexican automotive assembly plants is different from that in U.S. assembly plants to take advantage of lower labor costs in Mexico. These lower labor costs reflect the lower productivity of Mexican workers, which itself reflects (presumably) lower levels of human capital. Hence, the type of automotive plant chosen depends, to an important extent, on the skill composition of the workforce. Presumably the development or adoption of many technologies reflects the supply of factors complementary to the technology. After all, there is no point in developing a capital good that requires skill levels that exceed those of your workers.

Santiago-Rodriguez and Alcorta (2006) investigate the role of human resource management and development practices underpinning firms' performance in innovation in Mexico. The authors critically addressed some of the main challenges and attributes one may need to take into account to design and eventually implement a study about the nature of links between human resource management practices and firms' performance in innovation.

In a similar vein, Maloney and Perry (2005) studied the LAC and the high tech miracles have followed very different recipes of R&D, FDI, licensing, and education. They show that Latin America has followed a recipe that has relied little on R&D or licensing and too heavily on FDI. This is a perhaps worrying finding given the low rates of technological transfer with FDI, and it appears especially worrying given the generally passive approach to taking advantage of the technological benefits of FDI. In Mexico, for example, despite 30 years of the presence of IBM and HP in Guadalajara, there is little evidence of a knowledge cluster in computer technologies, at least as captured by the patenting data (see Maloney and Perry, 2005).

On the other side, Argentina and Uruguay are good examples of countries that have accumulated human capital (people with high-levels of education), but with low-levels of advanced technology. Ros (2000) pointed out that Latin American countries are paradigmatic cases of the need for further conditions of a sustained growth process. Most of them, even with very high initial levels of formal education, did not grow as fast as other countries that started with similar initial levels of human capital.

Further interesting research was conducted by Vonortas (2002) who empirically assessed science, technology, and innovation policy initiatives in Latin American countries. He found that R&D expenditures remain low

[5] *Maquiladoras* are export assembly plants in northern Mexico, producing parts and products for the United States.

with international standards, ranging from approximately 1% of GDP for Brazil, to 0.75% for Chile, 0.5% for Mexico, and 0.3% for Argentina. Moreover, their distribution was biased towards governments and universities and against the private sector. Inducing the private sector to innovate, and particularly the vast majority of small and medium sized enterprises that dominate economic activity, has proven a long and arduous task. Vonortas offers the examples from two of the largest and most advanced countries in the region (Brazil and Mexico), a medium-size, high-growth, relatively advanced country (Chile), a smaller-size developing country with significant pockets of economic activity (Panama), and three smaller-size, developing countries (Honduras, Jamaica, Nicaragua).

In any case, in this thesis we do not want to enter into the debate surrounding, for example, how to conduct R&D, FDI and innovation policy initiatives. The purpose of this thesis is going to be clarified in the next sections, but as we mentioned our aim is to study the dynamic complementarities and the evolution of an economy composed of different types of economic agents interacting within the constraints of such complementarities and the economic states that obtain.

1.3 Strategic Behavior of Economic Agents

We argue that the empirical evidence about the mismatch between the complementarities of innovation and human capital can be explained by the strategic behavior of firms and workers, both of which must decide whether to invest or not to invest in R&D and human capital given the current state of the economy. The current state of the economy means the profile or distribution of different types of economic agents, i.e. the current distribution of high and low profile economic agents, which represents the profiles of firms and workers.

The strategic behavior can be explained as follows:

- Assume that potential workers imitate their neighbors, that is they decide to be high or low profile economic agents. Specifically, they decide whether to be a high-skilled worker (e.g, more educated, more able, more experienced) going to a training school or to be a low-skilled worker without incurring the costs of training. Such decisions are rational in the sense that they imitate the best performed strategy given the current state of the economy. On the other hand, firms' decisions depend on the composition of labor in the economy. That is, if the number of high-skilled workers is large enough then a firm decides to be an innovative firm and invests in R&D, otherwise a firm decides to be non-innovative and does not invest in R&D.
- Assume that you are a potential worker. First, you need to decide whether to study (say a degree in mathematics or economics or chem-

istry) or not and then to enter the labor market and to supply your labor. If you decide to study, then you will be a high-skilled worker whereas if you decide not to study you will be a low-skilled worker. The latter option occurs currently in most developing countries where many young people seemingly prefer not to spend more time studying in order to enter the labor market immediately and to offer whatever services they can in order to start receiving a wage, since they observe that a mathematician or chemist or economist is likely to be unemployed. If your decision is taken according to an imitation rule, that is, if you can observe that your neighbors are earning more when they enter the labor market without specializing, then you will imitate them and you will decide not to study. But the converse will happen if you observe that your neighbors are specializing and then are supplying high-skilled labor which gives the maximum expected payoff. In any case, the decisions of potential workers whether to become qualified or not (training or not training) depends on the probability of being hired by innovative and non-innovative firms, that is, it depends on the complementarity between R&D and human capital. At the same time, firms decide to invest in R&D departments just when it is profitable and they base this decision on having a certain number of high-skilled labor to harness new technologies. But if the local economy is highly composed by low-skilled labor, then firms decide not to invest in R&D departments and they may import the necessary technology to produce and to compete.

Hence, we aim to analyze what is likely to occur in developing countries where the population of firms and workers are characterized by a mismatch of high and low profile economic agents, i.e., a mismatch of innovative and non-innovative firms with high-skilled and low-skilled workers.

1.4 Outline

The thesis is divided in two parts: Part I, **Evolutionary Game Theory**, is composed of two chapters: **Chapter 2** summarizes the main notions of evolutionary game theory such as: population games, evolutionarily stable strategies (ESS), the ESS against the field, the replicator dynamics, and the relationships between evolutionarily stable strategies and the steady states of the replicator dynamic system. Moreover, we present the notion of a behavioral rule as a crucial concept of the individuals' behavior.

Chapter 3 studies an evolutionary game of two asymmetric populations where in each round a player of population 1 is paired with a member of population 2. We present two imitation models. In the first model dissatisfaction drives imitation. In the second model agents imitate the successful. In the first model we use a simple reviewing rule, while in the second model we use a proportional imitation rule where switching depends on agents comparing their payoffs to others' payoffs. We show that such imitative behavior can be approximated by a replicator dynamic system. We characterize the evolutionarily stable strategies for a two asymmetric populations normal form game and we show that a mixed strategy is evolutionarily stable if and only if it is a strict Nash equilibrium. In addition, we characterize the evolutionarily stable strategies of our model. Moreover, we extended the model to the application of a specific behavioral rule where a reviewing strategist, i, who decides to change her current strategy must consider the probability to imitate a strategy that performs better than her current strategy and the probability of meeting the agent who uses such a strategy. From this investigation, we offer one clear conclusion: whom an agent imitates is more important than how an agent imitates.

Part II, **Evolutionary Dynamics of Poverty Traps**, comprises two chapters: **Chapter 4** develops an evolutionary coordination game of signaling by economic agents such that the agents' rational behavior may result in a poverty trap. We argue that poverty traps exist and occur because of strategic complementarities between profiles of agents (for example: low-skilled workers, no innovative firms, scarcity of human capital and R&D). First, we introduce a coordination game between "leaders" and "followers" with different profiles. Second, we study the game as an evolutionary game of the complementarity between the profiles of economic agents. We find the self-confirming equilibria and the ESSs against the field. We conclude that the possibility of either high-level or low-level equilibria implies that economic agents acting under identical settings may experience either an adequate living standard or deprivation (growth or crisis), and these depends only on the initial conditions. Then, if the current state of the economy is composed mainly of low-profile economic agents, the economy converges to the low-level equilibrium since it is rational for every player to imitate those strategists with low-profiles. But if the state of the economy changes and there is a sufficiently high number of high-profile agents, then players imitate those agents with high-profiles, and the economy converges to the high-level equilibrium.

Chapter 5 considers a model where workers' decisions are driven by imitative behavior and firms' decisions depend on the number of high-skilled workers. We analyze the dynamic complementarities between innovative firms and high-skilled workers. We show that when firms invest in R&D to become an innovative firm, they are successful only in the presence of sufficiently high number of high-skilled workers (as in Redding, 1996). At the same time workers are encouraged to increase their skills when a large number of firms make investments in high-technology. On the contrary, firms that do not invest in R&D, do not look for high-skilled workers, and therefore make the accumulation of skills unprofitable. We show that there exists a threshold number of innovative firms above which it becomes advantageous to accumulate human capital or high-skilled workers. This is the mechanism that allows an economy to move beyond the poverty trap. Then, we show that if, in a given economy, the percentage of innovative firms is under a certain threshold value then the economy evolves to a poverty trap where the number of high-skilled workers decreases to zero, and thus it becomes better for firms not to invest in R&D. But we also show that if the initial percentage of innovative firms is higher than the threshold value, then, where workers are following an imitation rule, the economy will evolve to a high level equilibrium. In fact, the high level equilibrium is a steady state of a dynamical system characterized by the fact that there coexist a percentage of non-innovative firms with a percentage of innovative firms and a percentage of high-skilled workers and a percentage of low-skilled workers. This result shows the real experience of many developing countries in which there is a mismatch between investment in R&D and Human Capital accumulation, which are the engine of the sustained economic growth.

Moreover, the low level (dynamic) equilibrium (the poverty trap) correspond with a Pareto dominated Nash equilibrium of a two population normal form game while not one of the possible high level (dynamic) equilibrium correspond to the Pareto dominant equilibrium of this game.

Finally, the **Section 5.7** extends the above model. But now, we study the imitative behavior of both agents, firms and workers. Therefore, we present an imitation game where the nature of interactions among agents creates a potential for multiple equilibria. We find the replicator dynamic system and the ESSs. Moreover, we show how an economy may move beyond the poverty trap by 'playing against the field'. That is, when the economy is already in the basin of attraction of the low-level equilibrium, then the agents play against such a state of the economy by imitating the high-profile economic agents. An economy in the basin of attraction of a poverty trap can be improved by exogenous changes or by external interventions. We study the replicator dynamic with payoff taxation as the way in which a policymaker can make intervention in the economic for encourage the firms and workers to be of high-profiles, and then to overcome the poverty trap. To the best of our knowledge, this paper constitutes the first attempt to apply evolutionary game theory to poverty traps.

We conclude that the imitative behavior of the economic agents can explain the evolution of the economy toward the high or low equilibria. Then, if being a high-profile economic agent is the successful strategy because the current state of the economy encourage you to rise your profiles, then imitation facilitates the choice of education (or human capital accumulation), and firms in turn decide to invest in R&D. The crucial parameters that encourage agents to become high-profile agents are education costs (training costs), skill premia and income taxes as a policy to incentive the investment in R&D.

Part I

Evolutionary Game Theory

CHAPTER 2

EVOLUTIONARY GAMES

Noncooperative game theory has become a standard tool in modelling conflict between rational agents, where the Nash equilibrium is the cornerstone in predicting the outcome of the game. In a Nash equilibrium each player's strategy maximizes her payoff given the strategies played by the other players. No player, therefore, has an incentive to deviate from the Nash outcome since it is the best situation for each player and for the group itself. But in many situations it appears that the Nash equilibrium is not unique, though it is, certainly, very desirable to have the knowledge of uniqueness of an equilibrium, a priori, before actually computing and searching for all equilibria (see Accinelli and Carrera, 2009). Since there may be multiple equilibria we may have problems of equilibrium selection, notwithstanding which we may not now the dynamics by which specific equilibria are reached because the Nash equilibrium concept is essentially static.

Since we face the question of what to do when there are multiple equilibria, with particular attention devoted to characterizing properties that might help distinguish equilibria worthy of our attention from less interesting equilibria. The possibility that different refinements will choose different equilibria is only one of the difficulties facing the refinements program.

However, the constant in conventional game theory, including equilibrium refinements and the equilibrium selection literature, is the belief that players are rational, and that this rationality is common knowledge. The common knowledge of rationality is often informally regarded as a necessary condition for there to be any hope that equilibrium will appear. Roughly speaking the problems upon which an equilibrium refinement is built are often introduced with an appeal to the possibility that players might make mistakes, or that one can only understand "complete rationality as a limiting case of incomplete rationality (Selten, 1975:35)." Thus, the resulting problems typically affect the rules of the game (as in Selten's trembling hand perfection), the preferences of the players and the beliefs of the players. The common practice is to ask a player to stubbornly persists in the belief that her opponent is rational, even after observing a tremble that would provide good evidence to the contrary if trembles really represented departures from rationality.

A seminal refinement of the Nash equilibrium concept is the notion of an Evolutionarily Stable Strategy (ESS). An ESS is stable against mutants and is asymptotically stable with respect to the replicator dynamics. This latter result shows that rational behavior is not necessary to obtain such a sophisticated equilibrium. A population of players that inherited the behavior of their parents is able to reach an ESS when the offspring is determined by the fitness of the players.

This brings us to the question of what can be learned from evolutionary game theory in studying the evolution of populations and the individual behavior of its members. Evolutionary games consider: i) a selection process that favors some varieties over others, and ii) a process that creates this variety, called the mutation process. In evolutionary game theory, the varieties in question are strategies in a game. In nature, the basic selection mechanism is biological survival and reproduction, and the mutation process is basically genetic. In the market place, the basic selection mechanism is economic survival, and the mutation process is experimentation, externalities and mistakes. In both cases there is also an element of individual and social learning.

The book "Evolution and the Theory of Games" by Maynard Smith (1982) explicitly introduced evolutionary selection pressure in a game theoretic setting. The notion of evolution of strategies in a repeatedly played game closely resembles certain models in which players learn from past behavior, thus facilitating the shift of attention in the field towards evolutionary models.

Hence, the point of departure for an evolutionary model is the belief that people are not always rational. Rather than springing into life as the result of a perfectly rational reasoning process in which each player, armed with the common knowledge of perfect rationality, solves the game, in evolutionary games strategies emerge from a trial-and-error learning process in which players find that some strategies perform better than others, after which they decide to adopt or simply to imitate such strategies. The agents may do very little reasoning in the course of this learning process. Instead, they simply take actions, sometimes with great contemplation and sometimes with no thought at all. Their behavior is driven by rules of thumb, social norms, conventions, analogies to similar situations, or by other possibly more complex systems for converting stimuli into actions. In adopting this approach, evolutionary game theory assumes that the behavior driving the process by which agents adjust their strategies may not be perfectly rational, even though it may lead to "rational" equilibrium behavior.

We recommend the readers the following book references on Evolutionary Game Theory: Gintis (2009), Hofbauer & Sigmund (2002), Vega-Redondo (1996), Van Damme (1991) and Weibull (1995).

2.1 Strategic Game

We proceed to describe a population game, but to do so we must first recall the notion of a strategic or normal form game.

Let $I = \{1, 2, ...N\}$ be a set of players, for each player $i \in I$ let S_i be her (finite) set of allowable actions, called the action set.

The deterministic choice of an action $s_i \in S_i$ by player $i \in I$, is called **a**

pure strategy for i.

A vector $s = (s_1, ..., s_n) \in \times_{i \in I} S_i$, where $s_i \in S_i$ is the pure strategy adopted by player $i \in I$, is called **a pure strategy profile** or a configuration of the players. The space of all the pure strategy profiles in the game is thus the Cartesian product $S = \times_{i \in I} S_i$ of the players' action sets (usually called the configuration space).

For any configuration $s \in S$ and any player $i \in I$, let $\pi_i(s)$ be a real number indicating the payoff to player i upon the adoption of this configuration by all the players of the game. That is, a player function for each player is,

$$\pi_i : S \to \mathbb{R}$$

The finite collection of the real numbers $\{\pi_i(s) : s \in S\}$ defines **player *i's* payoff function**. Let $\pi(s) = (\pi_i(s))_{i \in I}$ be the vector function of all the players' payoffs.

Thus, *a **strategic game** is described by a triplet* $\Gamma = (I, S, \pi)$ *where* I *is the set of players,* S *is their configuration space and* π *is the vector function of all the players' payoffs.*

A **mixed strategy** for player $i \in I$ is a probability distribution (as opposed to a deterministic choice that is indicated by a pure strategy) over her action set S_i. We may represent any mixed strategy as a vector

$$x_i = (x_i(s_i, 1), x_i(s_i, 2), ..., x_i(s_i, m_i)),$$

where $m_i = |S_i|$, $\forall j \in [m_i]$, $s_{i,j} \in S_i$ is the j^{th} allowable action for player i, and $x_i(s_{i,j})$ is the probability that action $s_{i,j}$ is adopted by player i. To simplify notation, we shall represent this vector as $x_i = (x_{i,1}, x_{i,2}, ..., x_{i,m_i})$. Of course, $\forall i \in I$, $\sum_{j \in [m_i]} x_{i,j} = 1$ and $\forall j \in [mi]$, $x_{i,j} \in [0, 1]$. Since for each player i all probabilities are nonnegative and sum up to one, **the mixed strategies set** of player i is the set

$$\Delta_i \equiv \left\{ x_i \in \mathbb{R}^{m_i}_{\geq 0} : \sum\nolimits_{j \in [m_i]} = 1 \right\}.$$

The vertices or corner of this simplex are

$$e_i^1 = (1, 0, ..., 0), ..., e_j = (0, ..., 1, ..., 0), ..., e_i^n = (0, 0, ..., 1).$$

Hence, a pure strategy is a concentrated distribution of probabilities. Pure strategies are then just special or "extreme" mixed strategies in which the probability of a specific action is equal to one and all other probabilities equal zero.

A **mixed strategies profile** is a vector $X = (X_1, ..., X_n)$ whose components are themselves mixed strategies of the players, i.e., $\forall i \in I$, $X_i \in \Delta_i$. We denote by $\Theta = \times_{i \in I} \Delta_i \subset \mathbb{R}^m$ the Cartesian product of mixed strategies sets of all the players, which is called the mixed strategies space of the game ($m = m_1 + \cdots + m_n$).

We write (x_i, y_{-i}) for the strategy profile in which player $i \in I$ plays strategy $x_i \in \Delta_i$ and all others according to the profile $y \in \Theta$.

Player's payoff $i \in I$ **associated with the mixed-strategy profile** x is given by **the expected value**,

$$u_i(x) = \sum_{s \in S} x(s)\, \pi_i(s),$$

where $x(s) = \prod_{i=1}^{n} x_{is_i}$ is the product of probabilities assigned by each player's *mixed strategy* $x_i \in \Delta_i$ to the pure strategy $s_i \in S$.

The payoff $u_i(x)$ is a linear function of each player's mixed strategy. That is, consider that player j plays $s_j = k$ which is equivalent to playing the mixed strategy e_j^k, then the payoff of i when j uses her $k-$th pure strategy is $u_i\left(e_j^k, x_{-j}\right)$. Hence, for any $x \in \Theta$,

$$u_i(x) = \sum_{k=1}^{n_j} u_i\left(e_i^k, x_{-i}\right) x_{ik}.$$

The combined function $u : \mathbb{R}^m \to \mathbb{R}^n$ defined by,

$$u(x) = (u_1(x), ..., u_n(x)),$$

is the combined payoff function of the game. Hence, **the mixed extension of a game** $\Gamma = (I, S, \pi)$ **is given by** $\Upsilon = (I, \Theta, u)$.

2.2 Population Game

Evolutionary game theory considers populations of decision makers, while analysing the player profiles within these populations, instead of single players. We can therefore identify a population game, where N large populations strategically interact, as an $N-$player form game, where each player has a large population behind him.

Let us consider an "economy" consisting of N, $N < \infty$, different populations. Formally we can describe our model of the populated economy as a countable set of populations $P = \{1, ..., N\}$. Each population is assumed to form a continuum of positive mass, m_i, $1 \leq i \leq N$. If we look at an arbitrary population we simply write i for this population, and i is an index that can vary between 1 and N. Each population is endowed with a finite set of pure strategies, which we can identify with the set S_i. Strategic interactions might take place between and within populations.

In a given game Γ, agents from each of the N player populations are randomly drawn and matched to play the game in question.

Then, **a strategy profile** $x = (x_1, ..., x_k)$ represents:

- The percentage of agents adopting one of the possible k strategies,
- or the percentage of times that the typical agent uses each possible strategy.

In biology, this is genetically programmed, and the payoff corresponds to inclusive fitness or the expected number of offspring. But in economics, agents that play a game many times can consciously switch strategies or behavior.

Let S_i be a finite set of pure strategies corresponding to a player population $i \in P$. Then,

Definition 1 *A **population state or population profile** is an element of the* $(n_k - 1)$*-dimensional simplex*[1]

$$\Delta S_i = \left\{ x = (x_1, ..., x_k) \in \mathbb{R}^n : \sum_{i=1}^{k} x_i = 1 \right\}$$

generated by the pure strategies $\left\{ e_i^1, ..., e_i^k, ..., e_i^n \right\}$*. For all* $i \in \{1, 2, ..., n\}$ *the component* $x_i \equiv \frac{x_{k_i}}{m_i}$ *is the proportion of* $i-$*strategists in the player population* k*, or the frequency with which strategy* e_i^k *is observed in population* k*. Hence, a population profile is a vector* x *that gives the probability* $x(s)$ *with which strategy* $s \in S_i$ *is played in a given population.*

In what follows, we assume that the population masses stay constant, so that we do not have to think about possible effects arising from demographic dynamics. Moreover will assume that all populations are of the same size, so every agent in every population is drawn with the same probability. Thus, we can concentrate on Definition 1 to discuss population states.

Moreover, from a modelling view point, we distinguish between two types of population games:

1. **Games against the field**. These games have the following characteristics: i) there is no specific opponent for a given agent, and ii) payoffs depend on what everyone in the population is doing.

2. **Pairwise contest game**. Describe situations where a given agent plays against an opponent that has been randomly selected (by Nature).

[1] Consider a finite collection of points in euclidean $\mathbb{R}^n$, given by $\{x_1, ..., x_m\}$. Such a set is affinely independent if $\sum_{i=1}^{m} \lambda_i x_i = 0$ and $\sum_{i=1}^{m} \lambda_i = 0$ imply that $\lambda_i = 0, 1 \leq i \leq m$. An $(m-1)$-dimensional simplex is the set of all strictly positive convex combinationsof an m element set of affinely independent points.

$$\Delta(x_1, ..., x_n) := \left\{ \sum_{i=1}^{m} \lambda_i x_i | \forall i = 1, ..., m : \lambda_i \geqslant 0 \wedge \sum_{i=1}^{m} \lambda_i = 1 \right\}$$

See Border (1985).

2.2.1 Games against the field

Consider an agent of any given population where the current population state is x. The population profile x is generated by the agent's strategy σ. If such an agent plays with σ, then, her payoff is denoted by,

$$\pi(\sigma, x) = \sum_{s \in S} \sigma(s)\pi(s, x). \tag{2.1}$$

This payoff represents the number of descendents: either through reproduction or imitation. An agent's payoffs therefore determine the process of evolution in a given population.

We are interested in individual behavior that attains payoffs such that there is no better response for the agent. That is, a necessary condition of evolutionary stability is,

$$\sigma^* \in \arg\max_{\sigma \in \Delta} \pi(\sigma, x^*). \tag{2.2}$$

So, at an equilibrium, the strategy adopted by agents must be the best response to the population state that it generates. The stability condition is therefore of great importance.

Consider a population where (initially) all the agents adopt a strategy σ^*. Now suppose that a mutation occurs and a small proportion ϵ of agents use some other strategy σ. After the mutation occurs a new population arises which is denoted by x_ϵ,

$$x_\epsilon = ((1-\epsilon)\sigma^* + \epsilon\sigma), \tag{2.3}$$

which is called post-entry population.

2.2.1.1 Evolutionarily stable strategies

A key concept in evolutionary game theory is that of an **evolutionarily stable strategy** (ESS), a concept conceived by Maynard Smith and Price (1973), see also Maynard Smith (1972, 1974). Such a strategy is robust to evolutionary pressures in a specific sense: a population playing such a strategy is uninvadable by any other strategy. Uninvadability is a useful characterization of evolutionary stability, and indeed its original definition is that a strategy σ^* is evolutionarily stable if and only if (a) it is a best response to itself, (b) it is a better response to all other best responses than these are to themselves. By definition, no alternative best response exists for any player population if the profile in question σ^* happens to be a strict Nash equilibrium, so such profiles should qualify.

Definition 2 ***A mixed strategy is an ESS*** *if there exists $\bar{\epsilon}$ such that, for every $0 < \epsilon < \bar{\epsilon}$ and every $\sigma \neq \sigma^*$, then,*

$$\pi(\sigma^*, x_\epsilon) > \pi(\sigma, x_\epsilon) \tag{2.4}$$

In other words σ^* is an ESS if a mutant that adopt any other strategy σ produces fewer offspring in the post-entry population, provided that the population of mutants is sufficiently small.

Evolutionary forces select against mutation if and only if $\pi(\sigma^*, x_\epsilon) > \pi(\sigma, x_\epsilon)$, the post-entry payoff is lower than that of the incumbent strategy.

2.2.2 Pairwise contest game

A pairwise contest game describes a situation in which a given agent plays against an opponent that is randomly selected from the population and for which the payoffs depend solely on what both agents do. Then,

$$\pi(\sigma, x) = \sum_{s \in S} \sum_{s\prime \in S\prime} \sigma(s) x(s\prime) \pi(s, s\prime). \tag{2.5}$$

Consider n genetically defined possible behaviors, i.e. there are $n-$pure strategies, $S = (s_1, ..., s_n)$ for each agent in the population. We say that the behavior of the agents generate the profile or state of the population in the sense that:

- all agents have a mixed behavior $\sigma = (x_1, x_2, ..., x_n)$ or,
- a proportion x_i of the agents in the population follow the behavior s_i, i.e. they are $i-$strategists.

Consider a population following σ^*. We are interested in knowing the payoff given by $\pi(\sigma, \sigma^*)$.

For the characteristics of the current game our interest is in symmetric Nash equilibrium. Recall that every symmetric two player game has a symmetric Nash equilibrium (σ^*, σ^*). We represent this kind of equilibria by Δ^{NE}.

Suppose then that agents initially play according to a mixed strategy σ^* (or that σ^* represents the proportion of agents playing each pure strategy), so in both cases: $\sigma^* = x$. Let x^* be a profile generated by a population of agents who all of them adopt the strategy σ^*, so, $x^* = \sigma^*$.

Let us denote the set of ESS, in a pairwise contest game, by:

$$\Delta^{ESS} := \left\{ \sigma^* \in \Delta : (\sigma^*, \sigma^*) \in \Delta^{NE} \right\}. \tag{2.6}$$

The following proposition characterize the set of ESS in a pairwise contest game.

Definition 3 *A strategy σ^* is ESS, if and only if $\forall\ \sigma \neq \sigma^*$ either:*

1. $\pi(\sigma^*, \sigma^*) > \pi(\sigma, \sigma^*)$, *or*
2. $\pi(\sigma^*, \sigma^*) = \pi(\sigma, \sigma^*)$ *and* $\pi(\sigma^*, \sigma) > \pi(\sigma, \sigma)$.

Corollary 1 *In a pairwise contest population (symmetric) game,*

$$\Delta^{ESS} := \left\{ x \in \Delta^{NE} : \pi(y, y) < \pi(x, y) \quad \forall\, y \in \beta(x), y \neq x \right\}. \tag{2.7}$$

where $\beta(x)$ is the combined correspondence of best response.

Hence, the concept of ESS is a refinement of NE, because if $\sigma^* \in \Delta^{ESS}$ then:

1. $\sigma^* \in \Delta^{NE}$ and
2. if $\pi(\sigma, \sigma^*) = \pi(\sigma^*, \sigma^*)$ then $\pi(\sigma^*, \sigma) > \pi(\sigma, \sigma)$ (the stability condition).

The following corollary holds:

Corollary 2 *if (x, x) is a strict NE then (x, x) is ESS.*

2.3 On the notion of ESS in asymmetric games

Asymmetric games are more realistic in economics because economic agents play different roles such as residents and migrants (or firms and workers), and customarily there are differences in the structures and methods of competition. In this thesis we focus on asymmetric games and our baseline model studied in Chapter (3) offers a model of imitation in two asymmetric populations where we characterize imitation using the ESS concept and Nash equilibrium for such classes of games.

One way to characterize the ESS in asymmetric games is to define an homogeneous set of players who in each period are paired randomly. One of the pair being randomly assigned to be player 1, and the other to be player 2. We may call this the symmetric version of the asymmetric game. However, an ESS in the symmetric version of an asymmetric game must be a strict Nash equilibrium, that is, each type in the asymmetric game must use exactly one pure strategy (see Selten, 1980 and more details in Gintis, 2009).

Therefore, to apply the ESS concept to asymmetric games, we now think of agents, when called upon to play the game, being randomly (with equal probabilities) assigned to be either player i or player j. The agents are informed of the role to which they have been assigned and are allowed to condition their choice upon this assignment.

Let us consider two polymorphic populations, $\tau \in \{1, 2\}$, and at each period $t \in [0, \infty)$, every member of each population is randomly matched with agents from the other population to play a bilateral finite game. Formally, the game is described by:

1. The strategy spaces of each type are denoted by S^1 and S^2, with respective cardinalities $m_1, m_2 \in \mathbb{N}$. Correspondingly, $\sum^1 = \Delta^{m_1 - 1}$ and $\sum^2 = \Delta^{m_2 - 1}$ stand for their respective spaces of mixed strategies, with generic elements $\sigma^1 \in \sum^1$ and $\sigma^2 \in \sum^2$.

2. The payoff matrices $A, B \in \mathbb{R}^{m_1 \times m_2}$, here, $a_{ij} \equiv \pi^1(s_i^1, s_j^2)$ and $b_{ij} \equiv \pi^2(s_i^1, s_j^2)$ stand for the payoffs obtained by a player of type 1 (population 1) and a player of type 2 (population 2), where players of type 1 adopt strategy s_i^1 and players of type 2 adopt strategy s_j^2.

3. We assume that agents adopt only pure strategies, and hence the spaces Δ^{m_1-1} and Δ^{m_2-1} are the set of possible population states, i.e. population profiles specifying the fraction of agents playing each of the different pure strategies. Such population states will be generically denoted by x^1 and x^2, where x_i^τ specifies the fraction of agents of population $\tau = 1, 2$ which adopt strategy $s_i^\tau \in S^\tau$.

4. Under the assumption of random matching, for which we abuse the previous notation, we can denote by $\pi^\tau\left(s_i^\tau, x^{\tau\prime}\right)$, $\tau, \tau\prime = 1, 2,\quad \tau \neq \tau\prime$,the expected payoff of playing strategy s_i^τ for an agent of population τ when the strategy profile prevailing in the other population is the continuum, such expected values are also identified with the corresponding average magnitudes.

Before we introduce the notion of ESS, let us consider:

$$\Delta^\tau = \left\{ x \in \mathbb{R}_+^{k^\tau}; \sum_{i=1}^{k^\tau} x_i = 1 \right\} \text{ for all } \tau \in \{1, 2\}$$

where k^τ is the number of pure strategies (behavior) for the population τ.[2]

Definition 4 *A strategy $x \in \Delta^\tau$ is an ESS in asymmetric games, for a population τ if and only if*

$$\pi^\tau(x_i, y_{-i}) \geqslant \pi^\tau(z_i, y_{-i}) \quad \forall\ z \in \Delta^\tau$$

and for all $y' \in \Delta$, $y' \neq y$ there exists some $\bar{\epsilon}_{y'} \in (0, 1)$ such that for all $\epsilon \in (0, \bar{\epsilon}_{y'})$ and with $w_\epsilon = \epsilon y' + (1 - \epsilon)y$,

$$\pi^\tau(x_i, w_\epsilon) > \pi^\tau(y_i, w_\epsilon), \quad \forall\ \tau \in \{1, 2\}\,.$$

Intuitively, we say that x is an ESS if and only if after a mutation in the population $-\tau$ continues to be a best response for the post-entry population, w_ϵ. However, an ESS of an asymmetric game must be a strict Nash equilibrium, i.e. both types of players, from each population τ, must

[2] Vega-Redondo (1996) defines a strategy $s_i^{\tau *}$ being ESS in asymmetric games if given any other strategy s_i^τ, there exists some $\bar{\varepsilon} > 0$ such that if $0 < \varepsilon \leq \bar{\varepsilon}$,

$$\pi^\tau\left(s_i^{\tau *}, (1-\varepsilon)s_i^{\tau *} + \varepsilon s_i^\tau\right) > \pi^\tau\left(s_i^\tau, (1-\varepsilon)s_i^{\tau *} + \varepsilon s_i^\tau\right).$$

be monomorphic in equilibrium, there being only one type of player in each population.

Selten's (1980) result shows that an asymmetric game has no mixed ESS or every ESS in the asymmetric game is a pure strategy (more details are presented in Gaunersdorfer, Hofbauer and Sigmund, 1991; Nachbar, 1990). Therefore, a strictly mixed Nash equilibrium of asymmetric games is not an evolutionary equilibrium under the replicator dynamic (see Hofbauer and Sigmund, 1998).

2.3.1 The ESS in the asymmetric two-population game

Let Γ be a normal-form game with player set composed by individuals that comprise two populations labeled by $\tau = \{1,2\}$. Each population splits in different clubs denoted by n_i^τ, $i = 1, ..., \tau_k$. The split depends on the strategy agents play or the behavior that agents follow. Strategies are in correspondence with the clubs, individuals belonging to the n_i^τ club will be called n_i-strategists. Thus, the set S_τ of pure strategies are: $S_1 = \{n_1^1, ..., n_k^1\}$ and $S_M = \{n_1^2, ..., n_k^2\}$. Individuals belong only to one club in each period of time, but but at every period they can move from one club to another.

For each population τ we represent the set of mixed strategy by:

$$\Delta^\tau = \left\{ x \in \mathbb{R}^{k_\tau} : \sum_{j=1}^{k_\tau} x_j = 1,\ x_j \geq 0,\ j = 1, ..., n_i \right\}$$

Note that, a profile distribution $x = (x_1, ..., x_{k_\tau}) \in \Delta^\tau$ can bee seen as the individual behavior of a player spending a part of his time, given by x_j, in the n_j-club, hence x represents the population state as the vector of individuals' share belonging to each club $i = 1, ..., \tau_k$ $\forall\ \tau \in \{1,2\}$.

The normal form representation for our described game, is given by the next matrix payoff:

$1 \diagdown 2$	y_1	$\cdots$	y_{k_M}
x_1	a_{11}, b_{11}	$\cdots$	a_{1k_M}, b_{1k_M}
$\vdots$	$\vdots$	$\cdots$	$\vdots$
x_{k_R}	a_{k_R1}, b_{k_R1}	$\cdots$	$a_{k_Rk_M}, b_{k_Rk_M}$

(2.8)

where a_{ij} denotes the payoff of an $i-$strategist from population 1 facing a $j-$strategist from population 2. Analogously for the entries b_{ij}.

The matching between individuals from different population is given in a random way. We use the notation:

$$E^1(n_i^1 \mid y) = \sum_{j=1}^{k_M} a_{ij} y_j,\ \forall\ n_i^1 \in S^1$$

to represent the $i-$strategist's expected payoff who belongs to the n_i-club from population 1 given that the typical (fitness) strategist from the other population follows a mixed strategy y, that is to say, the clubs' distribution from the opposite population, 2, is given by y.

Analogously, the expected payoff of the $i-$strategist belonging to n_i-club from population M is given by:

$$E^2(n_i^2/x) = \sum_{j=1}^{k_R} b_{ij}x_j,\ \forall n_i^2 \in S^2$$

where clubs' distribution from the other population, 1, is given by x. Rational individuals follow the strategic profile that maximize his expected payoff.

2.3.1.1 The definition of the evolutionarily stable strategy

We are interested in the evolution and in the stability of both populations playing the game. Then, we analyze if small changes in the individuals' behavior from one population imply changes in the behavior of the individuals from the other population.

Consider the two-population normal form game:

$$\Gamma = \{\tau \in \{1,2\}\,,\ \{S^\tau\}\,,\ \{A = (a_{ij})\,,\, B = (b_{ij})\}\}$$

where each population splits into clubs denoted by $n_i^\tau\ \ \forall\ \tau = \{1,2\}$ and $i = 1, ..., \tau_k$. Hence:

- The population 1 is: $\bigcup_{i=1}^{k_1} n_i^1$, and $\forall\ h \neq j\ n_h^1 \bigcap n_j^1 = \emptyset$.
- The population 2 is: $\bigcup_{i=1}^{k_2} n_i^2$, and $\forall\ h \neq j\ n_h^2 \bigcap n_j^2 = \emptyset$.

Let $p \in \Delta^\tau$ be the profile distribution of individuals' behavior from population 1, in a given period of time t_0, and that in the same time, the profile distribution of individuals' behavior in population 2 is $q \in \Delta^\tau$. Assume that in a post-period of time $t_1 > t_0$ a small mutation affects the individuals' behavior from population 2. Hence, the profile distribution from population 2 after the mutation, is denoted by the offspring:

$$q_\epsilon = ((1-\epsilon)q + \epsilon\bar{q},$$

which is called the fitness of the post-entry population. Analogously, the profile distribution from population 1 after suffering a small mutation is:

$$p_\epsilon = ((1-\epsilon)p + \epsilon\bar{p}.$$

Now, we can state the next definition:

Definition 5 *Let $(p^*, q^*) \in \Delta^1 \times \Delta^2$ be a profile of mixed strategies. We say that the profile (p^*, q^*) is an $\mathcal{ESS}$ for an asymmetric two-population normal form game, if for each $(\bar{p}, \bar{q}) \in \Delta^1 \times \Delta^2$ there exists $\bar{\epsilon}$ such that:*

$$1) \quad E^1(p^*/q_\epsilon) > E^1(p/q_\epsilon) \; \forall \, p \in \Delta^1$$

$$2) \quad E^2(q^*/p_\epsilon) > E^2(q/p_\epsilon) \; \forall \, q \in \Delta^2.$$

for all $0 < \epsilon \leq \bar{\epsilon}$, and being p_ϵ, q_ϵ their respective post-entry populations.

Hence, from the above definition, it follows that if (p^*, q^*) is an $\mathcal{ESS}$, then, the next assertions are verified:

1. The profile (p^*, q^*) is a $\mathcal{NE}$.
2. If there exists some $p \in \Delta^1$ such that $E^1(p/q^*) = E^1(p^*/q^*)$ then $E^1(p^*/\bar{q}) > E^1(p/\bar{q})$.
3. If there exists some $q \in \Delta^2$ such that $E^2(q/p^*) = E^2(q^*/p^*)$ then $E^2(q^*/\bar{p}) > E^2(q/\bar{p})$.

So, individuals' behavior who adopt an $\mathcal{ESS}$ performance better than the mutant individuals' behavior given by the post-entry population.

The evolutive properties of the ESS and its relationship with the set of Nash equilibria and the stationary states of the replicator dynamics for the case of symmetric games are well known. Let us introduce the symmetric (one population) version for the described asymmetric two-population game.

Definition 6 *The asymmetric two-population normal form game Γ has a corresponding symmetric one-population game characterized by:*

1. *The big population: $P = 1 \cup 2$.*
2. *Individuals from the big population P face their own population.*
3. *Let $n^1_1, ..., n^1_{k_1}$, $n^2_1, ..., n^2_{k_2}$ be the set of pure strategy for P.*
4. *The matrix payoff for the big population P is:*

$$P = \begin{bmatrix} 0 & A \\ B^T & 0 \end{bmatrix}$$

Hence, the numbered list item 1-4 characterizes the symmetric version Γ^s of the asymmetric game Γ.

Then, for each asymmetric two-population game Γ, there exists a corresponding symmetric version Γ^s.

Hence, as Gintis (2009:244) pointed out:

> "...it makes no sense to say that a mutant meets its own type when the game is asymmetric, so the ESS criterion has no meaning. The obvious way around this problem is to define a homogeneous set of players who in each period are paired randomly, one of the pair being randomly assigned to be player 1, and the other to be player 2."

In this way, Gintis (2009) defines the "symmetric version" of the asymmetric evolutionary game. Selten (1980) shows that an evolutionarily stable strategy in the symmetric version of an asymmetric evolutionary game must be a strict Nash equilibrium.

2.4 The Replicator Dynamics

The replicator dynamics (RD) explicitly model a selection process, specifying how population shares associated with different pure strategies in a game evolve over time. The mathematical formulation of the replicator dynamics is due to Taylor and Jonker (1978). They imagine a large population of agents who are randomly matched over time to play a finite symmetric two-player game, just as in the setting for evolutionary stability. However, here agents only play pure strategies.

Consider a population in which agents, called replicators, use a preprogrammed strategy, and pass this behavior to their descendents. Let the population state be a point $x = (x_1, ..., x_n)$ in the set Δ of mixed-strategy profiles, where each component x_i is a point in the corresponding mixed-strategy simplex Δ_i, representing the distribution of agents in population i across the pure strategies available to that type of player. The vector x_i may thus be thought of as the state of player population $i \in I$ at time t, where $x_{ih} \in [0, 1]$ is the proportion of agents in population i who are currently programmed to play pure strategy $h \in S_i$.

Hence, a population state is a distribution x over pure strategies. Such a state is mathematically equivalent to a mixed strategy in the game. If the payoffs in the game represent biological fitness, i.e., the number of offspring, and each child inherits its one parent's strategy, then the number of agents using pure strategy i will (in a large population) grow exponentially at a rate that equals the (expected) payoff $\pi(s_i, x)$ to pure strategy i when played against the mixed strategy x that represents the current strategy distribution in the population. It follows that *the growth rate of the population share using any pure strategy i equals the difference between the strategy's payoff and the average payoff in the population.* The latter is identical to the (expected) payoff $\bar{\pi}(x, x)$ to mixed strategy x when played against itself. These are the single-population replicator dynamics for symmetric two-player games,

$$\dot{x}_i = [\pi(s_i, x) - \bar{\pi}(x, x)]\, x_i. \tag{2.9}$$

In other words, *the proportion of agents using strategy s_i increases (decreases) if its payoff is bigger (smaller) than the average payoff of the population.*

Note that the best pure responses to the current population state x have the highest growth rate in the population, the second-best pure responses have the second-highest growth rate, and so on. Although more successful pure strategies grow faster than less successful ones, the average payoff in the population need not grow over time. The reason for this possibility is that if an agent is replaced by an agent using a better strategy, then the opponents meeting this new agent may receive lower payoffs.

Moreover, the RD defined for a two population model is:

$$\dot{x}_i^\tau(t) = x_i^\tau(t)\left[\pi^\tau\left(s_i^\tau, x^{\tau\prime}(t)\right) - \sum_{j=1}^{m_\tau} x_j^\tau(t)\pi^\tau\left(s_j^\tau, x^{\tau\prime}(t)\right)\right], \qquad (2.10)$$

for each $i, j = 1, 2, ..., m_\tau$, where populations $\tau, \tau\prime = 1, 2, \ \tau \neq \tau\prime$.

A population state $x \in \Delta$ is stationary in the RD if and only if, for each player population i, every pure strategy $s_i^\tau \in S^\tau$ in use earns the same payoff. Hence the common set of stationary states is

$$\Delta^o = \left\{x \in \Delta : \pi^\tau\left(s_i^\tau, x^{\tau\prime}\right) = \pi_i(x) \quad \forall i \in I, \ \ \tau, \tau\prime = 1, 2, \ \tau \neq \tau\prime\right\}. \quad (2.11)$$

Consequently all interior stationary states are Nash equilibria, and all Nash equilibria are stationary: $\Delta^o \cap int(\Delta) \subset \Delta^{NE} \subset \Delta^o$.

Definition 7 *A* ***fixed point or stationary state****, of the replicator dynamics is a state satisfying $\dot{x}_i = 0 \ \forall i$.*

A fixed point, describe a population that are no longer evolving. That is, a solution of the dynamical system $x(t) = x^* \ \forall t$. The following proposition holds:

Proposition 1 *If x^* is an interior fixed point, i.e., $x_i^* > 0 \ \forall i \in I$ (all strategy is presented in the population) of the replicator dynamics, then, $x^* \in \Delta^{NE}$.*

Proof. Note that if $x^* >> 0$, then, $\dot{x} = 0$ if and only if $\pi(s_i, x^*) = \pi(x^*, x^*)$. Hence all pure strategy must earn the same payoff, therefore, (x^*, x^*) is a NE. ■

Moreover, it is well known that a fixed point of a dynamical system is said to be **asymptotically stable** or an attractor point if any small deviation from this state are eliminated by the dynamics when $t \to \infty$.

2.4.1 ESSs and attractors

An ESS is a static concept, but it can become a dynamic concept by applying the replicator dynamic equation (see Taylor and Jonker, 1978), and it can be verified that if there is an ESS then it is an attractor point. The converse is not true: an attractor is not necessarily an ESS because locally the flow on Δ may spiral elliptically towards the attractor and it is not covered by the notion of ESS due to the linearity required in the definition.

The relationship between the replicator dynamic equation and evolutionarily stable strategies is that every ESS is an asymptotically stable point of the replicator dynamic, but asymptotically stable states need not be an ESS.

Zeeman (1992) shows the following result:

Theorem 1 *An ESS is an attractor point of the replicator dynamic, but not conversely.*

This result is formally proved by Zeeman (1992) and it was first proved by Taylor and Jonker (1978) and Hofbauer et al. (1979) under the assumption that the ESS was regular, and concluded that the attractor was hyperbolic. Recall that a point is an attractor of the flow on Δ (the n-simplex representing populations with different proportions playing the various strategies) if it is the $\omega-$limit of a neighborhood, and the $\alpha-$limit of itself only. Its basin of attraction is the open set of points of which it is the $\omega-$limit. It is hyperbolic if its eigenvalues have negative real part. Moreover, the theorem shows that from the point of view of smooth dynamics an attractor is a more general notion than an ESS, and a better characterization of the resistance to mutation.

In the next chapter we offer evolutionary games of economic agents. Then, to characterize the ESS as attractor points of the replicator dynamics, and to characterize the high- and low-level equilibria for such games, we may consider the following theorem:

Theorem 2 (ESS as attractor point of the replicator dynamics) *An ESS corresponds to an asymptotically stable fixed point in the replicator dynamics. That is, if σ^* is an ESS, then the population with $\mathbf{x}^* = \boldsymbol{\sigma}^*$ is asymptotically stable.*

Proof. If σ^* is an ESS then, by definition, there exists an $\bar{\epsilon}$ such that for all $\epsilon < \bar{\epsilon}$

$$\pi(\sigma^*, \sigma_\epsilon) > \pi(\sigma, \sigma_\epsilon) \quad \forall\, \sigma \neq \sigma^*$$

where $\sigma_\epsilon = (1-\epsilon)\sigma^* + \epsilon\sigma'$. In particular, this hold for $\sigma = \sigma_\epsilon$, so $\pi(\sigma^*, \sigma_\epsilon) > \pi(\sigma_\epsilon, \sigma_\epsilon)$. This implies that in the replicator dynamics we have, for $x^* = \sigma^*$, $x = (1-\epsilon)x^* + \epsilon x'$ and all $\epsilon < \bar{\epsilon}$

$$\pi(\sigma^*, x) > \bar{\pi}(x, x)$$

Now consider the relative entropy function

$$\mathbf{V}(\mathbf{x}) = -\sum_{i=1}^{k} x_i^* \ln\left(\frac{x_i}{x_i^*}\right)$$

Clearly $\mathbf{V}(\mathbf{x}^*) = 0$ and (using Jensen's inequality $\mathbf{E}f(x) \geqslant f(\mathbf{E}x)$ for any convex function, such as a logarithm)

$$\begin{aligned}\mathbf{V}(\mathbf{x}) &= -\sum_{i=1}^{k} x_i^* \ln\left(\frac{x_i}{x_i^*}\right) \\ &\geqslant -\ln\left(\sum_{i=1}^{k} x_i^* \frac{x_i}{x_i^*}\right) \\ &= -\ln\left(\sum_{i=1}^{k} x_i\right) \\ &= -\ln(1) \\ &= 0.\end{aligned}$$

The time derivative of $\mathbf{V}(\mathbf{x})$ along solution trajectories of the replicator dynamics is

$$\begin{aligned}\frac{d}{dt}\mathbf{V}(\mathbf{x}) &= \sum_{i=1}^{k} \frac{\partial \mathbf{V}}{\partial x_i}\dot{x}_i \\ &= -\sum_{i=1}^{k} \frac{x_i^*}{x_i}\dot{x}_i \\ &= -\sum_{i=1}^{k} \frac{x_i^*}{x_i}\left[\pi(s_i, x) - \bar{\pi}(x, x)\right] x_i \\ &= -\left[\pi(\sigma^*, x) - \bar{\pi}(x, x)\right].\end{aligned}$$

If σ^* is an ESS, then we established above that there is a region near to $\mathbf{x}^*$ where $[\pi(\sigma^*, x) - \bar{\pi}(x, x)] > 0$ for $\mathbf{x} \neq \mathbf{x}^*$. Hence,

$$\frac{d\mathbf{V}}{dt} < 0$$

for population states sufficiently close to the fixed point. $\mathbf{V}(\mathbf{x})$ is therefore a strict Lyapounov function in this region, and the fixed point $\mathbf{x}^*$ is asymptotically stable. ■

In general, there may be asymptotically stable fixed points in the replicator dynamics which do not correspond to an ESS. If the derivative of the relative entropy function for a fixed point (taken along solution trajectories) is positive, then the fixed point is unstable. If the derivative is zero,then the fixed point is neither asymptotically stable nor unstable: the evolution of the population is periodic around the fixed point.

2.5 Behavioral Rules

Behavioral rules driven by imitation have a long tradition in the literature of evolutionary game theory. One of the best known evolutionary models, the replicator dynamics, describes an evolutionary process which is driven purely by imitation of other as.

In any N player game there are N populations, one population for each type of player, from which we randomly draw agents who are programmed to play some pure strategy available to the type of player. Let these agents play the game. **As time proceeds agents in the populations are allowed to change their pure strategy. This is embodied by so-called behavioral rules**, such behavioral rules generate a system of differential equations, which describe the evolution of the relative frequency with which some pure strategy occurs in a population.

There is one differential equation for each pure strategy available to a population and every differential equation describes the evolution of the population share that this pure strategy has, that is the number x_i^τ for all $1 \leq \tau \leq N$ and $1 \leq i \leq n_\tau$.

Definition 8 *A* ***behavioral rule*** *is a map from currently aggregate behavior to conditional switch rates. The map is given by two basic elements:*

1. *The time rate $r_i(x)$ at which agents review their strategy choice. This time rate depends on the performance of the agent's pure strategy and other aspects of the current population state.*[3]
2. *The probability $p_{ij}(x)$, that a reviewing $i-$strategist will switch to some pure strategy j. The vector of this probabilities is written as: $p_i(x) = (p_{i1}(x), ..., p_{ik}(x))$, and it is a distribution on the set K of pure strategies. So, $p_i(x) \in \Delta$. This distribution may depend on the current performance of the strategies and other aspects of the population state.*

In a finite population one may imagine that the reviews times of an agent are the arrival time of a Poisson process with arrival time $r_i(x)$, and that at each such time the agents selects a pure strategy according to the probability distribution $p_i(x)$ over the set K.

Recall that, a Poisson process is characterized by:

- The number of changes in non-overlapping intervals are independent for all intervals.
- The probability of exactly one change in a sufficiently small interval h is $p = vh$, where $v = r_i(x)$ is the probability of one change.

[3]This is the "behavioural rule with inertia" (see Bjornerstedt and Weibull, 1996; Weibull, 1995 and Schlag, 1998; 1999) that allows an agent to reconsider her action with probability $r \in (0, 1)$ each round.

- The probability of two or more changes in sufficiently small interval since h is essentially 0.

Consider that all agents' Poisson processes are independent, the aggregate process in the subpopulation of $i-$strategists is itself a Poisson Process, with arrival rate $\lambda_i = x_i r_i(x)$. Consider independence of switches across agents, and consider the process of switches from strategy i to strategy j as a Poisson Process with arrival rate: $\lambda_{ij} = x_i r_i p_{ij}$.

Assuming a continuum of agents and, by the law of large numbers we model these aggregate stochastic process as a deterministic flow:

- The outflow from subpopulation i thus is:

$$\sum_{j \neq i} x_i r_i(x) p_{ij}(x).$$

- The inflow to this subpopulation is:

$$\sum_{j \neq i} x_j r_j(x) p_{ji}(x).$$

With the use of behavioral rules we can define an evolutionary dynamic as an inflow-outflow model. The inflow into pure strategy e_i captures the mass of agents that abandon their currently employed pure strategy and play e_i until the next switching opportunity. The outflow term is the mass of as that currently play e_i, but now decide to play some different pure strategy e_j in K. Inflow and outflow affect the number $\dot{x}_i$. Inflows make $\dot{x}_i$ grow, while outflows lower this strategy frequency. The net flow is the difference between inflow and outflow and determines whether the frequency with which we observe pure strategy e_i increases or decreases.

Therefore, rearranging terms, we obtain:

$$\dot{x}_i = \sum_{j \in K} x_j r_j(x) p_{ji}(x) - \sum_{i \in K} x_i r_i(x) p_{ij}(x). \tag{2.12}$$

Hence, every behavioral rule can generate an evolutionary dynamic and they take agents' behaviors as the starting point. This explicit microfoundation of the evolution of the populations makes behavioral rules a very attractive tool. In principle any function that is in accordance with Definition (8) can generate an evolutionary dynamic.

A population dynamic (2.12) will be called imitative if there are a least two different strategies and at least one agent following one of these strategies assesses, with a give probability, whether he should change his behavior. The final decision depends on the relationship between the benefits the agent obtains and the benefits obtained by agents following a different strategy.

Certainly there are some properties that we would like "good" behavioral rules to share. One such property can be seen technically, but also reflects a fact emerging from dynamics within large populations. A good behavioral rule is Lipschitz continuous in payoffs and social states. That is, to guarantee that this system of differential equations (2.12) induces a well defined dynamic on the space Δ we assume that $r_i : X \rightarrow [0, 1]$ and $p_i : X \in \Delta$ are Lipschitz continuous functions. Then there exists in an open set X containing Δ one and only one solution through any initial state $x_0 \in \Delta$ and such that a solution trajectory never leaves Δ. The state space Δ is forward invariant in this dynamics (2.12).

In fact, if agents had perfect information about all payoffs yielded by other pure strategies, and if they knew the state of the population perfectly, behavioral adjustments would be much faster, possibly leading to discontinuous switches. But perfect and complete information is rare in large populations, and if no central authority exists that can distributes information fast enough or sufficiently far to reach every agent in society. Hence, Lipschitz-continuity reflects an assumption that people have limited knowledge about payoffs and population states, which has some appeal if we look at large populations.

In the next chapter, we look at the replicator dynamic driven by imitative behavior in asymmetric populations. The behavioral rule is such that players acquire their behavioral response in part by copying the behaviors of those who, in similar situations, they perceive as successful by some standard, or by acting to maximize their payoffs given their beliefs how others will act. But others influences are also work, including conformism or dissatisfaction.

CHAPTER 3

IMITATION IN TWO ASYMMETRIC POPULATIONS

To adopt an asymmetric approach is crucial in economics. Consider the following examples: economic models with incumbents and entrants in oligopolistic markets, social theory where we need to consider the relationships between migrants and residents with non-homogeneous behaviors, or tourism economics where residents and tourists constitute two non-homogeneous populations with different attitudes towards - and perceptions about - tourism development efforts or environmental quality.

Then, let us consider two asymmetric populations where in each round an economic agent of population 1 is paired with a member of population 2 and each one of them should decide whether to cooperate or not. Hence, we propose two imitation models:

1. Agents follow a pure imitation rule; they adopt the strategy of *"the first person they meet on the street"*. Agents randomly select another agent from the population and adopt the strategy of that agent. This simplistic hypothesis captures behaviors encountered in several populations.
2. Agents imitate successful strategies. Agents take account of their limited cognitive capacities and use imperfectly observed local information to update their behaviors. We assume that the probability an agent adopts a certain strategy by reviewing agents to imitate correlates positively with the payoff currently expected by changing to that strategy. Though each agent imperfectly observes local information, successful strategies will tend to spread while unsuccessful strategies will tend to die out.

We show that these behavioral rules result in an adjustment process that can be approximated by a dynamic system that coincides with the replicator dynamics.

We proceed as follows. Section 3.1 describes the model. Section 3.2 introduces the pure imitation model where each agent implements her own strategy by imitating because dissatisfaction. Subsection 3.2.1 discusses the properties on dynamic and Nash equilibria. Section 3.3 presents a model on imitation of successful agents. Section 3.4 characterizes the ESS of our model. Section 3.5 extends the above model with a modified behavioral rule. Section 3.6 draws some concluding remarks.

3.1 The Model

Let us assume that agents, at a given period of time and in a given territory, comprise two populations: residents, R, and migrants, M. Each population splits into two clubs.[1] The split depends on the strategy agents play or the behavior that agents follow.

Suppose that these strategies are: *to cooperate or not with a member of the other population,* $\{c, nc\}$. Those who have a cooperative behavior follow an interaction directed toward a common goal which is mutually beneficial or towards a common purpose or benefit, i.e., joint action. Additionally, to cooperate or not is in fact more profitable when the members of each population are coordinated playing as $\{c, c\}$ or $\{nc, nc\}$ than in any other situation.

Let $x^\tau \in R^2_+$, be the vector $x^\tau = (x^\tau_c, x^\tau_{nc})$ $\forall$ $\tau \in \{R, M\}$, normalized to one, $x^\tau_c + x^\tau_{nc} = 1$, where each entry is the population share of agents in their respective club.

Each period an $i-$strategist, $i \in \{c, nc\}$, from population $\tau \in \{R, M\}$, reviews her strategy with probability $r^\tau_i(x)$ assessing whether she should change her current strategy, or not, where $x = (x^M, x^R)$.

Hence, $r^\tau_i(x)$ is the time rate at which agents review their behavior (strategy) choice, which depends on the current performance of the agent's behavior and of the other aspects relatives to the current population state x.

Let $p^\tau_{ij}(x)$ be the probability that a reviewing i-strategist really changes to some pure strategy $j \neq i$, $\forall j \in \{c, nc\}$. Thus, $r^\tau_i(x)p^\tau_{ij}(x)$ is the probability that an agent changes from the $i-$th to $j-$th club. In the following, $e_c = (1, 0)$ and $e_{nc} = (0, 1)$ will indicate vectors of pure strategies, c or nc, independently from population τ.

Thus, the *outflow* from club i in population τ is $q^\tau_i r^\tau_i(x)p^\tau_{ij}(x)$ and the *inflow* is $q^\tau_j r^\tau_j(x)p^\tau_{ji}(x)$, where $q^\tau_i = q^\tau x^\tau_i$ is the number of $i-$strategist agents from population τ and q^τ represents the whole population τ, hence $q^\tau = q^\tau_i + q^\tau_j$.

For any given population τ (assuming that the size of the population τ is constant) by the law of large numbers we can model these processes as deterministic flows.[2]

Rearranging terms, $\forall$ $i, j \in \{c, nc\}$, $j \neq i$, and $\tau \in \{R, M\}$ we get:

$$\dot{x}^\tau_i = r^\tau_j(x)p^\tau_{ji}(x)x^\tau_j - r^\tau_i(x)p^\tau_{ij}(x)x^\tau_i. \tag{3.1}$$

[1]A club can be defined as a voluntary group deriving mutual benefits from sharing some facts like: production costs, the members' characteristics, or a good characterized by excludable benefits (see Sandler and Tschirhart, 1997). In this thesis, a club is composed by the agents (players) who follow the same type or strategic profile.

[2]Since we consider a finite population setting the review times of an $i-$strategist constitute the arrival times of a Poisson proces with arrival rate $r^\tau_i(x)$, and therefore with such arrival times, the strategist adopts a pure strategy j according to $p^\tau_{ij}(x)$. Hence, for any strategist $i \neq j \in \{c, nc\}$ if their choices are statistically independent random variables, the aggregate arrival rate of the Poisson process of agents switching from pure strategy i to j is $r^\tau_i(x)p^\tau_{ij}(x)x^\tau_i$.

System (3.1) represents the interaction between agents with different behaviors, $i \in \{c, nc\}$, from asymmetric populations $\tau \in \{R, M\}$, under a certain environmental state, $x = (x^M, x^R)$. In fact, (3.1) shows the dynamics of polymorphic populations with agents changing their own behavior under imitation pressure. Furthermore, (3.1) allows us to analyze and to characterize the possible steady states of these populations.

Hence, the aim of this model is to capture an evolutionary stable equilibrium in which all members of the two different populations adopt a behavior that is the best possible given the behavior of the agent's own population and the characteristic of the other population. This is precisely a Nash equilibrium for our two population-player game.

Definition 9 *We represent by x a distribution[3] of probabilities on the possible behaviors of the agents on a given population τ Sometimes, following the language of the game theory, we call this vector x a strategy, or more precisely a mixed strategy.*

Define the real numbers:

$$u^M(x, y) = \sum_{j=1}^{n} \sum_{i=1}^{n} u^M(e_i, y) x_i y_j$$

and

$$u^R(y, x) = \sum_{i=1}^{n} \sum_{j=1}^{n} u^R(e_j, x) x_i y_j$$

where $u^M(e_i, y)$ is the agent's expected payoff from population M following the $i-th$ behavior, given that population R has the distribution y over the set of her own possible behaviors. Analogously for $u^R(e_j, x)$. Recall that asymmetry corresponds to migrants (M) and residents (R).

Definition 10 *In a two population-player normal form game with n different behaviors in each population, a pair of distributions (a strategy profile) (x, y) over the sets of different behaviors in each population, is a Nash equilibrium if the expected value $u^M(x, y) \geq u^M(z, y)$ for all $z \in \Delta$ and $u^R(y, x) \geq u^R(w, , x)$ for all $w \in \Delta$.*

Intuitively the definition means that x is a best response of population-player M when population-player R displays y, and in turns y is a best response of population-player R, given that player M is displaying x. $\Delta = \left\{x \in R_+^n : \sum_{i=1}^n x_i = 1\right\}$ is the set of possible distributions over the set of the different behaviors, (the share of each behavior) or pure strategies, in a given population.

[3]A distribution is a vector $x \in R_+^n$, $x = (x_1, ..., x_n)$ where $x_i \geq 0 \;\; \sum_{i=1}^n x_i \;\; i = 1, ...n$ being x_i the probability that an agent in a given population observes the $i-th$ behavior. Or, in the language of game theory, it is the probability that agents play the $i-th$ pure strategy .

Consider the following example. Imagine a country where two different populations inhabit the same area. Call them ‘migrants’ and ‘residents’. Both populations conform by disjoint "clubs" of agents with different behaviors, and the behavior of such agents may change by imitation. The main question here is *what kind of behavior will survive in the population if agents change because of imitative pressure?*

3.2 Imitation by Dissatisfaction

We introduce here the first evolutionary model - a simple imitation model. Each agent observes the performance of one other agent (see for instance, Alos-Ferrer and Weidenholzer, 2006; Bjornerstedt and Weibull, 1995 and Schlag, 1998; 1999). An agent’s decision to stick to a strategy/club or to change strategy is a function of the type of agent she encounters in her own population. For a model of pure imitation all reviewing agents adopt the strategy of *the first person that they meet in the street*, picking this person at random from the population.

We consider that:

(i) An agent’s decision depends upon the utility associated to her behavior, given the composition of the other population, represented by the notation $u^{\tau}(e_i, x^{-\tau})$ (where, τ represents the population which the agent following the $i-th$ behavior belongs and $-\tau \in \{R, M\},\ -\tau \neq \tau$) and on the characteristics of populations represented by $x = (x^R, x^M)$. So:

$$r_i^{\tau}(x) = f_i^{\tau}(u^{\tau}(e_i, x^{-\tau}), x). \tag{3.2}$$

We interpret the function $f_i^{\tau}(u^{\tau}(e_i, x^{-\tau}), x)$ as the propensity of a member of the i-th club considering switching membership as a function of the expected utility gains from switching. Agents with less successful strategies review their strategy at a higher rate than agents with more successful strategies.

(ii) Having opted for a change, an agent will adopt the strategy followed by the first population-player to be encountered (her neighbor), i.e., for any $\tau \in \{R, M\}$:

$$p(i \to j/\text{i considers a change to j}) = p_{ij}^{\tau} = x_j^{\tau},\ i, j \in \{c, nc\}. \tag{3.3}$$

Considering *(i)* and *(ii)*, equation (3.1) can be rewritten as:

$$\dot{x}_i^{\tau} = x_j^{\tau} f_j^{\tau}(u^{\tau}(e_j, x^{-\tau}))x_i^{\tau} - x_i^{\tau} f_i^{\tau}(u^{\tau}(e_i, x^{-\tau}))x_j^{\tau} \tag{3.4}$$

or

$$\dot{x}_i^{\tau} = (1 - x_i^{\tau})x_i^{\tau}\left[f_j^{\tau}(u^{\tau}(e_j, x^{-\tau})) - f_i^{\tau}(u^{\tau}(e_i, x^{-\tau}))\right]. \tag{3.5}$$

This is the general form of the dynamic system representing the evolution of a two-population and four club structure (two “clubs” in each population). It provides a system of four simultaneous equations with four state variables, where each state variable is the population share of the club members. However, given the normalization rule, $x_c^\tau + x_{nc}^\tau = 1$, for each $\tau = \{R, M\}$, equation (3.5) can be reduced to two equations with two independent state variables. Taking advantage of this property, from now onwards we use variables x_c^R and x_{nc}^M with their respective equations.

To grapple with the problem, let us assume, consistent with Weibull (1995), that f_i^τ is population specific, but the same across all its components independent of club membership. Assume, furthermore, that it is linear in utility levels. Thus, the propensity to switch will be decreasing in the level of the utility, i.e. $\forall\ i \in \{c, nc\}\ \tau \in \{R, M\}$:

$$f_i^\tau(u^\tau(e_i, x^{-\tau})) = \alpha^\tau - \boldsymbol{\beta}^\tau u^\tau(e_i, x^{-\tau}) \in [0, 1]$$

with $\alpha^\tau, \beta^\tau \geq 0$ and $\frac{\alpha^\tau}{\beta^\tau} \geq u^\tau(e_i, x^{-\tau})$. To get a full linear form, we assume that:

$$u^\tau(e_i, x^{-\tau}) = e_i A^\tau x^{-\tau},$$

In other words, utility is a linear function of both variables, through a population-specific matrix of weights or constant coefficients, $A^\tau \in \mathcal{M}_{2\times 2}$, ($\tau \in \{R, M\}$). This latter assumption implies that utility levels reflect population-specific (and therefore in principle different) properties, i.e., broadly speaking preference structures over their outcomes. This reduces the previous model to a much simpler version:

$$\dot{x}_m^\tau = \boldsymbol{\beta}^\tau x_m^\tau (1 - x_c^\tau) \left[(1, -1) A^\tau x_c^{-\tau}\right], \tag{3.6}$$

or in full:

$$\begin{aligned} \dot{x}_c^R &= \beta^R x_c^R (1 - x_c^R)(a^R x_c^M + b^R) \\ \dot{x}_c^M &= \beta^M x_c^M (1 - x_c^M)(a^M x_c^R + b^M) \end{aligned} \tag{3.7}$$

whose coefficients a^M and b^R depend upon the entries of the two population-specific matrices, A^M and A^R. The system (3.7) is a mere replicator dynamics.

3.2.1 Dynamic stability and ESS

System (3.7) admits five stationary states or dynamic equilibria, i.e.

$$(0, 0),\ (0, 1),\ (1, 0),\ (1, 1) \text{ and a positive interior equilibrium } (\bar{x}_c^R, \bar{x}_c^M)$$

where

$$\bar{x}_c^R = -\frac{b^R}{a^R}, \quad \bar{x}_c^M = -\frac{b^M}{a^M}.$$

In fact, the interesting case is when $\bar{P} = (\bar{x}_c^R, \bar{x}_c^M)$ is an equilibrium lying in the interior of the square $\mathcal{C} = [0, 1] \times [0, 1]$, which occurs when:

$$0 < -\frac{b^R}{a^R} < 1 \text{ and } 0 < -\frac{b^M}{\boldsymbol{a}^M} < 1$$

We can proceed to inquire about the stability of the five equilibria.

Recall Zeeman's (1992) result that an equilibrium of the replicator dynamics equations is an evolutionary equilibrium (equivalent to the locally asymptotically stable point in dynamic systems), which is an ESS (see for more details Shone, 2003).

Proposition 2 *The steady states* $(1,1)$ *and* $(0,0)$ *are asymptotically stable equilibria, and then ESS while* $(1,0)$ *and* $(0,1)$ *are non-stable nodes and* $(\bar{x}_c^R, \bar{x}_c^M)$ *is a saddle point when the consistent coefficients range are,*

$$\left\{ -\frac{1}{2} < \left(b^R,b^M\right) < 0 < \left(a^R, a^M\right) \right\}.$$

Proof. We can judge whether the five equilibria are ESSs via analyzing Jacobean Matrix of system. The Jacobean associated to the system (3.7) is:

$$J = \begin{bmatrix} \beta^R(1-2x_c^R)(a^R x_c^M + b^R) & \beta^R a^R x_c^R(1-x_c^R) \\ \beta^M a^M x_c^M (1-x_c^M) & \beta^M(1-2x_c^M)(a^M x_c^R + b^M) \end{bmatrix}$$

such values, of course, depend on the population specific matrices. Recall that equilibria fitting $\det(J) > 0$ and $tr(J) < 0$ are asymptotically stable, thus they are ESSs of the game. We thus have the following cases:

1. $x_c^R = x_c^M = 1$, the evaluated Jacobian in this case is,

$$J = \begin{bmatrix} -\beta^R(a^R + b^R) & 0 \\ 0 & -\beta^M(a^M + b^M) \end{bmatrix}.$$

   When $\left(b^R, b^M\right)$ are positive and $\left(a^R, a^M\right)$ negative numbers. The determinant is:

$$\begin{aligned} \det(J) &= \left(-\beta^R(a^R + b^R)\right) \cdot \left(-\beta^M(a^M + b^M)\right) > 0 \\ tr(J) &= -\beta^R(a^R+b^R) - \beta^M(a^M+b^M) < 0, \end{aligned}$$

   and then the equilibrium point $(1,1)$ is a stable node. Otherwise, if $\left(b^R, b^M\right)$ are negative and $\left(a^R, a^M\right)$positive numbers, then, $\det(J) > 0$ and $tr(J) < 0$, and then the equilibrium point $(1,1)$ is asymptotically stable. Therefore, $(1,1)$ is always a stable node.

2. $x_m^R = x_m^M = 0$, the evaluated Jacobian is

$$J = \begin{bmatrix} \beta^R b^R & 0 \\ 0 & \beta^M b^M \end{bmatrix}. \tag{3.8}$$

Note that, if $\left(b^R, b^M\right)$ are positive and $\left(a^R, a^M\right)$ negative numbers, it implies that

$$\begin{aligned} \det(J) &= \left(\beta^R b^R\right) \cdot \left(\beta^M b^M\right) > 0 \\ tr(J) &= \left(\beta^R b^R\right) + \left(\beta^M b^M\right) > 0, \end{aligned}$$

and then this equilibrium point $(0,0)$ is a non-stable node. With $\left(b^R, b^M\right)$ negative and $\left(a^R, a^M\right)$ positive numbers, $\det(J) > 0$ and $tr(J) < 0$. Therefore, the equilibrium point $(0,0)$ is an asymptotically stable.

3. $x_m^R = 1$, $x_m^M = 0$, the evaluated Jacobian is

$$J = \begin{bmatrix} -\beta^R b^R & 0 \\ 0 & \beta^M (a^M + b^M) \end{bmatrix}. \tag{3.9}$$

If $\left(b^R, b^M\right)$ are positive and $\left(a^R, a^M\right)$ negative numbers, it implies that

$$\begin{aligned} \det(J) &= \left(-\beta^R b^R\right) \cdot \left(\beta^M (a^M + b^M)\right) > 0 \\ tr(J) &= \left(\beta^R b^R\right) + \left(\beta^M b^M\right) < 0, \end{aligned}$$

and then the equilibrium point $(1,0)$ is an asymptotically stable node. Otherwise, if $\left(b^R, b^M\right)$ are negative and $\left(a^R, a^M\right)$ positive numbers, then, $\det(J) > 0$ and $tr(J) > 0$, and therefore the equilibrium point $(1,0)$ is an unstable node, which is consistent with the fact that $(0,0)$ and (1,1) are asymptotically stable nodes.

4. $x_m^R = 0$, $x_m^M = 1$, the evaluated Jacobian is

$$J = \begin{bmatrix} \beta^R (a^R + b^R) & 0 \\ 0 & -\beta^M b^M \end{bmatrix}. \tag{3.10}$$

If $\left(b^R, b^M\right)$ are positive and $\left(a^R, a^M\right)$negative numbers, it implies that

$$\det(J) = \left(\beta^R (a^R + b^R)\right) \cdot \left(-\beta^M b^M\right) > 0$$

$$tr(J) = \left(\beta^R (a^R + b^R)\right) + \left(-\beta^M b^M\right) < 0,$$

and then the equilibrium point $(0,1)$ is an asymptotically stable node. Otherwise, if $\left(b^R, b^M\right)$ are negative and $\left(a^R, a^M\right)$ positive numbers, then, $\det(J) > 0$ and $tr(J) > 0$, and then this equilibrium point $(0,1)$ is an unstable node, which is consistent with the fact that (0,0) and (1,1) are asymptotically stable nodes.

5. The interior equilibrium $\bar{x}_m^R = -\frac{b^R}{a^R}$ and $\bar{x}_m^M = -\frac{b^M}{a^M}$. Evaluating this point in the Jacobian yields,

$$J = \begin{bmatrix} \beta^R(1+2\frac{b^R}{a^R})(-a^R\frac{b^M}{a^M}+b^R) & -\beta^R b^R(1+\frac{b^R}{a^R}) \\ \\ -\beta^M b^M(1+\frac{b^M}{a^M}) & \beta^M(1+2\frac{b^M}{a^M})(-a^M\frac{b^R}{a^R}+b^M) \end{bmatrix}. \tag{3.11}$$

If $-\frac{1}{2}< b^R< 0$ and $-\frac{1}{2}< b^M< 0$ and $\left(a^R, a^M\right)$ positive numbers, then, $\det(J) < 0$. Therefore, this equilibrium point $\left(\bar{x}_c^R, \bar{x}_c^M\right)$ is a saddle point.

■

These equilibria can be interpreted as follows:

- The trivial equilibrium obtains when none of the residents is inclined to mix with migrants, nor is any migrant inclined to mix with residents: $(0, 0)$
- Another equilibrium obtains at the opposite corner, where the sharing clubs involve all of their respective population. Here, reciprocal integration of the two populations is complete at $(1, 1)$. The two remaining border equilibria show a different club dominating the two populations and in a sense constitute a mismatch between strategies: $(0, 1)$, $(1, 0)$.
- Finally, we have the interior equilibrium. This equilibrium implies that a certain percentage of each population is well-disposed to the other population while the rest only accept marriage between members of their own populations: $\bar{P} = (\bar{x}_c^R, \bar{x}_c^M)$.

3.3 Adopting the Most Successful Strategy

The rule now states that an agent must choose an action with the best payoff with a probability proportional to the expected payoff. In other words, a migrant (resident) will change to a strategy played by another member of her population if and only if the alternative strategy brings greater expected utility, i.e. a greater payoff. We assume that every reviewing strategist samples at random from her own population and that she can observe, with some noise, an average payoff of the sampled agent's strategy.

Let us assume that a reviewing strategist, from population $\tau \in \{R, M\}$, with behavior (club) $j \in \{c, nc\}$, meets another agent with an alternative behavior (club) $i \neq j$, and that, she will change from j to i if and only if:

$$u^\tau(e_i, x^{-\tau}) > u^\tau(e_j, x^{-\tau}). \tag{3.12}$$

The utility of each agent depends on her own strategy and on the characteristics of the agents of the other population. Further, we assume that there is some uncertainty in the strategies' estimated utility, so that each reviewing strategist must estimate the value of $u^\tau(e_i, x^{-\tau})$.

Let D be the estimator for the difference $u^\tau(e_i, x^{-\tau}) - u^\tau(e_j, x^{-\tau})$. Let $P^\tau(\bar{D} \geq 0)$ be the probability that the estimated $\bar{D} = u^\tau(e_i, x^{-\tau}) - \bar{u}^\tau(e_j, x^{-\tau})$ is positive.

In such a way, let p_{ji}^τ be the probability that j changes to i. This probability is given by

$$p_{ji}^\tau = x_{ji}^\tau P^\tau(\bar{D} \geq 0),$$

where x_{ji}^τ is the probability that j-strategist meets an i-strategist, $\forall\ \tau \in \{R, M\}$. Let us assume that $P^\tau(\bar{D} \geq 0)$ depends upon the true value of the difference $u^\tau(e_i, x^{-\tau}) - u^\tau(e_j, x^{-\tau})$ which is unknown to the i-th agent. That is

$$P^\tau(\bar{D} \geq 0) = \phi^\tau\left(u^\tau(e_i, x^{-\tau}) - u^\tau(e_j, x^{-\tau})\right). \tag{3.13}$$

where $\phi : R \to [0, 1]$ is a differentiable probability distribution. Therefore, the probability that a j-strategist from population τ estimates a positive value $\bar{D}$ increases with the true value of the difference $u^\tau(e_j, x^{-\tau}) - u^\tau(e_i, x^{-\tau})$. For the sake of simplicity, let $u^\tau(e_i, x^\tau)$ be linear, i.e.,

$$u^\tau(e_i, x^{-\tau}) = e_i A^\tau x^{-\tau}. \tag{3.14}$$

Thus, from equations (3.13) and (3.14) the probability that a j-strategist changes to the i-th strategy is:

$$p_{ji}^\tau(u^\tau(e_i - e_j, x^{-\tau})) = \phi^\tau(u^\tau(e_i - e_j, x^{-\tau}))x_i^\tau, \tag{3.15}$$

Hence, the change in the share of the i-strategist will be given by the probability that a j-strategist becomes an i-strategist weighted by the relative number of j-strategists minus the probability that an i becomes a j-strategist, likewise weighted:

$$\dot{x}_i^\tau = \left[x_j^\tau p_{ji}^\tau - p_{ij}^\tau x_i^\tau\right] x_i^\tau.$$

In this case, the equation is:

$$\dot{x}_i^\tau = x_j^\tau x_i^\tau \left[\phi^\tau(u^\tau(e_j - e_i, x^{-\tau})) - \phi^\tau(u^\tau(e_i - e_j, x^{-\tau}))\right], \tag{3.16}$$

and a first order approximation is given by:[4]

$$\dot{x}_i^\tau = x_i^\tau x_j^\tau[\tau\phi^{\tau\prime}(0,)\left[u^\tau(e_j - e_i, x^{-\tau}) - u^\tau(e_j - e_i, x^{-\tau}))\right] =$$

$$= 2\phi^{\tau\prime}(0)u^\tau(e_i - x^\tau, x^{-\tau})x_i^\tau. \tag{3.17}$$

Hence, in a neighborhood of an interior stationary point, the dynamics are approximately represented by the replicator dynamics multiplied by a

[4]Note that for this dynamic the set of interior stationary state coincides with the set of interior Nash equilibria.

constant. Stability analysis of the local type can therefore be carried out using the linear part of the nonlinear system.[5]

In the special case where ϕ^τ is linear,

$$\phi^\tau = \lambda_\tau + \mu^\tau u^\tau(e_j - e_i, x^\tau)$$

where λ^τ and μ^τ verify:

$$0 \leq \lambda^\tau + \mu^\tau u(x^\tau, x^{-\tau}) \leq 1, \ \forall \, x \in \left\{ z \in R_+^2 : \max_{i=1,2} z_i \leq 1 \right\}.$$

Thus, for each behavior i and each population τ the following equation holds:

$$\dot{x}_i^\tau = 2\mu^\tau u^\tau(e_i - x^\tau, x^{-\tau}) x_i^\tau, \tag{3.18}$$

which is merely a replicator dynamic. Hence, stability analysis is similar to the analysis of the model of pure imitation driven by dissatisfaction. Thus, it is more important whom an agent imitates than how an agent imitates

3.4 Evolutionarily Stable Strategies

A strategy is an evolutionarily stable strategy (ESS) if a whole population playing that strategy cannot be invaded by a small group with a mutant genotype. Similarly, a cultural form is an ESS if, upon being adopted by all members of a society (firm, family, etc.), no small group of agents using an alternative cultural form can invade. As Gintis (2009) noted, we move from explaining the actions of agents to modeling the diffusion of forms of behavior ("strategies") in society.

We now introduce a definition of ESS in the framework of our model were there are two populations playing an asymmetric normal form game.

To introduce this concept in our model, let us consider the following representation: Let $N_{\tau j}$ be the total of $j-$strategists, $j \in \{c, nc\}$, in population, $\tau \in \{M, R\}$, and let $H = N_{Mc} + N_{Mnc} + N_{Rc} + N_{Rnc}$ be the total inhabitants in the country. Let us denote by $x_{1R} = \frac{N_{Rc}}{H}$, $x_{2R} = \frac{N_{Rnc}}{H}$, $x_{1M} = \frac{N_{Mc}}{H}$, $x_{2M} = \frac{N_{Mnc}}{H}$. Then, we denote by

$$\Delta = \left\{ x \in R_+^4 : x_{1R} + x_{2R} + x_{1M} + x_{2M} = 1 \right\}$$

the $(k-1)-$simplex of R_k in our case $k = 4$. Also, we introduce the symbolism: $x = (x^R, x^M)$ where $x^\tau = (x_{1\tau}, x_{2\tau}); \forall \tau \in \{M, R\}$.

Let us now introduce the following notation:

$$\bar{x}_c^R = \frac{H}{|R|} x_{1R}, \ \bar{x}_{nc}^R = \frac{H}{|R|} x_{2R}, \ \bar{x}_c^M = \frac{H}{|M|} x_{1M}, \ \bar{x}_{nc}^M = \frac{H}{|M|} x_{2M}$$

where $|R|$ is the total number of agents in population R, and $|M|$ is the total of agents in population M, and $\bar{x}^\tau = (\bar{x}_c^\tau, \bar{x}_{nc}^\tau), \ \forall \, \tau \in \{R, M\}$.

[5] If the equilibrium is non hyperbolic.

Definition 11 *We say that $x \in \Delta$ is an **evolutionarily stable strategy (ESS)**, if and only if for every $y \in \Delta$, $y \neq x$ there exists some $\bar{\epsilon}_y \in (0,1)$ such that for all $\epsilon \in (0, \bar{\epsilon}_y)$ and with $w = \epsilon y + (1-\epsilon)x$ where $w = (w^R, w^M) = \epsilon(y^R, y^M) + (1-\epsilon)(x^R, x^M)$, then:*

$$u^\tau(\bar{x}^\tau, \bar{w}^{-\tau}) > u^\tau(\bar{y}^\tau, \bar{w}^{-\tau}), \ \ \forall\, \tau \in \{R, M\}\,.$$

Intuitively, we say that a distribution x is an ESS if the incumbent strategy x^τ does better in the post-entry population than the alternative strategy y^τ $\forall \tau$. In our definition, agents of a given population do not play against agents in their own population, so the second order conditions in the definition of an ESS become superfluous.

The next theorem shows the equivalence between our definition of ESS in the framework of asymmetric normal form games and the definition of strict Nash equilibrium.[6]

Theorem 3 *If the distribution $x = (x_{1R}, x_{2R}, x_{1M}, x_{2M})$ is an ESS in the sense of **Definition 11** then, the profile of mixed strategies $\bar{x} = (\bar{x}^R, \bar{x}^M) = (\bar{x}^R_c, \bar{x}^R_{nc}, \bar{x}^M_c, \bar{x}^M_{nc})$ is a strict Nash equilibrium for the two population normal form game and conversely, for each strict Nash equilibrium $\bar{x}$, there exists a distribution x that is an ESS.*

Proof. Let $S^\tau, \tau \in \{M, R\}$ be the set of mixed strategies of the populations M and R. The theorem follows immediately from our definition of ESS (**Definition 11**), the definition of strict Nash equilibrium and the continuity of the functions $u_\tau : S^\tau \times S^{-\tau} \to R$. Note that from the definition of ESS it follows that $u_\tau(\bar{x}^\tau, \bar{x}^{-\tau}) > u_\tau(\bar{y}^\tau, \bar{x}^{-\tau})$ $\forall\, y^\tau \neq x^\tau \in S^\tau$ and $\forall \tau$ so, $\bar{x}^\tau$ is the only best response against $\bar{x}^{-\tau}$ $\tau \in \{M, R\}$. ■

Note that a distribution $x \in \Delta$ defines an interior stationary point for each one of the dynamical systems (3.18)if and only if

$$u^\tau(e_i, \bar{x}^{-\tau}) = u(\bar{x}^\tau, \bar{x}^{-\tau}).$$

This means that x^τ is a best response for population $\tau = \{M, R\}$ against $x^{-\tau}$ so, the profile $(x^\tau, x^{-\tau})$ is a Nash equilibrium. And conversely, if the profile $(x^\tau, x^{-\tau})$ is a Nash equilibrium then, it is a stationary point for these dynamical systems.

Theorem 4 *If the distribution $x = (x_{1R}, x_{2R}, x_{1M}, x_{2M})$ is an ESS in the sense of **Definition 11**, then the profile of mixed strategies $\bar{x} = (\bar{x}^R, \bar{x}^M)$ is a globally asymptotically stable stationary point of (3.7) or (3.18).*

Proof. Consider the relative entropy function $H_x : Q_x \to R$, where Q_x is the set of the distributions $y \in \Delta$ that assign probability positive to

[6] Recall that a Nash equilibrium $s = (x, y)$ is called strict if and only the profile s is the unique best reply against itself.

every state that has probability positive in x. In fact Q_x is the union of the interior of the simplex Δ an the minimal boundary face containing x.

$$H_x(y) = -\sum_{i\in C(x)} x_i \ln\left(\frac{y_i}{x_i}\right) = -\sum_{i\in C(x^M)} x_i^M \ln\left(\frac{y_i^M}{x_i^M}\right) - \sum_{i\in C(x^R)} x_i^R \ln\left(\frac{y_i^R}{x_i^R}\right)$$

By $C(x)$ we denote the support of x, i.e. the set of coordinates not equal to 0.

As it is easy to see, if x is ESS, then $H_x(y)$ verify:

1. $H_x(y) = 0$ if and only if $y = x$.
2. $\dot{H}_x(y) = -\sum_{i=1}^4 \dot{y}_i x_i =$

   $$= -k\left[\left(u^M(x^M, y^R) - u^M(y^M, y^R)\right) + \left(u^R(x^R, y^M) - u^R(y^R, y^M)\right)\right],$$

   where:

   $$k = \begin{cases} \beta^\tau \text{ if the system is (3.7),} \\ \\ 2\mu^\tau \text{ if the system is by (3.18).} \end{cases}$$

3. Thus, if x is an ESS, then $\dot{H}_x(y) < 0\ \forall\ y \in U \cap Q_x$ where U is a neighborhood of x.

■

In this context this theorem make know that, when a population has a distribution that is an ESS, if this composition is perturbed (in a relative neighborhood of x) then the performance of all strategies in this population decreases with respect to the original distribution x. In other words, if x is an ESS, then, not only is it the best distribution within each group, given the behavior of the population in the given country, it is also a stable strategy in the sense that every change in the distribution of the behavior within a group, implies worse payoff for the deviating group. Moreover, if x is an ESS and the corresponding $\bar{x}_c = (\bar{x}_c^R, \bar{x}_c^M)$ is an interior stationary point for the dynamical system given by (3.7) or by (3.18), it follows that for every solution $\xi(t, z_0)$ of these systems of equations, with initial conditions z_0 in the interior of the square $[0,1] \times [0,1]$, implies that $\xi(t, z_0)_{t\to\infty} \to \bar{z}$. This means that $\bar{z}$ is a globally asymptotically stable stationary point, in the sense of attracting all interior initial state. Hence, if the distribution of the population $x = (x_{1R}, x_{2R}, x_{1M}, x_{2M})$ in $t = t_0$ is perturbed to a new distribution y, then the evolution of this perturbed distribution, whose state in each time t is denoted by $\phi(t, y)$, where $\phi(t_0, y) = y$, evolves according to one of the dynamical systems (3.7) or (3.18) , y will always approach the initial distribution x.

3.5 The Specific Behavioral Rule

An evaluation rule that seems particularly natural in a context of simple imitation is the "average rule" where each strategy is evaluated according to the average payoff observed in the reference group (see J. Apesteguia et al., 2007).

Let us consider the **Definition 8 of a Behavioral Rule** but now we define p_{ij}^{τ} as the probability of finding a better performed strategy j with finding or meeting to such better performed agent, x_j^{τ}.

We assume that there is some uncertainty in the strategies' estimated utility, so that each reviewing strategist, $j \neq i \in \{c, nc\}$, should estimate their corresponding value of $u^{\tau}(\cdot)$. Let $\bar{u}^{\tau}(e_j, \cdot)$ and $\bar{u}^{\tau}(e_i, \cdot)$ be the estimators of the real values $u^{\tau}(e_j, \cdot)$ and $u^{\tau}(e_i, \cdot)$. Then, we propose the following statement:

Definition 12 (Specific Behavioral rule) *A reviewing strategist i changes to j with probability, p_{ij}^{τ}, equal to*

$$P[\bar{u}^{\tau}(e_j, \cdot) > \bar{u}^{\tau}(e_i, \cdot)]x_j^{\tau} = \begin{cases} \lambda u^{\tau}(e_j, x^{-\tau})x_j^{\tau} & if \;\; u^{\tau}(e_j, x^{-\tau}) > 0 \\ \\ 0 & if \;\; u^{\tau}(e_j, x^{-\tau}) \leq 0 \end{cases} \tag{3.19}$$

where $\lambda = \frac{1}{|u^{\tau}(e_i, x^{-\tau}) + u^{\tau}(e_j, x^{-\tau})|}$, $\forall\ -\tau \neq \tau \in \{R, M\}$ *and* $i \neq j \in \{c, nc\}$.

Therefore, $\forall\ i \neq j \in \{c, nc\}, -\tau \neq \tau \in \{R, M\}$, the equations (3.5 and 3.18) can be rewritten as,

$$\dot{x}_i^{\tau} = f_j^{\tau}(u^{\tau}(e_j, x^{-\tau}))\lambda u^{\tau}(e_i, x^{-\tau})x_i^{\tau}x_j^{\tau} - f_i^{\tau}(u^{\tau}(e_i, x^{-\tau}))\lambda u^{\tau}(e_j, x^{-\tau})x_j^{\tau}x_i^{\tau} \tag{3.20}$$

To simplify, let us label the functions $u^{\tau}(e_i, x^{-\tau}) = u_i^{\tau}$ and $u^{\tau}(e_j, x^{-\tau}) = u_j^{\tau}$, then, we can write,

$$\dot{x}_i^{\tau} = (1 - x_i^{\tau})x_i^{\tau}\lambda\left[f_j^{\tau}(u_j^{\tau})u_i^{\tau} - f_i^{\tau}(u_i^{\tau})u_j^{\tau}\right]. \tag{3.21}$$

Once again consider that f_i^{τ} is population specific, thus, the propensity to switch behavior is decreasing in the level of the utility (see Weibull,1995), that is,

$$f_i^{\tau}(u_i^{\tau}) = \alpha^{\tau} - \boldsymbol{\beta}^{\tau}u_i^{\tau}, \tag{3.22}$$

with $\alpha^{\tau}, \beta^{\tau} \geq 0$ and $\frac{\alpha^{\tau}}{\beta^{\tau}} \geq u_i^{\tau}$ assure that $f_i^{\tau}(u_i^{\tau}) \in [0, 1]$. Thus, with respect to the utility level of the $i-$strategist, u_i^{τ} increases her average reviewing rate, and $r_i^{\tau}(x) = f_i^{\tau}(u_i^{\tau})$ will decrease.

Therefore, considering the above behavioral rule (Definition 12) and equation (3.22), we obtain the system of replicator dynamic equations driven by imitation, i.e.,

$$\dot{x}_i^\tau = \lambda x_i^\tau (1 - x_i^\tau) \left[\alpha^\tau \left(\frac{u_i^\tau - u_j^\tau}{u_i^\tau + u_j^\tau} \right) \right]. \tag{3.23}$$

Or in full,

$$\begin{cases} \dot{x}_c^R = -\dot{x}_{nc}^R = \alpha^R x_c^R (1 - x_c^R) \left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R} \right) \\ \\ \dot{x}_c^M = -\dot{x}_{nc}^M = \alpha^M x_c^M (1 - x_c^M) \left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M} \right) \end{cases} \tag{3.24}$$

System (3.24) describes the replicator dynamic model as a process where only those strategies that are better than the population average can replicate. The informational assumptions embodied in this proposed behavioral rule are weak because the only thing a revising agent needs to know is the strategy of the randomly met opponent. The case where strategies propagate via imitation, where the payoffs are what drives the rate of imitation, reinforcement, and inhibition of behaviors: to cooperate c or not nc among agents of two populations; residents R and migrants M.

The system (3.24) $\left(\dot{x}_c^R, \dot{x}_c^M\right)$ admits five stationary states or dynamic equilibria, i.e.,

$$(0,0),\ (0,1),\ (1,0),\ (1,1) \text{ and a positive interior equilibrium } (\bar{x}_c^R, \bar{x}_c^M)$$

where

$$\bar{x}_c^R = \alpha^R, \quad \bar{x}_c^M = \alpha^M$$

is the mixed-strategy Nash equilibrium and, $\bar{P} = (\bar{x}_c^R, \bar{x}_c^M)$ lying in the interior of the square $\mathcal{C} = [0,1] \times [0,1]$, or $0 < \alpha^R < 1$ and $0 < \alpha^M < 1$.

3.5.1 Evolutionary dynamics

We should identify all the attractors and saddle points, as well as the basins of attraction. The volume of the space covered by a basin is proportional to the probability that a mixed-Nash equilibrium will be adopted. Thus, basins of attraction provide insights into which equilibria are more likely to be adopted depending on the state.

To study the dynamics, we should have a functional form of $u^\tau(e_i, x^{-\tau}) = u_i^\tau$ and $u^\tau(e_j, x^{-\tau}) = u_j^\tau$. For instance, $\forall\ i \neq j \in \{c, nc\}, -\tau \neq \tau \in \{R, M\}$. We can assume the linear form,

$$u_i^\tau = a^\tau x_i^{-\tau} - (1 - a^\tau)\, x_j^{-\tau} + b^\tau,$$

where $a^\tau \in [0,1]$ and $b^\tau > 0$, with strategic complementarities across populations $\frac{\partial u_i^\tau}{\partial x_i^{-\tau}} > 0$ and $\frac{\partial u_j^\tau}{\partial x_j^{-\tau}} > 0$.

The next proposition summarizes our main result,

Proposition 3 The steady states $(1,1)$ and $(0,0)$ are asymptotically stable equilibria, while $(1,0)$ and $(0,1)$ are non-stable nodes. The mixed-strategy Nash equilibrium $\bar{P} = (\bar{x}_c^R, \bar{x}_c^M)$ is a saddle point and, with the exception of a single curve through this point, all solution trajectories converge to $(1,1)$ or $(0,0)$.

Proof. We can judge whether the five equilibria are ESSs via analyzing Jacobean Matrix of system. The Jacobean associated to the system $\left(\dot{x}_c^R, \dot{x}_c^M\right)$ is,

$$J = \begin{bmatrix} \alpha^R - \left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R}\right) & \alpha^R x_c^R (1 - x_c^R) \cdot \frac{\partial\left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R}\right)}{\partial x_c^M} \\ \alpha^M x_c^M (1 - x_c^M) \cdot \frac{\partial\left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M}\right)}{\partial x_c^R} & \alpha^M - \left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M}\right) \end{bmatrix}$$

such values, of course, depend on the population specific matrices. Recall that equilibria fitting $\det(J) > 0$ and $tr(J) < 0$ are asymptotically stable, thus they are ESSs of the game.

We evaluate $J\left(\dot{x}_c^R, \dot{x}_c^M\right)$,

1. $x_c^R = x_c^M = 0$. The evaluated Jacobean in this case is given by, $J = \begin{bmatrix} \alpha^R - \left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R}\right) & 0 \\ 0 & \alpha^M - \left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M}\right) \end{bmatrix}$
It yields $\det(J) > 0$ and $tr(J) < 0$. Hence this equilibrium point (0,0) is an attractor and therefore ESS.

2. Similarly when $x_c^R = x_c^M = 1$, the evaluated Jacobean is, $J = \begin{bmatrix} \alpha^R - \left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R}\right) & 0 \\ 0 & \alpha^M - \left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M}\right) \end{bmatrix}$
and $\det(J) > 0$ and $tr(J) < 0$. Therefore, the cooperation equilibrium (1,1) is asymptotically stable and hence an ESS.

3. The interior equilibrium $\bar{x}_c^R = \alpha^R$ and $\bar{x}_c^M = \alpha^M$. Evaluating this point in the Jacobean yields,
$$J = \begin{bmatrix} \alpha^R - \left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R}\right) & \left(\alpha^R\right)^2 (1 - \alpha^R) \cdot \frac{\partial\left(\frac{u_c^R - u_{nc}^R}{u_c^R + u_{nc}^R}\right)}{\partial x_c^M} \\ \left(\alpha^M\right)^2 (1 - \alpha^M) \cdot \frac{\partial\left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M}\right)}{\partial x_c^R} & \alpha^M - \left(\frac{u_c^M - u_{nc}^M}{u_c^M + u_{nc}^M}\right) \end{bmatrix},$$
by strategic complementarities: $\det(J) < 0$, thus the equilibrium point $\bar{P} = (\bar{x}_c^R, \bar{x}_c^M)$ is a saddle point.

Therefore, both equilibria $(0,0)$ and $(1,1)$ are ESSs and they cannot be invaded by any possible mutation or mutant strategy. Moreover, $\bar{P} =$

$(\bar{x}_c^R, \bar{x}_c^M)$ is a threshold, since, it separates the basins of attraction of the low-level and high-level equilibria. ■

Figure 3 gives a graphic representation of the solution orbits of the standard two-population replicator dynamics $\left(\dot{x}_c^R, \dot{x}_c^M\right)$ driven by imitation,

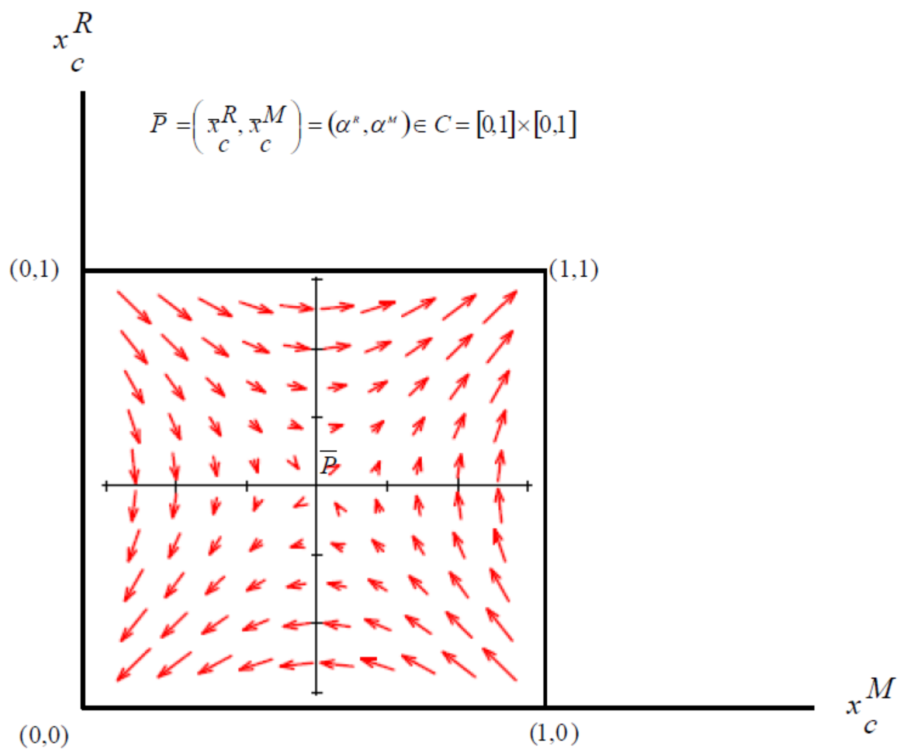

Figure 3. Solution orbits from $\left(\dot{x}_c^M, \dot{x}_c^R\right)$.

Note that, the hypothetical location of the point $\bar{P} = (\bar{x}_c^R, \bar{x}_c^M)$ looks at the center of the $\mathcal{C}-$square, but this is not necessarily its real location, since it depends on the values α^R and α^M which are the constant terms of the reviewing decision $r_i^\tau(x) = f_i^\tau(u_i^\tau)$ of switching behavior.

3.6 Concluding Remarks

We showed that imitative behavior can be represented by the replicator dynamics with two specific cases: (i) pure imitation driven by dissatisfaction where all reviewing agents imitate the first person they encounter, and (ii) the successful reviewing strategist chooses the actions with the best payoffs, with a probability proportional to the expected payoff.

As long as the linear approximation is mathematically valid, the second differs from the first, yet we obtain similar evolutionary dynamics for the two models. Similar conclusions can be applied to the relationship between ESS, Nash equilibrium and the Replicator Dynamics.

Hence, we conclude that, for imitation rules that use random sampling, **"whom an agent imitates is more important than how an agent imitates"**. We justify the extension of the simpler model based on a description of an observed behavior: agents choose an expected maximization of benefits, since *they do what others do when they can imitate successful strategies.*

Finally, we introduced the concept of ESS for a model of two asymmetric populations and we show that for every ESS there is a strict Nash equilibrium and conversely.

For further characterization results on the set of Evolutionary Stable Strategies and Nash equilibrium, we refer the reader to Balkenborg and Schlag (2007) and the texts by Van Damme (1991) or Vega-Redondo (1996) and Weibull (1995).

We presented a specific form of the behavioral rule, in which agents imitate according to the best performed strategy when they meet agents performing better than they do. We showed that imitation is represented by the replicator dynamic system since agents do what others do by imitating successful strategies. We characterized the evolutionarily stable strategies of the replicator dynamics. Thus we must add a caveat to our original conclusion, who an agent imitates is more important than how an agent imitates, but the specific 'how' an agent adopts can affect their relative success too.

Part II

Evolutionary Dynamics of Poverty Traps

CHAPTER 4

THE EVOLUTION OF POVERTY TRAPS

This chapter proposes a coordination game between two different types of economic agents: the high and low profile economic agents. The agents follow an imitative behavior supporting the "membership theory" and thus they adopt actions conforming to a convention. Samuel Bowles (2006) pointed out the notion of institutional poverty traps meaning that an institution is composed as one of a number of possible conventions. Consider equilibria in which members of a population typically act in ways that maximize payoffs given the actions taken by others and in which agents' beliefs about what others will do; these factors support continued adherence to conventional actions. A convention is thus an outcome in which it is in the interest of agents to adhere to the convention as long as they believe that most others will do the same. Because a convention is one of many possible mutual best responses, institutions are not environmentally determined, but rather are of human construction (but not necessarily of deliberate design). Conventions are self-enforcing, that is they are evolutionarily stable, as a result of the positive feedback associated with members' conforming to a common strategy. Consequently, institutions that take the form of conventions will display inertia, and transitions among them will occur rapidly but infrequently, displaying a pattern that biologists call punctuated equilibria (see Eldredge and Gould, 1972).

Hence, we aim to explain the notion of institutional poverty traps from a coordination game in an evolutionary game theoretical framework.

4.1 The Game

Let us allow economic agents to be either leaders, (1), or followers, (2), with two different types: high- or low-profile. The vectors $(H, L)\,;(h, l)$ are the strategy spaces, S_i, denoting high- and low-profiles types of leaders and followers, $i = 1, 2$.[1]

Let us assume that a contractual period is characterized by:

1. **Strategic complementarities**. In this economy, to profit the player 1 must employ 2 under strategic complementarity, in the sense that the $H-$type agent matching with an $h-$type is more profitable than matching with a $l-$type. Analogously, the $L-$type agent matching with a l-type is more profitable than matching with an $h-$type.

[1] Leader and follower games are described by imperfect information in which the leader moves fist knowing the type of the follower but then followers are the second movers without knowing the type of the leader (see Fudenberg and Tirole, 1991).

(a) A gross income of 1 being an $H-$type is U or u, and of being a $L-$type is V or v, which depends on matching with high- or low-profile followers.

(b) A gross income of 2 is W when hired by H and w when is hired by L.

(c) Agents 1 and 2 face income taxes γ and τ.

2. **Private information**. The leader knows the types of the followers, but the followers do not know the type of the leaders and they assume with probability σ that they will be hired by a leader of $H-$type and with probability $(1-\sigma)$ by a $L-$type.
3. Choosing between high- and low-profiles does not have any cost for the leader. While if the follower decides to become an $h-$type they incur a training cost or cost of education C, and if they decide to be a $l-$type does not incur any costs.
4. At the end of the contractual period the $H-$type leader must give a signal p to the $h-$type follower.[2] Such a signal can be viewed as giving some skill premium, bonus, or efficiency wage, which are perceived only by $h-$type followers and received at the end of the contractual period.

A normal-form representation of this game is presented in the following payoff matrix,

$2 \setminus 1$	**H**	**L**
h	$(1-\tau)\,W + p - C,\ (1-\gamma)\,U - W - p$	$(1-\tau)\,w - C,\ (1-\gamma)\,v - w$
l	$(1-\tau)\,W,\ (1-\gamma)\,u - W$	$(1-\tau)\,w,\ (1-\gamma)\,V - w$

Note that, the premium or bonus p, is bounded on,

- $p > C > 0$, incentives to be a high-profile follower and it is payoff dominant for a low-profile follower, or,

$$0 < C - (1-\tau)(w - W) < p.$$

  Which means that $h-$type followers must get a bonus greater than the net difference of wages to being high- or low-profile.

- The $H-$type leaders cannot give a bonus or efficiency wage greater than the net difference of gains to being high- or low-profile:

$$p < (1-\gamma)(U - u).$$

[2] Signaling games are incomplete information leader-follower games in which only the leader has private information. (see Fudenberg and Tirole, 1991).

The game has two pure Nash equilibria:

$$(H,h) = (\,(1,0)\,;(1,0)\,) \text{ and } (L,l) = ((0,1)\,;(0,1))\,,$$

the former is the payoff dominant while the latter is the risk dominant. There is a mixed strategy Nash equilibrium given by,

$$(\theta,(1-\theta);\ \sigma,(1-\sigma))\,,$$

where θ is the leader's probability of matching a high-type follower, and σ is the follower's probability of matching a high-type leader. That is,

$$\sigma = \frac{C}{p} \text{ and } \theta = \frac{(1-\gamma)(V-u)+W-w}{(1-\gamma)(U-u+V-v)-p}$$

Since our model can be studied as an extensive-form game, with a finite game tree consisting of nodes X. Let $z \in Z \subset X$ be the terminal nodes. A pure strategy for player $i \in (1,2)$, s_i, is an action at each node in $x \in X_i$, $s_i(x) \in S = \{(H,L)\,;(h,l)\}$.

We can now find the self-confirming equilibria (SCE), since they are based on the idea that players should have correct beliefs about probability distributions that they observe sufficiently often (Fudenberg & Levine, 1993; 2007). We suppose that all players know the structure of the extensive form, so that each player knows the space S of strategy profiles. Each player i receives a payoff in the stage game that depends on the terminal node. Player i's payoff function is denoted $\pi_i : Z \to \mathbb{R}$. Let $\Delta(\cdot)$ denote the space of probability distributions over a set. Then a mixed strategy profile is $\Phi \in \Delta(S_1) \times \Delta(S_2)$. Let the nodes that are reached with positive probability under Φ be denoted by $\bar{X}(\Phi)$.

A Nash equilibrium requires that players have correct beliefs about the strategies their opponents use to map their types to their actions. In order for repeated observations to lead players to learn the distribution of opponents' strategies, the signals observed at the end of each round of play must be sufficiently informative. Each player has a belief about his opponents' play. Let μ_i denote the belief of player i on the a probability distribution over feasible actions, $\sigma_i(x) \in \Delta(\cdot)$. Player i's belief is correct at an opponent j's node x if $\mu_i\left(\left\{\sigma_{-i}|\sigma_j(x) = \sigma^{\phi}(x)\right\}\right) = 1$.

Hence, SCE assumes that the players' inferences are consistent with their observations which are consistent with the Perfect Bayesian Equilibria (PBE).[3]

Definition 13 ***(Fudenberg & Levine, 2007).*** *Φ is a heterogenous self-confirming equilibrium if for each player $i = 1, 2$ and for each strategy s_i with $\hat{\phi}_i(s_i) > 0$ there are beliefs μ_i such that*

[3] Recall that, PBE involves optimal actions given beliefs and consistent beliefs in equilibrium (Fudenberg and Tirole, 1991). In cases where more than one PBE is possible, it is also appropriate to examine whether some can be ruled out. In some cases, PBEs rely on unreasonable beliefs that are technically sustainable (because they are off the equilibrium path of behavior) but unlikely to persist if people deviate slightly from equilibrium predictions.

- *s_i is a best response to $\mu_i(s_i)$ and*
- *$\mu_i(s_i)$ is correct at every node reached with positive probability under $\hat{\Phi}_{-i}$, $x \in \bar{X}(s_i, \Phi)$.*

Given that all information sets are always reached, self-confirming equilibria are equivalent to Nash equilibria. Thus σ cannot be chosen exogenously but must be determined in equilibrium. Formally, equilibrium $\{H, h\}$ is self-confirming for any value of large enough value of σ, since, it is supported by player 1 believing that player 2 will play h and player 2 believing that player 1 will play H. That is, assume that player 1 picks an action H and 2 believes on it with $\mu_2(\sigma) = \sigma$. Then, her expected payoff, E_h^2, is given by,

$$E_h^2 = \sigma((1-\tau)W + p) + (1-\sigma)(1-\tau)w - C.$$

Alternatively, when 2 is choosing l-type,

$$E_l^2 = (1-\tau)(\sigma W + (1-\sigma)w).$$

Thus, 2 prefers to be an $h-$type strategist if $E_h^2 > E_l^2$, and it happens if and only if σ is large enough, i.e.

$$\sigma \geq \frac{C}{p},$$

where $\frac{C}{p} \in (0,1)$ is interpreted as a ratio of "education costs-skill premium". Player 2's best response to $H-$type is h and $\sigma = 1$, which is consistent with the fact that $\sigma > \frac{C}{p}$, hence it is a SCE. Analogously, Player 2's best response to $L-$type 1 is l if $\sigma < \frac{C}{p}$ and $\sigma = 0$, which is consistent with $\sigma < \frac{C}{p}$, and then it is a SCE.

Therefore, equilibria $\left\{H, h, \sigma \geq \frac{C}{p}\right\}$ and $\left\{L, l, \sigma < \frac{C}{p}\right\}$ are heterogenous SCE.

The next section analyzes this game as an evolutionary process. Recall that the term evolutionary process means that more successful types tend to proliferate while less successful types tend to disappear, an assumption that applies equally well to learning by imitation and cultural evolution as well as to literal population replacement by natural selection.

4.2 The Evolutionary Game

The simplest setting in which to study learning is one in which agents' strategies are completely observed at the end of each round, and agents are randomly matched with a series of anonymous opponents so that the agents have no impact on what they observe. Hereafter, populations of leaders (1) and followers (2) are denoted by X^1 and X^2, and the populations are composed of a large number of agents that face the problem of selecting a profile $\{H, L\}$; $\{h, l\}$. Let us denote a fraction of agents of each sub-population as

$$x_i^k = \frac{X_i^k}{X^k}, \tag{4.1}$$

for all pairs $i \in \{(H, L)\,; (h, l)\}$ of sub-population $k \in \{1, 2\}$. That is, the share of $h-$type strategists is

$$x_h^2 = \frac{X_h^2}{X_h^2 + X_l^2} \tag{4.2}$$

where $X_h^2 + X_l^2 = X^2$ is assumed to be a constant. Assume that both populations are normalized to 1, $x_H^1 + x_L^1 = 1$ and $x_h^2 + x_l^2 = 1$. Hence, the probabilities $\sigma = x_H^1$ and $\theta = x_h^2$, and thus, the expected payoffs can be written as

$$\begin{aligned} E_h^2 &= x_H^1\left((1-\tau)W + p\right) + (1 - x_H^1)(1-\tau)w - C & (4.3)\\ E_l^2 &= x_H^1\,(1-\tau)\,W + (1 - x_H^1)\,(1-\tau)\,w & (4.4)\end{aligned}$$

$$\begin{aligned} E_H^1 &= x_h^2\left((1-\gamma)\,U - W - p\right) + (1 - x_h^2)\left((1-\gamma)\,u - W\right) & (4.5)\\ E_L^1 &= x_h^2\left((1-\gamma)\,v - w\right) + (1 - x_h^2)\left((1-\gamma)\,V - w\right) & (4.6)\end{aligned}$$

Let us consider the $n-$population replicator dynamics (Weibull, 1995:172) suggested by Taylor (1979) of the form,

$$\dot{x}_i^k = \left[E_i^k - \bar{E}^k\right] x_i^k, \tag{4.7}$$

where $x_i^k \in [0, 1]$ and $\dot{x}_i^k + \dot{x}_j^k = 0$ for all pairs $i \neq j \in \{(H, L)\,(h, l)\}$ of population $k \in \{1, 2\}$.[4] In other words, the growth rate $\frac{\dot{x}_i^k}{x_i^k}$ of the associated population share is equal its excess payoff, $E_i^k - \bar{E}^k$, over the average payoff in its player population:

$$\bar{E}^k = \frac{X_i^k}{X^k} \cdot E_i^k + \frac{X_j^k}{X^k} \cdot E_j^k. \tag{4.8}$$

In the following, we argue that if followers and leaders decide to imitate successful strategists, and if the state of the economy is such that playing high-profile is the successful strategy then the economy converges to the high-level equilibrium. Otherwise, if the state of the economy is one in which being low-profile is the successful strategy, then the economy will be caught in a poverty trap.

[4] It assures that the trajectory $x^k(t) = \left\{(x_i^k(t), x_j^k(t)),\ t_0 \leq t\right\}$ is bounded in the unit square $\mathbb{C} = [0, 1] \times [0, 1]$.

4.2.1 Replication by imitation

Consider an i-type agent, $i \in \{(H,L);(h,l)\}$, from population $k \in \{1,2\}$, who reviews her strategy with probability $r_i^k(x)$ to consider whether she should or should not change her current strategy, where $x = \left(\left(x_H^1, x_L^1\right), \left(x_h^2, x_l^2\right)\right)$ is the current profiles' distribution.

Assume that an agent's decision depends upon the expected payoff associated with her own behavior, s_i,[5] given the composition of the opposite population, labeled as $E_i^k(s_i, x^{-k})$, $\forall i \in \{h,l;H,L\}$, of sub-population $k, -k \in \{1,2\}, k \neq -k$.

Hence, $r_i^k(x)$ is the average time-rate at which an agent that currently uses strategy i, reviews her strategy choice. Then,

$$r_i^k(x) = f_i^k\left(E_i^k(s_i, x^{-k}), x\right) \in [0,1] \tag{4.9}$$

As before, the function $f_i^k(\cdot)$ is interpreted as the propensity of a member from the i-th club that considers switching membership as a function of the expected utility gains from such a choice. Agents with less successful strategies on average review their strategy at a higher rate than agents with more successful strategies.

Having opted for a change, the agent will adopt the strategy followed by the first person of her population to be encountered (her neighbor), i. e., for any $k \in \{1,2\}$, $p_{ij}^k(x)$ is the probability that a reviewing i-strategist changes to some pure strategy $j \neq i$, $\forall j \in \{(H,L);(h,l)\}$.

The *outflow* from club i in population k is $x_i^k r_i^k(x) p_{ij}^k(x)$ and the *inflow* is $x_j^k r_j^k(x) p_{ji}^k(x)$. Then, $\forall j \neq i \in \{(H,L);(h,l)\}$, $k \in \{1,2\}$, by the law of large numbers we model these processes as deterministic flows and, rearranging terms, we obtain

$$\dot{x}_i^k = x_k^j\left[f_j^k\left(E_j^k(s_j, x^{-k})\right)p_{ji}^k(x)\right] - x_i^k\left[f_i^k\left(E_i^k(s_i, x^{-k})\right)p_{ij}^k(x)\right], \tag{4.10}$$

System (4.10) represents the interaction between two groups of agents that imitate their neighbors.

By the normalization rule, $x_H^1 + x_L^1 = 1$ and $x_h^2 + x_l^2 = 1$, system (4.10) can be reduced to two equations with two independent state variables.

4.2.1.1 The specific behavioral rule

Once again, assume $f_i^k(\cdot)$ to be population specific and to be linear in utility levels (see Weibull, 1995). Thus, the propensity to switch behavior is decreasing in the level of the expected utility, that is, $\forall j \neq i \in \{(H,L);(h,l)\}$, $k \in \{1,2\}$, we get

$$f_i^k\left(E_i^k(s_i, x^{-k})\right) = \alpha^k - \boldsymbol{\beta}^k E_i^k(s_i, x^{-k}), \tag{4.11}$$

[5] $s_i = \{(H,L);(h,l)\}$ indicates vectors of pure strategies independently from population k.

where α^k, $\boldsymbol{\beta}^k \geq 0$ and $\frac{\alpha^k}{\boldsymbol{\beta}^k} \geq^k E_i^k(s_i, x^{-k})$ assures that $f_i^k(\cdot) \in [0,1]$. The parameter α^k can be interpreted as a degree of dissatisfaction and $\boldsymbol{\beta}^k$ measures the performance of the own payoff on reviewing the current strategy. As far as the payoff level of the $i-$strategist, $E_i^k(\cdot)$, increases her average reviewing rate, $r_i^k(x)$, will decrease.

Recall the "average rule" where each strategy is evaluated according to the average payoff observed in the reference group (see J. Apesteguia et al.,2007). Consider that economic agents do not know the exact payoffs of their corresponding neighbors, but they can compute some average payoffs in their neighborhoods and they can imitate the behavior that yields the highest average payoff.

Although an agent does not know all the true values of the payoff of the others, she can take a sample of such true values in order to estimate the average. Let $\tilde{E}_i^k$ and $\tilde{E}_j^k$ be the estimators for the true values E_i^k and E_j^k. Hence, the process of copying successful behaviors exhibits *payoff monotonic updating*, since strategies with above-average payoffs are adopted by others and thus increase their share in the population, that is, each $i-$strategist changes her strategy if and only if $\tilde{E}_i^k < \tilde{E}_j^k$.

Let us apply the behavioral rule given in the Definition 12 from Chapter 3 where a reviewing strategist, i, who decides to change her current strategy must take into consideration: (i) a probability of imitate one strategy which performs better off than her current strategy, $P[\tilde{E}_i^k(s_j, x^{-k}) < \tilde{E}_j^k(s_i, x^{-k})]$,and (ii) the probability of meeting the agent, x_j^k, who currently uses such strategy.

Therefore, for any pair $i \neq j \in \{(H,L);(h,l)\}$, $k \in \{1,2\}$,

$$p_{ij}^k(x) = \begin{cases} \lambda E_j^k(\cdot)\, x_j^\tau & \text{if } E_j^k(\cdot) > 0 \\ 0 & \text{if } E_j^k(\cdot) \leq 0 \end{cases}$$

and

$$p_{ji}^k(x) = \begin{cases} \lambda E_i^k(\cdot)\, x_i^\tau & \text{if } E_i^k(\cdot) > 0 \\ 0 & \text{if } E_i^k(\cdot) \leq 0 \end{cases}$$

where $\lambda = \frac{1}{E_i^k(\cdot)+E_j^k(\cdot)}$.

Hence, by the above considerations, system (4.10) becomes the system of replicator dynamic equations driven by imitation, i.e.,

$$\dot{x}_i^k = x_i^k(1-x_i^k)\lambda\left[\left(\alpha^k - \boldsymbol{\beta}^k E_j^k(\cdot)\right) - \left(\alpha^k - \boldsymbol{\beta}^k E_i^k(\cdot)\right)\right] \tag{4.12}$$

by substitution of the expected payoffs (equations 4.3-4.6), and after some algebraic manipulation, we get:

$$\begin{cases} \dot{x}_h^2 = -\dot{x}_l^2 = x_h^2\left(1-x_h^2\right)[A(\cdot)] \\ \\ \dot{x}_H^1 = -\dot{x}_1^L = x_H^1(1-x_H^1)\,[B(\cdot)] \end{cases}$$

where the functions $A(\cdot)$ and $B(\cdot)$ are:

- $A(\cdot) = \frac{\beta^2\left(px_H^1 - C\right)}{2\left(x_H^1 W + (1-x_H^1)w\right)(1-\tau) + px_H^1 - C}$,
- $B(\cdot) = \frac{\beta^1\left(x_h^2((v-U)(1-\gamma)-p) - \left(1-x_h^2\right)(1-\gamma)u - V\left(\left(1-x_h^2\right) - \left(1-x_h^2\right)\gamma\right) - W + w\right)}{x_h^2((1-\gamma)(U+v)-W-w-p) + (1-x_h^2)((1-\gamma)(V+u)-W-w)}$.

The system $\left(\dot{x}_h^2, \dot{x}_H^1\right)$ describes the case where strategies propagate via imitation, and expected payoffs drive the rate of imitation, reinforcement, and inhibition of behaviors of the high-profile agents from the populations of leaders 1 and followers 2.

The system $\left(\dot{x}_h^2, \dot{x}_H^1\right)$ admits five stationary states or dynamic equilibria, i.e.

$$(0,0),\ (0,1),\ (1,0),\ (1,1) \text{ and a positive interior equilibrium } \left(x_H^{1^*}, x_h^{2^*}\right),$$

where the interesting case is the equilibrium $\bar{P} = \left(x_H^{1^*}, x_h^{2^*}\right)$ given by:

$$x_H^{1^*} = \frac{C}{p},$$

$$x_h^{2^*} = \frac{(1-\gamma)(V-u) + W - w}{(1-\gamma)(U-u+V-v) - p}.$$

The trivial equilibrium is one where leaders and followers are all low-profile economic agents, that is, the pure strategic profile: $(L,l) = (0,1;0,1) = (0,0)$. On the other hand, at the opposite corner the case where all agents are high-profile, that is, the pure strategic profile: $(H,h) = (1,0;1,0) = (1,1)$. The two remaining border equilibria, which are not Nash equilibria, show a different club dominating the two populations and in a sense a mismatch between strategies $\{L,h\}$, $\{H,l\}$ or $(0,1)$, $(1,0)$. The interior equilibrium is composed of marriages among low- or high-profile economic agents: $\bar{P} = \left(x_H^{1^*}, x_h^{2^*}\right)$.

4.3 Analysing the Evolutionary Dynamics

To observe the dynamics of the game we calculate trajectories, i.e. how the mixed strategies change. We start with any pair of mixed strategies $\left(x_2^h, x_1^H\right)$ and calculate the dynamics given by the system $\left(\dot{x}_h^2, \dot{x}_H^1\right)$ as we progress along certain trajectory.

At the mixed-Nash equilibrium the dynamics of the two populations are zero, meaning that it does not pay for a leader or a follower to change to another mixed strategy. In this case, a trajectory either converges or diverges from a mixed-Nash equilibrium. When we have convergence we say that the equilibrium is an evolutionarily stable strategy and it is referred to as an attractor. Alternatively, where we have divergence, the equilibrium is unstable, or vulnerable to small fluctuations, and it is known as a saddle point. The region within which all trajectories converge to a particular attractor is the basin of attraction of that equilibrium. More formally,

Definition 14 ***A trajectory*** *is the change in mixed strategy starting from a particular mixed strategy and following the replicator dynamics.*

Definition 15 ***An attractor*** *is a mixed-Nash equilibrium towards which the replicator dynamics' (trajectories) converge.*

Definition 16 ***A saddle point*** *is a mixed-Nash equilibrium from which the replicator dynamics' (trajectories) diverge.*

Definition 17 ***A basin of attraction of a mixed-Nash equilibrium*** *is the space of mixed strategies from which a trajectory will converge to that equilibrium. The area of the basin corresponds to the probability that the equilibrium will be reached if we assume that it is equally likely for an agent to start at any mixed strategy.*

Zeeman (1992) shows that an ESS is asymptotically stable (AS) in the replicator dynamics such that trajectories do not necessarily have to settle at the equilibrium (which would be neutrally stable) to be an ESS (see Weibull (1995) for more details), but it is not necessarily true the converse of this statement, and AS does not imply ESS. For asymmetric games with strategic complementarities is more useful the notion of ESS against the field. **An evolutionarily stable strategy (ESS) against the field** *is a mixed-Nash equilibrium that cannot be invaded by any alternative strategy into the basin of attraction.*

Definition 18 *Consider a two-population normal form game where each population, $(i = 1, 2)$, has two possible behaviors (H, L) and (h, l) from the strategy spaces, S_i, denoting high- and low-profiles types of leaders and followers, $i = 1, 2$. Suppose that the profile distribution from population 1 is given by $x^1 = \left(x_H^1, x_L^1\right)$, then we say that the strategy $\bar{x}^2 = \left(\bar{x}_H^2, \bar{x}_L^2\right)$ is an ESS against the field of x^1 if there exists $\epsilon_{x^1} > 0$ such that:*

$$E^2\left(\bar{x}^2, \breve{x}^1\right) \geqslant E^2\left(x^2, \breve{x}^1\right)$$

for all $x^2 \in S_2$ where $\left|x^1 - \breve{x}^1\right| \leq \epsilon_{x^1}$.

Consider the Jacobean associated to the system $\left(\dot{x}_h^2, \dot{x}_H^1\right)$ given by,

$$J(\cdot) = \begin{pmatrix} \left(1 - 2x_h^2\right)\left(x_H^1 p - C\right) & x_h^2(1 - x_h^2)p \\ \\ x_H^1(1 - x_H^1)(1 - \gamma)\left(U - u + V - v\right) & \left(1 - 2x_H^1\right)\mathbb{k} \end{pmatrix}, \tag{4.13}$$

where $\mathbb{k} = x_h^2\left((1 - \gamma)\left(\Delta U + \Delta V\right)\right) - (1 - \gamma)\left(V + u\right) - W + w$.

Recall Theorem 1 (ESS as attractor point of the replicator dynamics) from Chapter 2. The following proposition summarizes our results:

Proposition 4 *By imitation of agents the evolutionary dynamics of the economy is as follows:*

i) Equilibria $(0,0)$ and $(1,1)$ are asymptotically stable points and ESS against the field into their basin of attraction.

ii) Equilibrium $\bar{P} = \left(x_H^{1^*}, x_h^{2^*}\right)$ is a threshold, since, it separates the basins of attraction of the low-level and high-level equilibria and, with the exception of a single curve through this point, all solution trajectories converge to the attractors.

Proof. Equilibria fitting $\det(J) > 0$ and $tr(J) < 0$ are asymptotically stable, thus from Definition (18) they are ESSs against the field. We evaluate $J\left(\dot{x}_h^2, \dot{x}_H^1\right)$,

1. $x_h^2 = x_H^1 = 0$. The evaluated Jacobean in this case is given by,

$$J = \begin{bmatrix} -C & 0 \\ 0 & -\left(\left(1-\gamma\right)\left(V+u\right)+W-w\right) \end{bmatrix}.$$

It yields $detJ = (W - w + (1-\gamma)(V+u))(C) > 0$ and $trJ < 0$. Hence this equilibrium point $(0,0)$ is an attractor and therefore an ESS.

2. $x_h^2 = x_H^1 = 1$. The evaluated Jacobean is given by,

$$J = \begin{bmatrix} -\left(p-C\right) & 0 \\ 0 & -\left(W-w+\left(1-\gamma\right)\left(U-2u-v\right)\right) \end{bmatrix}.$$

Thus, $detJ > 0$ and $trJ < 0$. Hence this equilibrium point $(1,1)$ is an attractor and an ESS.

3. $x_h^2 = 1$, $x_H^1 = 0$. The evaluated Jacobean is,

$$J = \begin{bmatrix} C & 0 \\ 0 & \left(1-\gamma\right)\left(U-2u-v\right)-W+w \end{bmatrix}.$$

Thus, $detJ > 0$ and $trJ > 0$. In this case, the equilibrium point $(1,0)$ is a repulsor.

4. $x_h^2 = 0$, $x_H^1 = 1$. The evaluated Jacobean in this case is,

$$J = \begin{bmatrix} p-C & 0 \\ 0 & \left(1-\gamma\right)\left(V+u\right)+W-w \end{bmatrix}.$$

Thus, $detJ > 0$ and $trJ > 0$. In this case, the equilibrium point $(0,1)$ is a repulsor.

Since, the $\mathcal{C}-$square is partitioned into four regions, the point $\left(x_h^{2^*}, x_H^{1^*}\right)$ is a saddle and the other four are local attractors or repulsors. The curve that converges to $\bar{P}$ is a set of critical values into the state of the $\mathcal{C}-$square with the following property: the optimal strategy is different depending on which side of the threshold the current state lies. Therefore, there is just a one-dimensional manifold (threshold level) which goes through $\bar{P}$. Such a threshold separates the basins of attraction into $(0,0)$ and $(1,1)$.

Therefore if the initial distribution of high-profile economic agents is lower that the threshold $\left(x_h^{2^*}, x_H^{1^*}\right)$, then the strategic profile $(L,l) = (0,1;0,1)$ is an ESS against the field of $\left(\bar{x}_h^2, \bar{x}_H^1\right)$ for all $\left(\bar{x}_h^2, \bar{x}_H^1\right) \neq \left(x_h^{2^*}, x_H^{1^*}\right) \in \Delta^k$. Otherwise when it has been overcome the threshold, the equilibrium $(H,h) = (1,0;1,0)$ is an ESS against the field. ∎

Hence, the imitative behavior of the economic agents leads the economy either to a low-level or to a high-level equilibrium and there exists a threshold number of high-profile economic agents, $\bar{P} = \left(x_1^{H^*}, x_2^{h^*}\right)$, that is necessary to overcome the poverty trap.

Figure 4 draws the evolutionary dynamics for the replicator dynamics by imitation $\left(x_h^{2^*}, x_H^{1^*}\right)$.

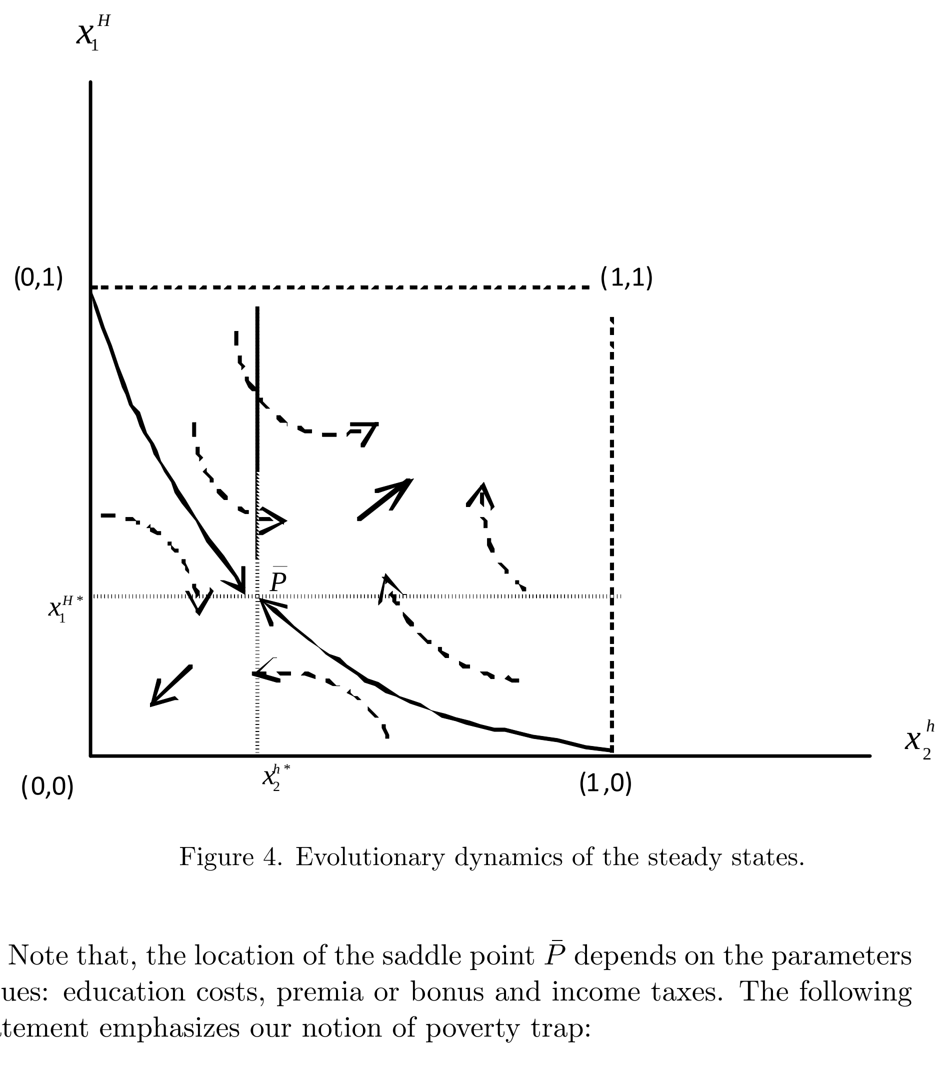

Figure 4. Evolutionary dynamics of the steady states.

Note that, the location of the saddle point $\bar{P}$ depends on the parameters values: education costs, premia or bonus and income taxes. The following statement emphasizes our notion of poverty trap:

Equilibrium $(0,0)$ is a poverty trap in the sense that an economy starting with a (sufficiently) low number of high-profile agents experiences a decreasing sequence of high-profile economic agents that eventually leads to no high-profile agents. This is local stability of the fixed point $(0,0)$.

4.4 To Overcome the Poverty Trap

For a large initial number of high-profile agents, greater than the level $\bar{P} = \left(x_1^{H^*}, x_2^{h^*}\right)$, the economy converges over time toward the sample path $t \to (1,1)$ of high-profile agents.

To reduce the basin of attraction of the low-level equilibrium $(0,0)$ either $x_1^{H^*}$ or $x_2^{h^*}$ should decrease, i.e.,

1. The ratio of education costs-skill premian, $\frac{C}{p}$, decreases if the training costs C fall or the value of the premia p rises.
2. With fixed training costs, C, if the followers' probability of matching with a high-type leader, σ, decreases, then the number of high-profile followers is decreasing too. To avoid this situation, the value of p must be larger than C. For instance, if $\sigma = \frac{1}{2} \geq \frac{C}{p}$ the bonus should be twice as large as the training costs, $p \geq 2C$. Hence, when the number of high-profile followers is small, then the skill premia p should be large enough in order to incentivise the other agents to switch their current behavior and to join the club of high-profile followers.
   (a) Decreasing the value of $x_1^{H^*}$: either training costs C should decrease, or the bonus p must increase, i.e.,
   $$\text{either, } \lim_{C \to 0} x_1^{H^*} \text{ or } \lim_{e \to \infty} x_1^{H^*} \Rightarrow x_1^{H^*} = 0.$$
   Hence, it fully expands the basin of attraction to $(1,1)$ which is the high-level equilibrium.
   (b) Moreover, the income taxes γ play a crucial role to decrease the value $x_2^{h^*}$:
      i. Assume that $\gamma \to 1$ (i.e., complete intervention of policy makers), then,
      $$\lim_{\gamma \to 1} x_2^{h^*} = \frac{W - w}{W} = 1 - \frac{w}{W}.$$
      ii. Instead if $\gamma \to 0$ (non-intervention), then,
      $$\lim_{\gamma \to 0} x_2^{h^*} = \frac{V - u + W - w}{p - (V - v) - (U - u)}.$$

iii. In fact, the value of γ which minimizes to zero $x_2^{h^*}$ is,

$$\gamma = \frac{u - V - (W - w)}{u - V} \Rightarrow x_2^{h^*} = 0,$$

which depends on the relations of followers' differential wages, $(W - w)$, and on the gross-utilities of the $H-$type leader, u, matching a $l-$type follower and the $L-$type leader, V, matching a $l-$type follower.

Hence, to overcome the poverty trap requires exogenous changes like reductions in training costs (or education costs) and increments of skill premia (or bonuses) and these can be implemented, for instance, by a fiscal policy on γ such that this encourages players to become high-profiles.

4.5 Concluding remarks

We studied an evolutionary game of the complementarity between the profiles of the economic agents. We show that the economy can be located in a low-level equilibrium and that an economy requires a threshold number of high-profile economic agents to overcome the poverty trap. Equilibrium $\{L, l\} = (0, 1; 0, 1)$ or $(0, 0)$, which is the low level equilibrium, is the most probable outcome in less developed countries during the early stages of development. It may often be interpreted as a poverty trap, as it is characterized by low levels of skills and technological profile. On the other hand, the high level equilibrium $\{H, h\} = (1, 0; 1, 0)$ or $(1, 1)$ is generally found in developed countries in which the existence of low-profile economic agents may be negligible. The possibility of either high-level or low-level equilibria implies that agents acting under identical settings may experience either adequate living standards or deprivation (growth or crisis), and their result depends only on their history or initial conditions, that is, the composition of the state of the economy depending on the initial number of high-profile agents.

The threshold level of the number of high-profile economic agents is a point $\bar{P} = \left(x_2^{h^*}, x_1^{H^*}\right)$ separating paths leading to stable high and low level steady-states. Thus, this model can explain the coexistence of countries with low growth with countries with high growth as a function of their respective initial conditions alone.

Economies where the number of economic agents surpasses the threshold level $\bar{P} = \left(x_2^{h^*}, x_1^{H^*}\right)$ can overcome the poverty trap, which is a latent possibility for any developing country. Such a threshold is mainly determined by training costs or educations costs, bonuses or skill premia, and income taxes. Therefore, a policy maker may improve the state of the economy by means of these parameters.

CHAPTER 5

DYNAMIC COMPLEMENTARITIES OF FIRMS AND WORKERS

The aim of this chapter is to investigate the equilibrium/disequilibrium dynamics in an economy where two types of firms are present, innovative and non-innovative, and two types of workers are available in the labour market, high-skilled and low-skilled. Firstly, we consider that worker's decision to be high or low-skilled is driven by an imitative behavior, meanwhile the initial distribution of innovative firms is taken as exogenous. But then, we consider that firms can also imitate in order to invest or not in innovation.

The model shows two possible equilibria: on one side there is the high-level equilibrium where a certain percentage of innovative firms is present in the economy together with a definite percentage of high-skilled workers; on the other side, if the initial percentage of innovative firms is below a certain threshold, then the economy moves towards a low-level trap equilibrium where only non-innovative firms are present employing only low-skilled workers.

Hence, the main purpose of the present chapter is to explain the way in which an economy comprising different types of firms and workers can overcome low level equilibria and to give some insights about which policies or main parameters a policy maker may consider in order to attain the high level equilibria and therefore sustained economic growth.

Even if our models are different to those studied by Nelson and Phelps (1966), Redding (1996) and Acemoglu (1997; 1998), our results correspond to theirs. But, we find that the main factors to overcome the low level equilibria when the agents act under imitative pressures, since they look at what their neighbors decide to do given the current state of the economy. Such factors are explained by the lowering the cost of education and increasing the skill premia through tax incentives. Acemoglu (2009:9) summarized our insights clearly:

> *Economic growth will only take place if the society creates the institutions and policies that encourage innovation, reallocation, investment, and education. But such institutions should not be taken for granted.*

Hence, it is the complementarity of human capital and innovation (R&D firms) and its institutional attainment through agents' behavior that constitutes a way to overcome low level equilibria.

We state a model of two "players" considered innovative/non-innovative firms and skilled/unskilled workers, and they represent crucial issues nowadays on which policy-makers have to evaluate how much to invest. As stated "the complementarity between R&D and human capital accumulation is widely accepted as an engine of sustained growth".

Then, two well known theories have to be considered. The first one, more conventional, is the "Skill-biased Technological Change" (see papers by Berman et al., 1994 in the US, Haskel and Heden, 1999 in the UK, and Machin and Van Reenen, 1998 extending to the continental Europe) where the investments in R&D, new products, new process, new technologies - even the ICTs, Information and Communication Technologies - increase the firms' demand for skilled workers, assuming they better know how to implement the new technologies (see the "absorptive capacity" by Cohen and Levinthal, 1990, p.131 where "an organization's absorptive capacity will depend on the absorptive capacities of its individual members"). In this framework, skilled labour is a necessary complement to R&D activities in reinforcing the absorptive capacity of a given organization and new technologies become more effective.

The second theory supports the endogeneity of the phenomenon, i.e. the endogenous skill-bias which suggests that skilled workers are responsible for inducing investments in new technologies in firms (see Kiley, 1999; Funk and Vogel, 2004).

Our approach is quite different from the above references, firstly because we consider a model where workers' decisions are driven by imitative behavior and firms' decisions depend on the number of high-skilled workers. Secondly, we apply evolutionary game theory. Thus, let us start with the game.

5.1 The Game

Consider a two-player normal form game: a worker and a firm. The potential worker needs to decide whether to improve her skills or not. It can be hard to understand why and how a skilled worker can change her behavior to "non-skilled". Of course, this would depend on the kind of skills or knowledge under consideration and how the idea is motivated. Hence, we speak about the decision of being (or not) in the "knowledge frontier". We assume that the workers's choice to be skilled or not can be taken by "imitation" - this means that every worker knows exactly other workers' economic conditions and takes a training course with a 100% probability of being successful (but, of course, entering a training course is not a guarantee to pass it and to find a skill-adequate job). Then, the firm needs to decide whether to invest in R&D or not to become innovative firms or not. If a firm invests in R&D it becomes innovative, if not, then not. Such decisions may depend on the current state of the economy. Thus if the economy is composed mainly of workers and firms with low-profiles, then potential workers and entrant firms decide not to invest in education and

R&D departments.

Hence, we consider an economy composed of two populations: workers, W, and firms, F. Let us assume that each population is divided in two clubs:

- Every now and then, the $W-$population divided into the $S-$club that invests in improving their individual skills or high-skilled workers, and the $NS-$club or low-skilled workers.
- Every now and then, the $F-$population divided into the $I-$club that invests in R&D or are innovative firms, and the $NI-$club of non-innovative firms.

The contractual period between types of firms and workers is described by:

- **Asymmetric information**. At the beginning of the contractual period, workers do not know the type of the firm that hired them. On the contrary, workers must certify their skill levels (for example they must show a CV), and therefore firms know the type of the worker. Then, let us just consider that a worker enter in a firm without understanding if the company invests in R&D or not, but it is more reasonable modeling that if an innovative firm is successful, the worker will get a premium.
- **Training costs and Investment costs**. To become a high-skilled worker has an associated cost. That is, the $S-$type strategist must invest in education by, say, going to a training school, at an associated cost denoted CE. To become an innovative firm implies that the firm carries out investment in R&D departments at an associated cost denoted by CI.
- **Gross income**. Let $B_i(j)$ be the gross-benefit of the $i-$firm hiring the $j-$worker, $\forall\ i = \{I, NI\}\,,\ j = \{S, NS\}$. For all firms, the $S-$type worker gets a salary $\bar{s}$, instead, the $NS-$type worker gets a salary $s < \bar{s}$, $s \geq 0$.
- **Skill premia**.[1] "Skill premia hypothesis" assumes that the innovative firms give premia to their workers (at the end of the contractual period). Assume that the innovative firms, I, give premia to their workers, given at the end of the contractual period, while $NI-$firms do not share these benefits.[2] Thus, high-skilled workers, S, receive a premium $\bar{p}$ and low-skilled workers, NS, receive a premium p, $(0 < p < \bar{p})$, when both are engaged with an innovative firm, I.

[1] A seminal paper about the notion of skill premia is due to Acemoglu (2003).

[2] Recall that workers do not know the type of contracting firm. So, at the beginning of the productive process each worker does not know if she is going to receive a premium or not. This information is revealed only at the end of the period, once she learns the type of contracting firm.

Thus, $CE > \bar{s}$, i.e., if there are no prizes there are no incentives to be high-skilled worker. Thus, $CE > \bar{s}$, i.e., if there are no prizes there are no incentives to be high-skilled worker. If $s \geq 0$ and $CE > \bar{s}$, $s \geq 0 > \bar{s} - C$.

Moreover, there are **strategic complementarities** between types of firms and workers[3]. Thus,

- If the firm is innovative, the payoff of the high-skilled worker is greater than the payoff of the low-skilled worker, i.e.: $\bar{s} + \bar{p} - C > s + p$.
- If the firm is non-innovative, then the payoff of the low-skilled worker is at least as great as the payoff of the high-skilled worker, i.e.: $s \geqslant \bar{s} - C$.
- If the worker is high-skilled, then the payoffs obtained by the innovative firm are greater than those obtained by the non-innovative firm, i.e., $B_I(S) - \bar{p} > B_{NI}(S)$.
- If the worker is low-skilled, then the benefits of the non-innovative firm are greater than those of the innovative firm, i.e.: $B_I(NS) - p < B_{NI}(NS)$.

In summary, we have a two population normal form game. The payoff matrix for the game is represented by,

$W \diagdown F$	**I**	**NI**
S	$\bar{s} + \bar{p} - CE,\ B_I(S) - (\bar{s} + \bar{p} + CI)$	$\bar{s} - CE,\ B_{NI}(S) - \bar{s}$
NS	$s + p,\ B_I(NS) - (s + p + CI)$	$s,\ B_{NI}(NS) - s$

The expected payoffs of the $S-$type and $NS-$type strategist worker, given the probabilities of being hired either by the I or NI firms are denoted by $E(S)$ and $E(NS)$:

$$E(S) = \text{prob}(I)\,[\bar{s} + \bar{p} - CE] + \text{prob}(NI)(\bar{s}) - CE \tag{5.1}$$

$$E(NS) = \text{prob}(I)\,[s + p] + \text{prob}(NI)s \tag{5.2}$$

where prob(I) represents the workers' probability of being hired by an innovative firm and prob(NI) being hired by a non-innovative firm. Hence, workers prefer to be the $S-$type strategist if $E(S) > E(NS)$, and conversely. This happens if and only if prob(I) is sufficiently large i.e.,

$$\text{prob}(I) > \frac{CE - (\bar{s} - s)}{(\bar{p} - p)}. \tag{5.3}$$

[3] Two references on strategic complementarities among types of firms and workers are: 1) Acemoglu (1999); when there is a sufficient fraction of workers who are skilled, firms find it profitable to create jobs specifically targeted at this group, and as a result, unskilled wages fall and skilled wages increase. 2) Acemoglu (1998); as the economy accumulates more skills, technical change responds to make new technologies more complementary to skilled labor.

Workers are indifferent between being a high-skilled or a low-skilled if and only if,[4]

$$\text{prob}(I) = \frac{CE - (\bar{s} - s)}{(\bar{p} - p)}. \tag{5.4}$$

Let us denote the firms' probability to employ the high-skilled worker by prob(S) and prob(NS) to employ a low-skilled one. Hence, a firm prefers to be innovative if and only if the expected payoff of being innovative is greater than the expected payoff of being non-innovative, i.e., $E(I) > E(NI)$ or,

$$\text{prob}(S) > \frac{B_I(NS) - B_{NI}(NS) - p - CI}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) + (\bar{p} - p)}. \tag{5.5}$$

Let us call prob(I) $= P_u$ and prob(S) $= \bar{x}_s$. Hence, a threshold level where both firms and workers prefer to be high-profile is $(\bar{x}_s, P_u)$. Therefore, the game has three Nash equilibria. Two in pure strategies: $A = \{S, I\}$ and $B = \{NS, NI\}$ and a mixed strategy Nash equilibrium given by:

$$NE = (\bar{x}_s, (1 - \bar{x}_s);\ P_u, (1 - P_u)),$$

it follows that equilibrium A Pareto-dominates equilibrium B, while B is the risk dominant equilibrium.

In the following sections, we study the dynamic complementarities for the types of firms and workers. We consider the workers' population dynamic, and from such population dynamics, we characterize the dynamic equilibria and we find the threshold value to overcome the low level equilibrium. Moreover, we consider the dynamics of the firms with labor inputs.

5.2 Dynamic Imitation of Workers

Hereafter, we consider populations of firms and workers normalized to a measure of 1. Hence, prob(I) $= PI = QI/Q$ where QI is number of innovative firms, and Q is total number of firms. Then, prob(NI) $= PNI = 1 - PI$.

Consider that potential workers may assess whether to change or not her current behavior. Let R_i be the probability that the $i-$strategist, $i \in \{S, NS\}$, raises the question about whether she must change her current behavior or not. Then, R_i denotes the average time-rate at which an individual worker, currently using strategy $i \in \{S, NS\}$, reviews her strategy choice.

Let P_{ij} be the probability that such reviewing worker will really change to the strategy $j \neq i$. Then,

$$P(i \to j) = R_i P_{ij}, \tag{5.6}$$

[4]Note that, $0 < \frac{CE-(\bar{s}-s)}{(\bar{p}-p)} < 1$ holds.

is the probability that a worker of the $i-th$ club changes to the $j-th$ club.[5] In the following, $e_S=(1,0)$ and $e_{NS}=(0,1)$ indicate the vector of pure strategies, S or NS.

Hence, the percentage expected flow of high-skilled workers, $\dot{X}_S$, will be equal to the percent probability of low-skilled changing to a high-skilled worker's club minus the percent probability of high-skilled changing to a low-skilled worker's club. Since we consider large populations, we invoke the law of large numbers and model these aggregate stochastic processes as deterministic flows, each such flow being set equal to the expected rate of the corresponding Poisson arrival process. Rearranging terms, we get the system of differential equations that characterizes the dynamic flow of workers,

$$\begin{aligned} \dot{X}_S &= R_{NS}P_{NSS}X_{NS} - R_S P_{SNS} X_S \\ \dot{X}_{NS} &= -\dot{X}_S \end{aligned} \tag{5.7}$$

where X_S is the fraction of high-skilled workers and X_{NS} is the fraction of low-skilled workers.

Let us state the following definition about imitative population dynamics.

Definition 19 *A population dynamic (5.7) will be called 'imitative' if there are at least two different strategies, and agents following one of these strategies assesses, with a given probability, whether he must change his own behavior or not, and the final decision depends on the relation between the benefits he obtains and the benefits others obtain by following a different behavior.*

An imitative dynamic, as defined by the equation system (5.7), makes sense if there are at least two different behaviors, one of them currently followed and the other as a behavior possibly to imitate. In this model if one of the two kinds of populations disappear there is no incentive to change.

We consider that reviewing workers evaluate their current strategy and they decide to imitate successful strategies. Assume that potential workers do not observe payoffs of individual neighbors, but they can, in some way, compute average payoffs in their neighborhoods and they imitate the behavior that leads to the highest average payoff.

[5] In a finite population one may imagine that review times of an S-strategist in population W are modelled as the arrival times of a Poisson process with average arrival rate, across such agents, R_S, and that at each such arrival time the agent selects a pure strategy according to the conditional probability distribution P_{SNS}. Assuming that all agents' Poisson processes are statistically independent, the probability that any two agents happen to review simultaneously is zero, and the aggregate reviewing time in the W player population among S-strategists is a Poisson process. If strategy choices are statistically independent random variables, the aggregate arrival rate of the Poisson process of agents who switch from one pure strategy S to another NS is $R_S P_{SNS}$.

Although the worker does not know all the true values of the payoff of the other workers, she can take a sample of true values to estimate the average. Let $\tilde{E}(i)$ and $\tilde{E}(j)$ be the estimators of those true values $E(i)$ and $E(j)$. Hence, each $i-$worker changes her current strategy if and only if $\tilde{E}(i) < \tilde{E}(j)$.

Consider that the probability that an $i-$type strategist becomes a $j-$type strategist is given by

$$P[\tilde{E}(j) - \tilde{E}(i) > 0],$$

and thus (5.7) can be written as,

$$\begin{aligned} \dot{X}_S &= R_{NS}P[\tilde{E}(NS) - \tilde{E}(S) < 0]X_{NS} - R_S P[\tilde{E}(NS) - \tilde{E}(S) > 0]X_S, \\ \dot{X}_{NS} &= -\dot{X}_S. \end{aligned} \tag{5.8}$$

Consider that the value $P[\tilde{E}(j) - \tilde{E}(i) > 0]$ increases proportionally to the true value $E(j)$ if $E(j) > 0$. This probability is equal to zero if $E(j) < 0$, i.e. $\forall\ i, j \in \{S, NS\}$, i.e.:

$$P[\tilde{E}(j) > \tilde{E}(i)] = \begin{cases} \lambda E(j) & if \;\; E(j) > 0 \\ 0 & if \;\; E(j) < 0 \end{cases} \tag{5.9}$$

where $\lambda = \frac{1}{|E(NS)+E(S)|}$.[6]

Let us assume a constant number of innovative firms, PI; and that salaries $(\bar{s}, s)$, premiums $(\bar{p}, p)$, and education costs CE are fixed or given in the economy. Then, $E(S)$ and $E(NS)$ are constant.

Recall that $E(NS) = (PI)(p) + s \geq 0$ while $E(S) = (PI)(\bar{p}) + \bar{s} - CE$ can be either positive or negative, depending on the values PI and CE. Given salaries and prizes, if CE is fixed, then $E(S) > 0$ if and only if $PI > \frac{CE-\bar{s}}{\bar{p}}$. Let us denote by,

$$\pi = \frac{CE - \bar{s}}{\bar{p}} \tag{5.10}$$

the percentage of innovative firms such that $E(S) = 0$.

In this way, the system (5.8) can take one of the following forms:

(I) **If** $E(S) < 0$**,** and then $P\left[\tilde{E}(S) - \tilde{E}(NS) > 0\right] = 0$.[7] The evolution of the high-skilled workers is given by the dynamical system,

$$\dot{X}_S = -R_S \lambda E(NS) X_S. \tag{5.11}$$

[6] If $E(j) < 0$, then $P\left[\tilde{E}(j) > \tilde{E}(i)\right] = 0$ and then it can be that the complementary probability $P\left[\tilde{E}(j) < \tilde{E}(i)\right] = 1$, otherwise this complementary probability is not well defined. Hence, $\lambda E(j) \in (0, 1]$.

[7] Note that the complementarity probability $P\left[E(NS) > E(S)\right] = \frac{E(NS)}{|E(S)+E(NS)|} \in (0, 1]$ is well defined.

- Its solution is,

$$X_S(t) = X_S(0)\exp\left(\frac{-R_S E(NS)}{|E(NS)+E(S)|}\right)t \tag{5.12}$$

where $X_S(0)$ is the fraction of the high-skill workers at time $t = 0$.[8]

- The population of high-skilled workers decreases. The only possibility to change this situation is to increase premia of high-skilled workers or to reduce education costs. Therefore, the population of high-skilled workers decreases and vanishes. But, this trend can be modified by changing the parameters of the model, which is a necessary condition. A policy maker can implement a policy to reduce the training costs (education costs) and to increase the prizes given to high-skilled workers.

(II) **If** $E(S) > 0$, then the dynamical system takes the form,

$$\begin{aligned} \dot{X}_S &= -\left[R_{NS}E(S) + R_S E(NS)\right]\lambda X_S + R_{NS}\lambda E(S). \\ \dot{X}_{NS} &= -\dot{X}_S. \end{aligned} \tag{5.13}$$

Let us label $A = \lambda\left[R_{NS}E(S) + R_S E(NS)\right]$ and $B = R_{NS}\lambda E(S)$.

- Then the solution of the differential equation (5.13) in this case is,

$$X_S(t) = \left(X_S(0) - \frac{B}{A}\right)\exp\left(-At\right) + \frac{B}{A}. \tag{5.14}$$

where $X_S(0)$ is the fraction of high-skilled workers at time $t = 0$ and,

$$\frac{B}{A} = \frac{R_{NS}E(S)}{R_{NS}E(S) + R_S E(NS)}.$$

- Note that the percentage of high-skilled workers converge to $\frac{B}{A}$. By substitution of expected payoffs, $E(\cdot)$, we get,

$$\frac{B}{A} = \frac{R_{NS}\left[(PI)\left(\bar{p}\right) + \bar{s} - CE\right]}{R_{NS}\left[(PI)\left(\bar{p}\right) + \bar{s} - CE\right] + R_S\left[(PI)\left(p\right) + s\right]}.$$

1. This value depends on the initially given percentage of innovative firms, and increases with this percentage. Considering the percentage B/A as a function of the initial percentage on innovative firms PI, it follows that this value increases with PI.

[8] Indeed, when $P\left[E(NS) > E(S)\right] = \frac{E(NS)}{|E(S)+E(NS)|} = 1$, and the expression (5.12) should be $X_S(t) = X_S(0)\exp\left(-R_S t\right)$.

2. Note that even in the case of a country where all firms are innovative, i.e., $PI = 1$, it does not follow the at the limit, all workers will become high-skilled. In this case, at the equilibrium the percentage of high-skilled workers is given by:

$$B/A = \frac{R_{NS}\left[\bar{p} + \bar{s} - CE\right]}{R_{NS}\left[\bar{p} + \bar{s} - CE\right] + R_S\left[p + s\right]}.$$

3. A particular interesting case occurs where $PI = P_u = \frac{CE-(\bar{s}-s)}{(\bar{p}-p)}$. In this case, the percentage of innovative firms is equal to the value where workers are indifferent between being high-skilled or low-skilled. Given that $P_u > \pi$, the economy evolves to a high level equilibrium where:

$$\frac{B}{A} = \frac{R_{NS}}{R_{NS} + R_S}.$$

is the limit percentage of high-skilled workers.

Provided that $E(NS) > 0$ (i.e., strictly positive), the results, although intuitive, depend completely on the fact that $E(S)$ can be negative, while $E(NS)$ can only be strictly positive. In simple words, while the probability of transition from low-skilled to high-skilled can be zero, the other way around is not possible (the transition from high-skilled to low-skilled cannot zero). The assumptions are such that all-skilled state is not even a stationary state.

5.3 Initial Conditions Matter

How does the initial number of innovative firms PI explain the economy's evolution path? Consider two countries, 1 and 2, where the respective percentage of innovative firms in $t = t_0$ are given by: $PI_1 > PI_2$. So, looking at the solution of (5.13), it follows that the population of high-skilled workers in country 1 is, for each $t > t_0$, larger than the population of high-skilled workers in country 2, i.e.,

$$X_{1S}(t) > X_{2S}(t),\ \forall t > t_0$$

So, the equilibrium state is higher in the country 1 than in the country 2, which means that the number of high-skilled workers in equilibrium is higher in the first country than in the second. Now, recall the threshold dynamic value, π, of the number of innovative firms corresponding to $E(S) = 0$, i.e., $\pi = \frac{CE-\bar{s}}{\bar{p}}$, such that:

1. if $PI > \pi$ then:
   - if $X_S(0) > \frac{B}{A}$, the population of high-skilled workers is decreasing and converges to $\frac{B}{A}$,

- if $X_S(0) < \frac{B}{A}$, the population of high-skilled workers is increasing and converges to $\frac{B}{A}$.
- In both cases the economy converges to the high level equilibrium.

2. if $PI < \pi$ then:

   - The population of high-skilled workers is decreasing over time to 0, and the economy is in a poverty trap, since it is the rational behavior of workers to be low-skilled, and for firms to decide to be non-innovative.

Figure 5 presents the above cases:

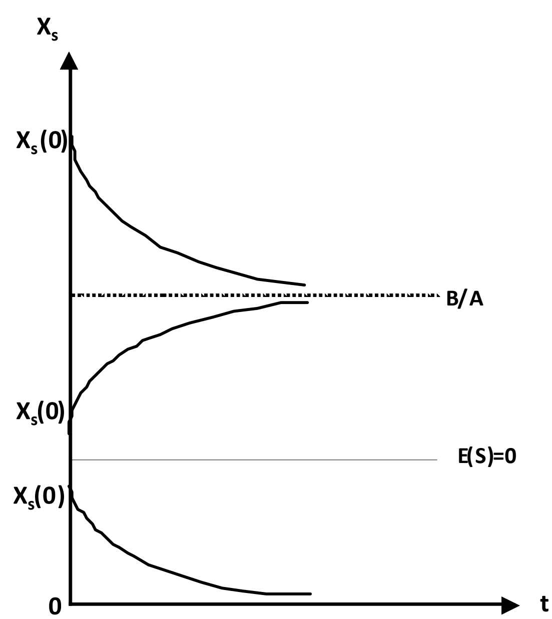

Figure 5. Convergence of high-skilled workers: above and below the threshold level.

The following theorem summarizes our results.

Theorem 5 *Consider the imitative dynamics, given by the system (5.7). There exists a threshold value,* $\pi = \frac{CE-\bar{s}}{\bar{p}}$, *such that*

1. *If the initial number of innovative firms* PI *is larger than this value, i.e.,* $PI > \pi$, *then the percentage of high-skilled workers* $X_S(t)$ *converges to* $\frac{B}{A}$.
2. *If the initial number of innovative firms* PI *verify,* $PI < \pi$, *then the percentage of high-skilled workers* $X_S(t)$ *converges to* 0.

Proof. **Case 1**. If $PI > \pi$, then $E(S) > 0$ and the evolution of the population follows the dynamical system (5.13). So, the population of high-skilled workers is given by $X_S(t) = X_S(0)e^{-At} + \frac{B}{A}$ and $X_S(t) \to \frac{B}{A}$.

Case 2. If $PI < \pi$, then $E(S) \leq 0$ and the evolution of the population of high-skilled workers follows the evolution given by (5.11) so, the population of high-skilled workers is decreasing to zero. ■

We can conclude from this theorem that:

- If $\pi < PI \leq 1$, then in the long-run the percentage of high-skilled workers in the economy will be $\frac{B}{A}$. It follows that non-innovative firms coexist with innovative firms and a percentage of high-skilled workers, that in the limit is equal to $\frac{B}{A}$, coexists with a group of low-skilled workers. However if $PI < \pi$, the economy is in the poverty trap and the corresponding stable equilibrium $(0, 1; 0, 1)$ is a Nash equilibrium

Furthermore, let us introduce a index of location of the economy, that is,

$$U = \frac{PI}{\pi}. \tag{5.15}$$

Then:

- If $U \leq 1$, then the economy is in a poverty trap, i.e., converges to the the low equilibrium where all workers are low-skilled and all firms are non-innovative. If $U > 1$, then the economy has overcome the poverty trap, and converges to a high level equilibrium.

Hence, an economy can be located either in a poverty trap or a high-level equilibrium, depending on the relationship between the number of innovative firms and the parameters (training costs and premia) of the model, this relation is summarized by the index U.

In our setup, an institutional policy that increases the value of U tends to shrink the basin of attraction of the low stationary state. A change in the parameters of the model, reducing education costs or increasing rewards to skill workers, may drive the economy out of the basin of attraction of the low state.

5.4 Dynamic Equilibria and Nash Equilibria

In our set up, there are no possibilities to observe the high Nash equilibrium (in pure strategies) $(S, I) = (1, 0; 1, 0)$ because it is not a dynamic equilibrium. On the contrary, the low Nash equilibrium $(NS, NI) = (0, 1; 0, 1)$ is an asymptotically stable equilibrium, and the poverty trap arises because of rational behavior by the agents. A worker's rational choice is to imitate the best performing strategy (in this case to become $NS-$type). A firm's rational choice is not to invest in a R&D department or not to innovate when the initial number of innovative firms PI is below the threshold value π.

We observe a specific possible mixed Nash equilibrium given by,

$$NE = (\bar{x}_s, (1-\bar{x}_s);\ P_u, (1-P_u)),$$

such that, given the distribution of the firms $(P_u, (1-P_u))$, the equality $E(S) = E(NS)$ is verified. And, given the distribution of the workers' population $(\bar{x}_s, (1-\bar{x}_s))$, the firms are indifferent between being innovative or not being innovative i.e., $E(I) = E(NI)$. Hence, in the mixed nash equilibrium, since the number of innovative firms is $PI = P_u = \frac{CE-(\bar{s}-s)}{(\bar{p}-p)} > \pi$, the population of high-skilled workers converges to $\frac{B}{A} = \frac{R_{NS}}{R_{NS}+R_S}$ which can be equal to $\bar{x}_s$ for the specific case in which the average time-rate at which an individual worker reviews her strategy choice, $R_i\ \forall i \in \{S, NS\}$, is defined such that $\frac{R_{NS}}{R_{NS}+R_S} = \bar{x}_s$.

Let us now consider the concept of an *evolutionarily stable strategy against the field* given a profile distribution of the firm's population, y. Let Δ^w be the set of distributions of the workers population, and Δ^F be the set of distributions of the firms. Let $x_w = (x_s, x_{ns}) \in \Delta^w$ be a population distribution of workers and $y_f = (y, 1-y) \in \Delta^F$ be a population distribution of firms.

Consider a perturbation in the distribution of the firms and let y_ϵ be the post-perturbation distribution. Let $\epsilon > 0$ be sufficiently small, such that the Euclidean distance $|y_f - y_\epsilon| < \epsilon$.

Definition 20 *We say that $x_w \in \Delta^w$ is an evolutionarily stable strategy (ESS) against the field given by y_f, if*

1. *The expected value $E(x_w/y_f) > E(x'_w/y_f)$ for all $x'_w \neq x_w \in \Delta^w$, and*
2. *$E(x_w/y_\epsilon) > E(x'_w/y_\epsilon)$ for all $x'_w \neq x_w \in \Delta^w$.*

Intuitively this means: (i) that x_w is the only best response against y_f and (ii) that this distribution does better against perturbations (in the distributions of the *field*) than all other distributions. In our case, when $y \leq \pi$ the distribution $x_w = (0, 1)$ –all workers are low-skilled– is an ESS against the field given by y_f.

5.5 About the Dynamic of the Firms

We can study the dynamics of the firms about innovate or not supported by the hypothesis of "Skill-Biased Technical Change". Recent studies develop different models to prove that high-skilled labor and high-technology firms are complements in order to obtain a high-level equilibrium (particularly see, Acemoglu, 1997; 1998). This is the well-known notion of skill-biased technical change. Skill-biased technical change implies a shift in the production technology that favors high-skilled over low-killed labor by increasing relative productivity of high-skilled labor and, therefore, its relative demand (see Acemoglu, 2002; Aghion, 2006; Hornstein et al., 2005).

To analyze strategic complementarities let us suppose that innovative firms are represented by the technological production function,

$$y = f(z, x_s, x_{ns}) \tag{5.16}$$

where z is the technology, x_s the number of high-skilled workers and x_{ns} the number of low-skilled workers engaged by the firm, and y the product obtained by the firm from these inputs.

We assume that R&D technology is a complementary input to high-skilled labor.[9] Hence the marginal product of the technology is an increasing function of the number of high-skilled workers.

Let $x_s(t)$ be the total number of high-skilled workers hired by innovative firms at the time t. Consider the time $t_0 < t_1$ and suppose that the number of high-skilled workers is increasing with time, i.e. $x_s(t_0) < x_s(t_1)$. Then, from the hypothesis about technology, it follows that,

$$\frac{\partial c(y, x_s(t_1))}{\partial y} \leq \frac{\partial c(y, x_s(t_0))}{\partial y}, \tag{5.17}$$

where $c(y, x_s)$ denotes the short run cost function. Hence, there exists $\bar{y}$ such that $c(y, x_s(t_0)) > c(y, x_s(t_1))$; $\forall y > \bar{y}$. Figure 5a draws a graphical representation.

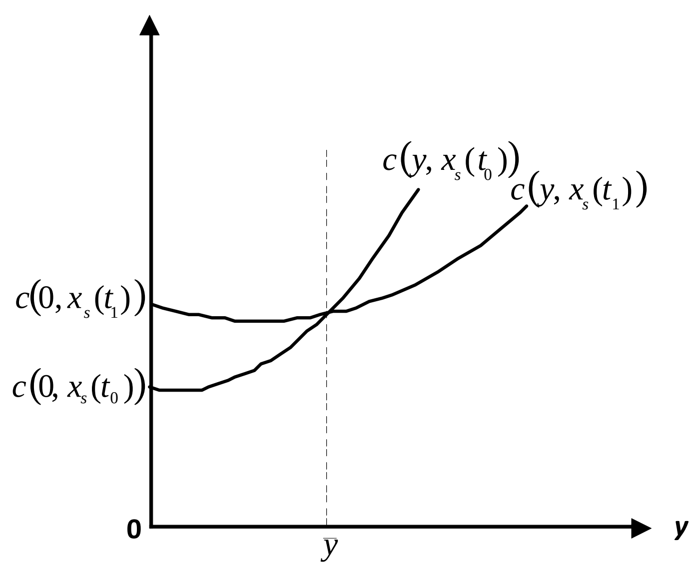

Figure 5a. Decreasing marginal costs for the innovative firms.

Thus, if the number of high-skilled workers is increasing, the cost of innovative firms decreases in the long-run. The innovative firms can obtain

[9]For instance $y = z^{\alpha} x_s^{\beta} + x_{ns}$ where $0 < \alpha, \beta < 1$.

positive profits, and they therefore have incentives to enter. Non-innovative firms, with negative profits in the long-run, wish to exit the industry. The following reason reinforces this conclusion about the evolution of the firms: *Innovative firms require high-skilled workers, whereas non-innovative firms require low-skilled workers*, but the number of low-skilled workers decreases when the number of innovative firms is sufficiently high. Hence, the demand for low-skilled workers will be lower when innovation appears. Hence, a positive net flux of low-skilled to high-skilled workers should be observed as a consequence of an accelerating process of innovation and, at the same time, this process is enhanced by the existence of an increasing demand for high-skilled labor.

5.5.1 Example

To understand the situation described above let us consider the following case:

Suppose that the firms are characterized by the technological function:

$$f(z, x_s, x_{ns}) = kz^{\alpha}x_s^{\beta} + x_{ns}^{\lambda}.$$

Where:

$$k = \begin{cases} H & \text{if the firm is innovative} \\ \\ h & \text{if the firm is non-innovative,} \end{cases}$$

$H > h > 0$, and α, β and λ are positive constants such that $\alpha + \beta = 1$ and $\lambda < 1$.

Assume that the feasible technology $z = \bar{z}$ is a given positive constant, and that skill premia (the bonus of being high-skilled worker) are denoted by pr. The short rum cost function is therefore:

$$C(x_{ns}, y, \bar{z}, \bar{x}_s) = (w_s + pr)x_s + w_{ns}\left[y - k\bar{z}^{\alpha}x_s^{\beta}\right]^{\frac{1}{\lambda}} \tag{5.18}$$

It follows that:

$$C'_y(x_{ns}, y, \bar{z}, \bar{x}_s) = w_{ns}\tfrac{1}{\lambda}\left[y - k\bar{z}^{\alpha}x_s^{\beta}\right]^{\frac{1}{\lambda}-1}.$$

$$C''_{y,x_{ns}}(x_{ns}, y, \bar{z}, \bar{x}_s) = -w_{ns}(\tfrac{1}{\lambda} - 1)\tfrac{1}{\lambda}\left[y - k\bar{z}^{\alpha}x_s^{\beta}\right]^{\frac{1}{\lambda}-2}k\bar{z}^{\alpha}x_s^{\beta-1} < 0.$$

Thus, for the case of innovative firms, it can be observed that the marginal cost decreases with x_s faster than for the case of non-innovative firms. So if in $t = t_0$ the fraction of innovative firms is greater than the threshold value π, then innovative firms can reduce their costs more quickly than non-innovative firms.

Assume that the market price for the product is p. If we assume that firms are competitive, then the optimal supply for each firm is given by:

$$Y_I^* = pHz^\alpha x_{Is}^* + x_{Ins}^*$$

$$Y_{NI}^* = phz^\alpha x_{NIs}^* + x_{NIns}^*$$

Where x_{is}^* and x_{ins}^*, $i \in \{I, NI\}$ denote the long run demand for inputs for innovative and non-innovative firms:

$$x_{Ins}^* = x_{NIns}^* = \left(\frac{w_{ns}}{\lambda p}\right)^{\frac{1}{\beta-1}}, \; x_{Is}^* = \left(\frac{w_s + pr}{\lambda pHz^\alpha\beta}\right)^{\frac{1}{\beta-1}}, \; x_{NIs}^* = \left(\frac{w_s + pr'}{\lambda phz^\alpha\beta}\right)^{\frac{1}{\beta-1}}.$$

Let $PI > \pi$ be the existing number of innovative firms in time $t = t_0$ and let $D(p)$ the demand of the market for the product. The total supply $S(p)$ of the innovative firms will be equal to:

$$S(p) = (PI)Y_I^*.$$

The number of non-innovative firms will be equal to:

$$\max\left\{\frac{D(p) - S(p)}{Y_{NI}^*}, 0\right\}.$$

Therefore, in the long run a percentage of innovative firms could coexist with a percentage of non-innovative firms. To see this, assume that there is an associated cost to become innovative firm (the investment cost on R&D departments), let $C(h, H)$ be this cost. So, an incumbent non-innovative firm has an incentive to become an innovative firm if and only if, the benefits are such that:

$$B(NI) < B(I) - C(h, H).$$

This possibility depends on, among other things, the segment of the market that the firm can obtain given the existing number of innovative firms in the market. When $B(I) - C(h, H) < B(NI)$ the firm prefers to continue being non-innovative.

5.5.2 A behavioral rule about to innovate or not

Consider that firms' decisions to innovate or not to innovate depend on a given number of high-skilled workers and on the investment costs of innovation.

Then, from the one-shot game the expected payoffs of innovative and non-innovative firms given by,

$$\begin{aligned} E(I) &= x_S\left(B_I(S) - \bar{s} - \bar{p}\right) + (1 - x_S)\left(B_I(NS) - s + p\right) \\ E(NI) &= x_S\left(B_{NI}(S) - \bar{s}\right) + (1 - x_S)\left(B_{NI}(NS) - s\right) \end{aligned}$$

where x_S is the share of high-skilled workers which is equal to the firm's probability of hiring a high-skilled worker. A firm decides to be innovative if and only if $E(I) > E(NI)$, i.e.,

$$x_S \geqslant \frac{B_I(NS) - B_{NI}(NS) - p}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) + (\bar{p} - p)}.$$

Nevertheless, the behavioral rule that firms use to decide be innovative or not should consider both: i) cost of investment, denoted by CI and, ii) the number of high-skilled workers that makes such an investment to be profitable in the long-run. It is the threshold dynamic value corresponding to $E(S) = 0$ or $\pi = \frac{CS - \bar{s}}{p}$.

Therefore, the investment in innovation is profitable when:

$$\pi - CI > 0.$$

Let us normalize the firms population to 1, $y_I + y_{NI} = 1$. As before, let R_{ij} be the probability that the $i-$strategist, $i \in \{I, NI\}$, raises the question about whether she must change her current behavior or not to $j \neq i \in \{I, NI\}$.

Hence, the dynamic flow of firms is given by,

$$\dot{Y}_I = R_{NII}(\pi - CI)E(I)(1 - Y_I) - R_{INI}(\pi - CI)E(NI)Y_I$$

$$\dot{Y}_{NI} = -\dot{Y}_I$$

Rearranging terms, we get:

$$\dot{Y}_I = R_{NII}(\pi - CI)E(I) - Y_I \left[R_{NII}(\pi - CI)E(I) + R_{INI}(\pi - CI)E(NI)\right]$$

Let us label

$$f = R_{NII}(\pi - CI)E(I)$$

and

$$g = \left[R_{NII}(\pi - CI)E(I) + R_{INI}(\pi - CI)E(NI)\right].$$

Then, the solution of $\dot{Y}_I$ is:

$$Y_I(t) = \left(Y_I(0) - \frac{f}{g}\right)\exp(-gt) + \frac{f}{g},$$

where:

$$\frac{f}{g} = \frac{R_{NII}E(I)}{R_{NII}E(I) + R_{INII}E(NI)},$$

is interpreted as:

> **the ratio of coexisted innovative and non innovative firms, that review their strategies, given a certain initial number of high-skilled workers x_s in the economy.**

Recall that the expected payoffs of innovative $E(I)$ and non innovative firms $E(NI)$ are defined as a function of x_s and $E(I) \geqslant 0$ if:

$$x_s \geqslant \frac{B_I(NS) - p - s - CI}{B_I(NS) - B_I(S) + \bar{p} + \bar{s} - p - s}.$$

Figure 5b shows the evolution of firms under the threshold x_s.

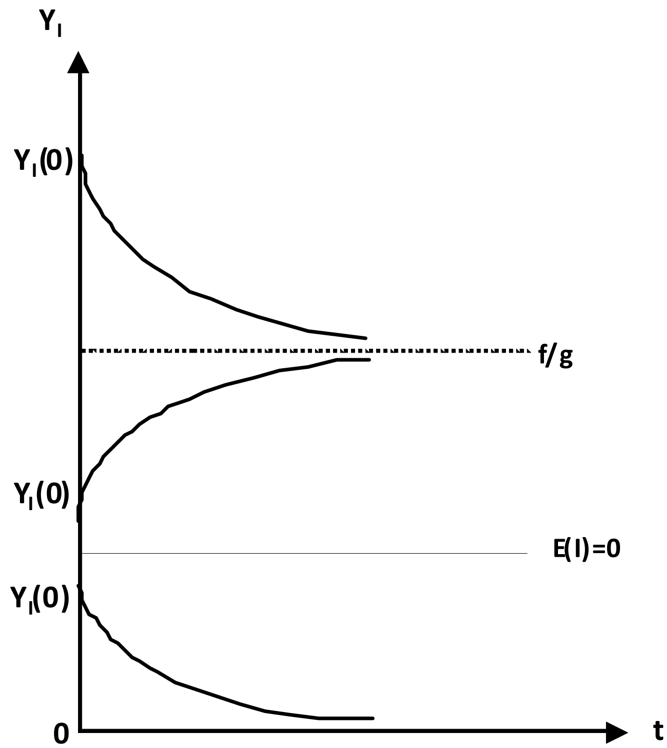

Figure 5b. Evolution of firms from above and below the threshold level.

5.6 Concluding remarks

We presented a game theoretical model concerning the strategic complementarities between different types of firms and workers. Workers follow an imitative behavior and firms decide to invest or not in R&D depending on the current number of high-skilled labor in the economy.

Similar to Accinelli et al. (2007), we conclude that to overcome a poverty trap it is necessary to surpass a threshold number of innovative firms. Once this threshold is reached, workers will have an incentive to improve their skills. Firms, in turn, can obtain more benefits by investing in R&D. The main problem to overcome a poverty trap is that all the agents involved act rationally, but the rational persistence of a poverty trap is inefficient. As we have shown, a threshold value from which the rationality change exists, that is to say to invoke a coordination problem, thus, above such threshold, the agents are able to coordinate to the high level equilibrium. And surpassing this value the agents rationally change their behavior and the economy moves to another, Pareto superior, equilibrium. Since there exists a continuum of possible high equilibria, one for each percentage of innovative firms between the threshold value π and 1, there exists a continuum of countries in high equilibriums but with different percentages of innovative firms and high-skilled workers.

The policy maker in developed country i.e.: in a country with a high level equilibrium, who wishes to improve their level of equilibrium or needs to improve the efficiency of the industry, may do this by designing a mechanism to transform the industry, say by favoring the substitution of non-innovative firms for innovative firms.

The benevolent policy maker in an under developed country, faces two options to overcome the poverty trap, either 1) to improve the efficiency of the industry favoring the substitution of non-innovative firms for innovative firms, or 2) to adopt a policy that decreases the threshold values given by π, in such way that new trajectories belong to the basin of attraction of a high equilibrium. This objective may be attained by reducing the training costs to become a high-skilled worker, or giving incentives to innovative firms to increase the rewards for high-skilled workers. From our model it can be seen that the closer a country gets to the technology threshold, the more investment in education enhances growth.

In summary, policy makers should find the right mechanism to allow agents to choose the most efficient behavior. Differences in the policies followed by institutions in different countries can help us to understand differences in the degree of development across countries and over time. A fundamental and enduring concern of a policy maker is the design of an incentive policy conducive to increasing the number of innovative firms and the possibilities that workers become highly skilled.

For further research, intuitively, by a suitable modification of the probability of changing (expression 5.9), a model of imitation based on the difference of expected payoffs, could have (at least) three sets of *stationary states*: all high-skilled, all low-skilled, and a mixed one. Moreover, the intuition says that the all high-skilled state should not be *asymptotically stable*, and the results, at the end, should be similar to those found in this paper. Another possibility is to keep expression 5.9 avoiding strictly negative expected payoffs (i.e., assume $E(S) > 0$ and $E(NS) > 0$). In this case, we would have always a mixed equilibrium, which would depend, among other things on the relative rates of change (R_S and R_{NS}).

5.7 The Replicator Dynamics for Firms and Workers

In this section, we extend the above model to extract the evolutionary dynamics from the replicator dynamic system, when both decisions of firms and workers are driven by imitation.

Hence, we characterize the steady states, the ESSs, and the threshold values required to move from the low level to the high level equilibrium.

5.7.1 The population game: replicator by imitation.

Consider that at each period of time $t \in [0, \infty)$ an agent from each population is randomly matched with agent from the other population to play a bilateral finite game. Recall that prob$(I) = PI = y_I$ is the worker's probability of being hired by an innovative firm, and prob$(S) = x_S$ is the firm's probability of hiring a high-skilled worker.

Let N_i^τ be the total of $i-$strategists, $i \in \{(I, NI); (S, NS)\}$, from the total population, $\tau \in \{F, W\}$, and both populations are normalized to 1, that is, $y_I + y_{NI} = 1$ and $x_S + x_{NS} = 1$.

Hence, the fractions of $S-$type strategists and $I-$type strategist are,

$$\begin{aligned} x_S &= \frac{N_S^W}{N_S^W + N_{NS}^W}, \\ y_I &= \frac{N_I^F}{N_I^F + N_{NI}^F}. \end{aligned}$$

In the following, the vectors $y_F = (y_I, y_{NI})$ and $x_W = (x_S, x_{NS})$ are distributions of agents playing a certain pure or mixed strategy in the space $\{(I, NI); (S, NS)\}$ from the two populations F and W.

Now, the expected payoffs are[10]:

$$\begin{cases} E(S) = y_I \bar{p} + \bar{s} - CE. \\ E(NS) = y_I p + s. \\ E(I) = x_S \left[B_I(S) - B_I(NS) - (\bar{s} + \bar{p}) + (s + p) \right] + B_I(NS) - (s + p). \\ E(NI) = x_S \left[B_{NI}(S) - B_{NI}(NS) - \bar{s} + s \right] + B_{NI}(NS) - s. \end{cases} \tag{5.19}$$

5.7.1.1 The specific behavioral rule.

Let us consider the **Definition 8** of **Behavioral Rule**. The *outflow* from club i in population τ is $q_i^\tau r_i^\tau(x) p_{ij}^\tau(x)$ and the *inflow* is $q_j^\tau r_j^\tau(x) p_{ji}^\tau(x)$, where $q_i^\tau = q^\tau x_i^\tau$ is the number of i-strategist agents from population τ and q^τ represents the whole population τ, hence $q^\tau = q_i^\tau + q_j^\tau$. Hence,

[10] Recall that, $E(NS) = y_I p + s$ is always positive but $E(S) = y_I \bar{p} + \bar{s} - CE$ can be positive or negative depending on y_I and CE. We interpreted $E(S) < 0$ as the case in which the probability of being hired by an innovative firm tends to zero, and then, cost of education is greater than salaries, i.e., $\lim_{\phi_I \to 0} E(S) < 0$ if $CE > \bar{s}$.

population τ (assuming that the size of the population τ is constant) by the law of large numbers we can model these processes as a deterministic flow. Rearranging terms, for each pair $i, j \in \{(I, NI); (S, NS)\}$, $j \neq i$, from population $\tau \in \{F, W\}$, we get,

$$\dot{x}_i^\tau = r_j^\tau(x) p_{ji}^\tau(x) x_j^\tau - r_i^\tau(x) p_{ij}^\tau(x) x_i^\tau. \tag{5.20}$$

Consider that the an agent's decision depends upon the expected payoff associated with their own behavior, given the composition of the other population, $E^\tau(e_i, x^{-\tau})$ (where, τ represents the population to which the agent following the $i-th$ behavior belongs, and $-\tau \in \{F, W\}$, $-\tau \neq \tau$), and depends on the characteristics of the populations represented by $x = (y^f, x^W)$. Therefore, we may assume,

$$r_i^\tau(x) = f_i^\tau(E^\tau(e_i, x^{-\tau}), x). \tag{5.21}$$

Function $f_i^\tau(E^\tau(e_i, x^{-\tau}), x)$ is interpreted as the propensity of a member from the i-th club to switch membership as a function of the expected utility gains from such a switch. Agents with less successful strategies on average review their strategy at a higher rate than agents with more successful strategies.

Assume f_i^τ is population-specific, but the same across all its components independent of club membership, and assume, furthermore, that it is linear in utility levels (see Weibull, 1995). Thus, the propensity to switch behavior will be decreasing in the level of utility,i.e.,

$$f_i^\tau(E^\tau(e_i, x^{-\tau})) = \alpha^\tau - \boldsymbol{\beta}^\tau E^\tau(e_i, x^{-\tau}) \in [0, 1]. \tag{5.22}$$

where $\alpha^\tau, \beta^\tau \geq 0$ and $\frac{\alpha^\tau}{\beta^\tau} \geq E^\tau(e_i, x^{-\tau})$.

To simplify the notation, for each pair $i, j \in \{(I, NI); (S, NS)\}$, $j \neq i$ let us simply label:

- $r_i^\tau(x) = r_i$ and $r_j^\tau(x) = r_j$,
- $p_{ji}^\tau(x) = p_{ji}$ and $p_{ij}^\tau(x) = p_{ij}$,
- $E^\tau(e_i, x^{-\tau}) = E\,(i)$ and $E^\tau(e_j, x^{-\tau}) = E\,(j)$.

From equations (5.21-5.22), and after some algebraic manipulations, system (5.20) becomes the replicator dynamic driven by imitation, i.e.,

$$\dot{x}_i^\tau = x_i^\tau\,(1 - x_i^\tau)\,[\lambda\,(\alpha^\tau + \boldsymbol{\beta}^\tau)\,(E(i) - E(j))]\,. \tag{5.23}$$

where $x_i^\tau \in [0, 1]$ and $\dot{x}_i^\tau + \dot{x}_j^\tau = 0$ for each pair $i, j \in \{(I, NI); (S, NS)\}$, $j \neq i$ from population $\tau \in \{F, W\}$, [11]; $\lambda = \frac{1}{|E(i)+E(j)|}$, $\alpha^\tau, \beta^\tau \geq 0$ and $\frac{\alpha^\tau}{\beta^\tau} \geq E(\cdot)$.

[11]This condition ensures that the trajectory $x_k(t) = \left\{(x_k^i(t), x_k^j(t)),\ t_0 \leq t\right\}$ is bounded in the unit square $\mathbb{C} = [0, 1] \times [0, 1]$.

In particular, if we consider the profile distribution of workers, $x_W = (x_S, x_{NS})$, and firms, $y_F = (y_I, y_{NI})$, then we can model the flow of high-skilled workers and innovative firms over time: $\dot{x}_S$ and $\dot{y}_I$.

Since we consider large populations, we invoke the law of large numbers and model these aggregate stochastic processes as deterministic flows, each such flow being set equal to the expected rate of the corresponding Poisson arrival process. Hence, rearranging terms, we obtain the system of differential equations that characterizes the dynamic flow of workers,

$$\begin{aligned} \dot{x}_S &= r_{NS} p_{NSS} x_{NS} - r_S p_{SNS} x_S \\ \dot{x}_{NS} &= -\dot{x}_S \end{aligned} \tag{5.24}$$

and the differential equations that characterizes the dynamic flow of firms,

$$\begin{aligned} \dot{y}_I &= r_{NI} p_{NII} y_{NI} - r_I p_{INI} y_I \\ \dot{y}_{NI} &= -\dot{y}_I \end{aligned} \tag{5.25}$$

An agent does not know the true values of the expected payoffs other agents receive, but she can take a sample of such true values in order to estimate the average. Let $\tilde{E}(i)$ and $\tilde{E}(j)$ be the estimators for the true values $E(i)$ and $E(j)$, $\forall i, j \in \{(I, NI);(S, NS)\}$, $j \neq i$. Hence, each $i-$strategist changes her current strategy if and only if $P[\tilde{E}(i) < \tilde{E}(j)]$.

From Chapter 3 Definition 12 we apply the **Specific Behavioral Rule** where a reviewing worker, i, who decides to change her current strategy must take into consideration both a probability of imitating at least one strategy that performs better than her current strategy, $P[\tilde{E}(i) < \tilde{E}(j)]$, and the probability of meeting an agent, x_j^τ, who uses such strategy, i.e.:

- *A reviewing strategist i changes to j with a probability, p_{ij}, equal to getting a positive average rule times the probability of finding a $j-$strategist in the whole population, i.e. $p_{ij} = P[\tilde{E}(j) - \tilde{E}(i) > 0]x_j$, where $P[\tilde{E}(j) - \tilde{E}(i) > 0]$ increases proportionally to the true value $E(j)$, i.e. $\forall i, j \in \{(I, NI);(S, NS)\}$, $j \neq i$,*

$$P[\tilde{E}(j) > \tilde{E}(i)]x_j^\tau = \begin{cases} \lambda E(j) x_j^\tau & if \;\; E(j) > 0 \\ 0 & if \;\; E(j) \leq 0 \end{cases} \tag{5.26}$$

where $\lambda = \frac{1}{|E(i)+E(j)|}$.

Now, we simply introduce the specific behavioral rule (5.26), the substitution of expected payoffs (5.19), and, after some algebraic manipulation from equations (5.24-5.25), we can obtain the **worker's replicator dynamic (RD)** system and the **firm's replicator dynamic (RD)** system driven by imitation:

$$\dot{x}_S = -\dot{x}_{NS} = x_S (1 - x_S) A(\cdot). \tag{5.27}$$

$$\dot{y}_I = -\dot{y}_{NI} = y_I (1 - y_I) B(\cdot). \tag{5.28}$$

Where the functions A and B are defined by,

$$\begin{aligned} A(x_S, y_I) &= \alpha^W \frac{y_I (\bar{p} - p) + s - \bar{s} + CE}{y_I (p + \bar{p}) + s + \bar{s} - CE}, \\ B(x_S, y_I) &= \alpha^F \left[\frac{B_I(NS) - B_{NI}(NS) + x_S (\Delta B_I + \Delta B_{NI} - \Delta P) - p}{B_I(NS) + B_{NI}(NS) + x_S (\Delta B_I - \Delta B_{NI} - \Delta P - 2s - 2\bar{s}) - p + 2s} \right], \end{aligned}$$

where: $\Delta B_I = B_I(S) - B_I(NS)$, $\Delta B_{NI} = B_{NI}(NS) - B_{NI}(S)$ and $\Delta P = \bar{p} - p$.

5.7.2 Stability and equilibria analysis

The RD system, $(\dot{y}_I, \dot{x}_S)$, admits five stationary states or dynamic equilibria, i.e. $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$ and $(\hat{y}_I, \hat{x}_S)$. Where:

$$\begin{aligned} \hat{y}_I &= \frac{CE - (\bar{s} - s)}{\bar{p} - p}, \\ \hat{x}_S &= \frac{B_I(S) - B_{NI}(NS) + p}{B_I(NS) - B_I(S) + B_{NI}(S) - B_{NI}(NS) - p + \bar{p}}. \end{aligned}$$

In fact, the interesting case is when $G = (\hat{y}_I, \hat{x}_S)$ is an equilibrium lying in the interior of the square $\mathcal{C} = [0,1] \times [0,1]$, which occurs when $0 < \hat{y}_I < 1$ and $0 < \hat{x}_S < 1$.

Recall that every ESS is a locally asymptotically stable point of the RD system and its basin of attraction contains a neighborhood of the relative interior of the lowest dimensional face of the simplex. Therefore:

Theorem 6 *The evolutionary dynamics of firms and workers driven by imitation is as follows:*

i) Equilibria $(1,0)$ and $(0,1)$ are nodal sources and unstable.

ii) Equilibria $(0,0)$ and $(1,1)$ are asymptotically stable points or nodal sinks, and therefore ESSs against the field.

iii) Equilibrium $G = (\hat{y}_I, \hat{x}_S)$ is a saddle point, and therefore a threshold since it separates the basins of attraction of the low-level and high-level equilibria.

A graphic representation is given by the following figure:

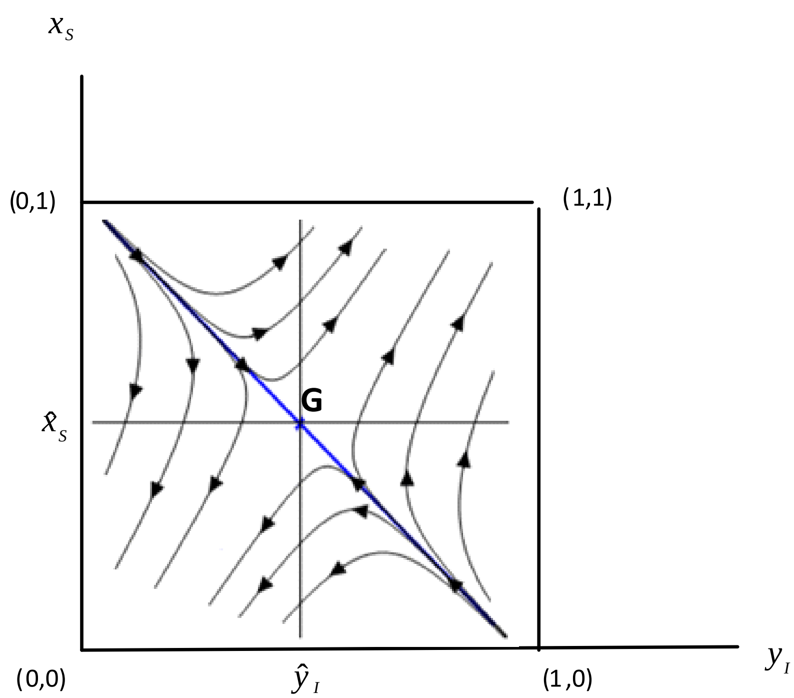

Figure 6. Vector field of the replicator system $(\dot{y}_I, \dot{x}_S)$.

The point G is hypothetically centered but it is not necessarily always the case since it depends on the parameters.

These equilibria can be interpreted as follows:

- Equilibrium $(0, 0)$ is the low-level equilibrium in which firms and workers decide a low-profile $\{NI, NS\}$.
- Equilibrium $(0, 1)$ and $(1, 0)$ is is a mismatch of profiles $\{NI, S\}$, $\{I, NS\}$.
- Equilibrium $(1, 1)$ is the high-level equilibrium in which firms and workers decide a high-profile $\{I, S\}$.

Therefore, it is the rational behavior of the economic agents, firms and workers acting under imitation, which can lead the economy to a poverty trap, and a threshold to overcome it exists.

5.7.3 How to overcome a poverty trap: going for education costs

The market alone is incapable of overcoming this kind of poverty trap. Policy makers should intervene, for instance, by providing some kind of financial incentive for R&D investment or by imposing a minimum period of schooling.

Note that the historical dependence of the above model, since the results depend on the initial conditions, that is, the profile distributions, y_F and x_W. Therefore, an optimal strategy of the dynamic replicator system converges towards two distinct attractors, $(0,0)$ and $(1,1)$.

Let us denote by $y^f = (y_I^f, y_{NI}^f)$ and $x^w = (x_S^w, x_{NS}^w)$ the initial distributions of firms and workers populations. Then, the following cases may arise:

1. In an economy composed of one small fraction of high-skilled workers, $x_S^w < \hat{x}_S^w$, a firm will decide to be non-innovative. Hence, to overcome the poverty trap, $\{NS, NI\}$, the distribution of workers must increase the fraction of high-skilled workers:
   - This can happen either if, given gross benefits of firms (determined in the market), $B_i(j)$, $\forall\ i \in \{I, NI\}\ j \in \{S, NS\}$, then, the premia of low-skilled workers is equal to

$$p = B_{NI}(NS) - B_I(S),$$

which means that an innovative firm must offer a premium which encourages the low-skilled workers to supply their labor or become employed in the R&D sectors. In turn, due to imitation and to strategic complementarities, the low-skilled worker must becomes a high-skilled worker.

This consideration can be analyzed as a "game against the field" in which there is no specific "opponent" to a given agent and payoffs depend on what everyone in the population is doing.

Consider the population of workers. A payoff matrix for the population of workers in the game against the field is given by,

Invader $\diagdown$ $W-$population	**S**	**NS**
S	$\pi(S,S)$	$\pi(S,NS)$
NS	$\pi(NS,S)$	$\pi(NS,NS)$

where $\pi(i,j)$ is the invader's payoff or fitness of playing the strategy $i \in \{S, NS\}$ against the strategy $j \in \{S, NS\}$ in the $W-$population.

2. If the fraction of innovative firms, y_I^f, is smaller than the threshold value $\hat{y}_I^f$, i.e.,

$$y_I^f < \hat{y}_I^f = \frac{CE - (\bar{s} - s)}{\bar{p} - p}, \tag{5.29}$$

then all potential workers prefer not to become high-skilled, and thus the initial distribution of workers $x^w(t_0) = (x_S^w(t_0), x_{NS}^w(t_0)) \to (0,1)$ in pure strategies. Suppose now, that a small part of the non-innovative firms become innovative, $y^{'f} = (y_I^{'f}, y_{NI}^{'f})$ where $y(t_0) < y_I^{'f} < y_I^f$, then workers prefer to be low-skilled and the population of the high-skilled workers continues to decrease. In this case a rational worker will choose to be a low-skilled worker. So, we need a big change in the distribution of the population of firms to obtain that workers prefer to be one of high-skilled. The threshold value is $\hat{y}_I^f$, if the new distribution verify that $y_I^f > \hat{y}_I^f$, then the rational worker chooses to be high-skilled.

We can define the ESS against de field in the next:

Definition 21 *Consider a two population normal form game where each population A and B has two possible behaviors,* 1 *and* 2*. Suppose that the distribution of the population A is given by* $y_A = (y_{1A}, y_{2A})$*, we say that the the strategy* $x_B^* = (x_{1B}^*, x_{2B}^*)$ *is an ESS against the field if there exists an* ϵ *such that if*

$$\pi_B(x_B^*, y_A') \geq \pi_B(x_B, y_A')$$

for all $x_B \in S_B$ *where* $|y_A - y_A'| \leq \epsilon$*. Therefore, if the distribution of the initial population of firms is given by* $y^f = (y_I^f, y_{NI}^f)$ *where* $y_I^f < \hat{y}_I^f$ *then* $x^w = (0, 1)$ *is an ESS against the field.*

To overcome a poverty trap requires that we change the rationally determined outcome, this is possible to do by implementing a policy that decreases the costs to become a high-skilled worker, or, given the incentive to become an innovative firm, to increase rewards to skill workers, or to choose a policy that would increase the number of innovative firms above ϵ, i.e. to increase the number of innovative firms in such way that the number of innovative firms overcomes the threshold $\hat{y}_I^f$.

Therefore, to reverse the inequality (5.29), and to overcome the poverty trap:

- Either costs of education CE must decrease or ΔP must increase, and thus a perturbation, σ_ϵ, performs better in the basin of attraction of the low level equilibrium (playing against the field), and the economy can now overcome the poverty trap. We thus consider the perturbation as a shock that compels agents to adopt new strategies, i.e. alternative strategies will invade the equilibrium as a consequence of the exogenous shock and, because the economy is in the basin of attraction of the higher-level equilibrium, the economy will converge to the higher-level equilibrium.

Decreasing CE means investing in human capital, which implies an increasing ΔP. This means that workers with greater profiles should get the largest possible premium, $\bar{p}$. In this way, the number of innovative firms (R&D) and the number of high-skilled workers (human capital) are large enough to overcome the poverty trap, i.e. $\left(x_S^w, y_I^f\right) > \left(\hat{x}_S^w, \hat{y}_I^f\right)$.

Any situation in the game against the field can be modified by "somebody" who is able to modify the profile distributions y_F and x_W and the "somebody", namely a central planner, or policy maker or even the nature of each strategist who is wondering whether their current behavior, is the optimal response for the long run outcome of the economy as a whole - the economy becomes more efficient.

5.7.3.1 Replicator dynamics with payoff taxation

Another policy option is to subsidize education (to reduce the value of CE) or give fiscal incentives to those firms that are implementing the skill premia, and it can happens by a well-implemented income tax.

In this vein, let us consider the following:

1. Let us consider the replicator dynamics from equation (4.7) described in the above Chapter 4, but now we follow the notation for an economy composed by the populations of firms and workers (see the above Section 5.1 and Subsection 5.7.1). The expected payoffs $E(\cdot)$ are defined by the expressions (5.19).

2. Let N_i^τ be the total of $i-$strategists, $i \in \{(I, NI);(S, NS)\}$, from the population $\tau \in \{F, W\}$, and let $H = \sum_i N_i^\tau$ be the total of inhabitants in the whole economy.

   The mass (or number) of player-population, firms and workers, that adopt a strategy i, $\forall i \in \{(I, NI);(S, NS)\}$, is given by:

   $$m_i = \frac{N_i^\tau}{H}$$

   or:

   $$m_I = \frac{N_I^F}{H},\ m_{NI} = \frac{N_{NI}^F}{H},\ m_S = \frac{N_S^W}{H},\ m_{NS} = \frac{N_{NS}^W}{H}.$$

   Then, we denote by, $\Delta = \left\{m \in R_+^k : \sum_{i=1}^k m_i = 1\right\}$ the simplex of R_k in our case $k = 4$.

3. The two populations, F and W, are normalized to 1. Hence, the share of $S-$type strategists and $I-$type strategist are, respectively:

   $$x_S = \frac{N_S^W}{N_S^W + N_{NS}^W}, \quad (5.30)$$

   $$y_I = \frac{N_I^F}{N_I^F + N_{NI}^F}. \quad (5.31)$$

4. Let us assume that the policy maker imposes some income taxations at rate $\gamma \in [0, 1]$ and $\delta \in [0, 1]$ for firms and workers, respectively. When ($\gamma = 0$, $\delta = 0$) means *non-intervention* and ($\gamma = 1$, $\delta = 1$) is just *complete intervention*.

5. Such taxations are imposed to each player-population $i \in \{(I, NI); (S, NS)\}$, and then the total revenue collected in the economy is:

$$\gamma\left[m_I E(I) + m_{NI} E(NI)\right] + \delta\left[m_S E(S) + m_{NS} E(NS)\right].$$

6. In the sequel, $y_F = (y_I, y_{NI})$ and $x_W = (x_S, x_{NS})$ are the vectors of profile distributions of individuals playing a certain pure or mixed strategy in the space $\{(I, NI); (S, NS)\}$, for each population, $\tau \in \{F, W\}$.
7. Let us denote by $\rho^* = (y_I^*, x_S^*) \in \Delta$ a *target state* of profile distributions such that the government considers as a "desirable population state", in the sense that ρ^* has already to overcome the threshold level $G = (\hat{y}_I, \hat{x}_S)$, i.e. the desirable vector of profile distributions are such that:
$$\rho^* = (y_I^*, x_S^*) > \left(\hat{\phi}_I, \hat{\sigma}_S\right).$$
8. We consider that a good implemented policy taxation by the government is characterized for the more subsidies provided, and this is the case of the strategic profile (ϕ_I^*, σ_S^*). Then, the subsidy for each population, $\tau \in \{F, W\}$, which is provided to the player-population profile of innovative firms and high-skilled workers, (I, S), is assumed to be:
$$y_I^* \gamma m_{NI} E(NI)$$
$$x_S^* \delta m_{NS} E(NS)$$
9. Hence, the reallocated subsidy to the strategic profile (I, S) is equally-divided to innovative firms and high-skilled workers. That is, the subsidy which is provided to each player-population with the strategic profile (I, S) is given by:
$$\begin{aligned} \frac{\gamma y_I^* m_{NI} E(NI)}{m_I} &= \gamma\left(\frac{y_I^*}{y_I}\right) y_{NI} E(NI) \\ \frac{\alpha x_S^* m_{NS} E(NS)}{m_S} &= \alpha\left(\frac{x_S^*}{x_S}\right) x_{NS} E(NS) \end{aligned}$$

Thus, a revenue function for firms and workers with rate taxations and subsidies is given by the following equations:

$$\begin{aligned} &(1-\gamma) E(I) + \gamma\left(\frac{y_I^*}{y_I}\right) \bar{E}^F(i), \quad \forall i = (I, NI) \\ &(1-\delta) E(S) + \delta\left(\frac{x_S^*}{x_S}\right) \bar{E}^W(i), \quad \forall i = (S, NS) \end{aligned} \tag{5.32}$$

where $\bar{E}^\tau(i)$ is the average payoff, $\tau \in \{F, W\}$.

Substituting (5.32) for the players' expected payoffs $E(i)$ of replicator equation (4.7), it leads for each pair $i, j \in \{(I, NI); (S, NS)\},\ j \neq i$, of population $\tau \in \{F, W\}$, to get the replicator dynamics with rate taxations and subsidies as the following system:

$$\begin{cases} \dot{y}_I = (1-\gamma)\, y_I \left\{E(I) - \bar{E}^F(i)\right\} + \gamma\,(y_I^* - y_I)\,\bar{E}^F(i) \\ \\ \dot{x}_I = (1-\delta)\, x_S \left\{E(S) - \bar{E}^W(i)\right\} + \delta\,(x_S^* - x_I)\,\bar{E}^W(i) \end{cases} \tag{5.33}$$

where $\gamma, \delta \in [0,1]$ are the taxation rates. The next proposition states the result:

Proposition 5 *The target state $\rho^* = (y_I^*,\ x_S^*) \in \Delta$ is globally asymptotically stable equilibrium point for the system (5.33) if the policy maker adopts the complete intervention* $(\gamma = 1,\ \delta = 1)$.

Therefore, a policy maker should intervene in the economy to implement a good tax policy such that it encourages potential workers to become high-skilled and potential firms to become innovative. In particular, education costs should be equal to the net difference (after taxes) in salaries from the high and low-skilled worker. If such a policy intervention occurs, then the basin of attraction for the high-level equilibrium expands (the basin of attraction of the low-level equilibrium contract).

5.8 Concluding Remarks

We conclude that to overcome a poverty trap it is necessary to surpass a threshold number of innovative firms. Workers will therefore have incentives to improve their skills. Firms, in turn, can obtain more benefits by investing in R&D. We showed the strategic foundations of high and low level equilibria when firms and workers imitate and thus they pick up the best performing strategy given the current state of the economy.

A poverty trap arises because of imitation by the agents when the state of the economy lacks both R&D and human capital. For low numbers of high-skilled labor, innovation will not be profitable, not just in terms of final output but also in terms of the generated rate of technical change. Only after a country is sufficiently developed in terms of its supply and demand for high-skilled labor, will deliberate R&D be undertaken.

The main obstacle to overcoming a poverty trap is agents' own rationality, the rationally occurring poverty trap is inefficient because it comprises rationally low-skilled worker and rationality non-innovative firms. As we showed, there exists a threshold value after which there is a rational motivation to change behavior: surpassing this value the firms and workers will rationally change their behavior such that the Pareto superior equilibrium will eventually obtain.

When innovation or R&D becomes economically feasible, the dedicated research effort generates a long-run economic growth where the number of innovative firms and high-skilled workers is large enough, and therefore the economy converges to the high-level equilibrium $(1, 1)$.

To overcome a poverty trap:

1. *It is necessary to surpass a threshold number of innovative firms and high-skilled workers. Low-skilled workers can be provided with such incentives by being given optimal premia to improve their skills by innovative firms. Hence, firms, in turn, can obtain more benefits being* $I-$*type strategists.*
2. *If the economy does not reach this threshold value, a policy maker should implement an incentive-based policy to reach the high-level equilibrium of innovative firms and high-skilled workers, for instance, a policy intended to lower the cost of attaining skills. The market alone is incapable of overcoming this poverty trap, and policy makers should intervene. For instance, policy makers could provide a tax scheme that encourages R&D investment, or they could impose a minimum level of high-quality education for the whole population.*

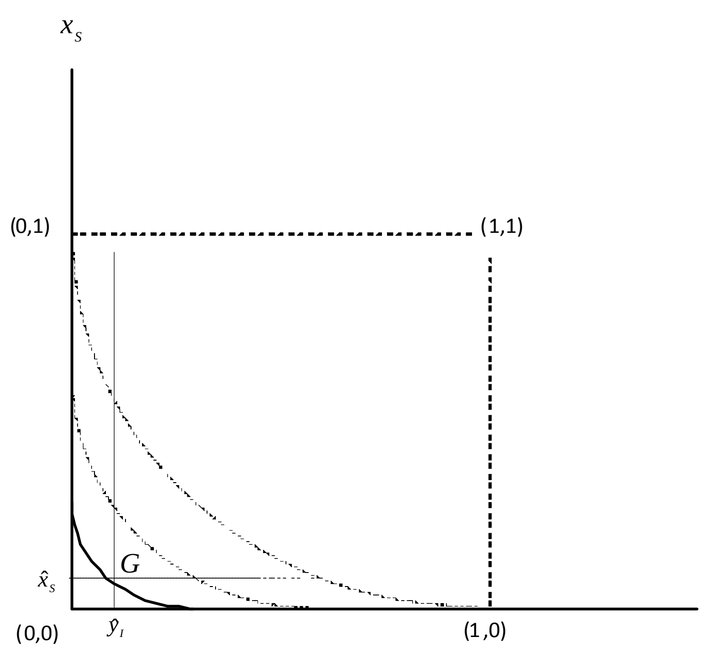

Figure 6a. Decreasing the value of G through education costs, skill premia and payoff taxation.

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