Three Essays on Persuasion and Collective Decision Making

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To my Sister

Acknowledgements
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Abstract

The aim of this thesis is to discuss the role of endogenous information in collective decision making. In particular, we study the outcome of collective decisions in which, prior to casting a vote, individuals observe the realization of a signal whose information content is designed by a player, the Persuader, who tries to persuade the collective that the current state of the world is the one in which his favorite alternative is the best choice.

In the *first chapter* we introduce a simple collective decision problem that we exploit to discuss the role of information and voting behavior on the light of the Bayesian Persuasion Model (Kamenica and Gentzkow, 2011). This introductory chapter attempts to survey the recent contribution regarding information manipulation in the context of collective decisions in order to frame the remaining chapters in this broad literature.

In the *second chapter* we start the analysis considering a collective decision characterized by a fixed number of privately informed individuals. We demonstrate that Persuader's optimal behavior follows the logic of Kamenica and Gentzkow (2011) and, in equilibrium, the Persuader makes individuals indifferent with respect to their private information. We relate our results to the delicate issue arose in the Jury decision making literature that studies the efficiency of unanimous verdicts and we show that, in equilibrium, unanimous verdicts and majority verdicts correspond.

In the *third chapter* we shift our attention on large collective decisions. Individuals are recruited by Nature in a large Poisson game (Myerson, 1998a) and we characterize the outcome of the majority decision in the spirit of the Condorcet Jury Theorem. We consider three different environments, that we call societies, characterized by the following primitives, i) the way in which the signal is conveyed, i.e., private vs public signals, ii) voters' type. We show that regardless from Persuader's attempt to manipulate the outcome of the collective decision, the equilibrium voting behavior of privately informed voters satisfies the Condorcet Jury Theorem. On the contrary, whenever information is publicly supplied, the majority completely obeys the suggestion embedded by the public signal. Finally, we consider how rational voters respond to the presence of a small fraction of independent voters whose behavior is predetermined regardless from the design of information. We demonstrate that, in large elections, as the fraction of not controllable voters goes to zero, the outcome of the game

converges towards an efficient majority decision characterized by a public signal that tends to be perfectly informative.

In the *fourth chapter* we consider a large election in a Poisson framework and we study the manipulability of voluntary voting behavior. We show that, in equilibrium, swing's voters are strategically induced to abstain whenever the fraction of partisan supporting Persuader's favorite alternative is sufficiently large. Nevertheless, Persuader never benefits from marginally increasing the share of his supporters.

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CHAPTER 1

Persuasion and Collective Decisions

According to Merriam-Webster dictionary, persuasion is defined as *the act of causing people to do or believe something.* For a rational and Bayesian decision maker, what she believes determines her action. In a recent work, Kamenica and Gentzkow (2011) develop a *Bayesian Persuasion* model that precisely reflects that idea of persuasion. The model basically relies on the observation that *actions* or *decisions* are the expression of the information environment in which a decision maker finds herself. Were the information environment be manipulable, so would be decision maker's actions. In Kamenica and Gentzkow (2011) there is a receiver who must choose an action whose payoff depends on the actual state of the world that is uncertain to her. On the other hand, prior to making a decision, she observes a signal that is strategically designed by a sender. In particular, sender would like to induce the receiver to take a particular action regardless from the true state of the world. In this sense, sender wants to persuade the decision maker that the actual state of the world is the one in which his favorite action is the optimal choice. Although decision maker knows that sender is trying to persuade her, the information content of the signal is correlated with the true state of the world. For this reason, she might find convenient to change her action with respect to what she would have decided without additional information.

The aim of this chapter is to review the Bayesian Persuasion model on the light of other contributions that extended the model to collective decisions. As a matter of fact, many economic decisions are taken collectively and they rely as much as possible on information in order to guarantee that the best possible decision is made. Nevertheless, information is rarely neutral and committees are no less susceptible to persuasion than single decision makers[1]. During elections, information is released so as to convince voters that a candidate fares batter than another, during a trial, the prosecutor advances evidence suggesting that the defendant is guilty. The Condorcet Jury Theorem states the conditions under which majority decisions are expected to be efficient and asymptotically correct. Nevertheless, the Theorem fails to embed in its formulation the source of information. That is to say, the Theorem assumes that individuals observe some sufficiently precise information though it is not discussed who and why supplies that information. As we will argue throughout the chapter, the possibility to design the information environment of a committee crucially affects the working of the Theorem.

This chapter is organized as follows. We introduce a simple decision problem that would serve as an example throughout the discussion. We discuss the role of information and the behavior of strategic decision makers. Then, we introduce the Bayesian Persuasion model and we show, exploiting the simple example we introduced, how the strategic design of information affects the quality of collective decisions that rely on the supplied information. This introductory chapter can be viewed as a short survey on collective decisions and Bayesian persuasion that contextualizes the following chapters in this broad literature.

1.1 The Decision Problem

The decision problem encompasses a variety of situations. A decision maker has to take an action whose payoff depends on the realization of a state variable. Let $\Omega = \{\alpha, \beta\}$ denotes the set of states of the world and $X = \{A, B\}$ the action set or *alternatives*. The decision maker wants to match her action with the unknown state of the world. She gets a payoff of 0 whenever $x = A$ and $\omega = \alpha$ or $x = B$ and $\omega = \beta$. On the contrary, whenever her action does not match the state of the world she suffers an utility loss. Formally, let $u(x, \omega)$ denotes decision maker's utility. Then:

$$u(x, \omega) = \begin{cases} 0 & \text{if } x = A, \omega = \alpha \text{ or } x = B, \omega = \beta \\ -q & \text{if } x = A,\ \omega = \beta, \\ -(1-q) & \text{if } x = B,\ \omega = \alpha. \end{cases}$$

Throughout the chapter we make the following assumptions:

[1] McCloskey and Klamer (1995) argue that a considerable amount of resources are devoted to persuading activities. DellaVigna and Gentzkow (2009) provide empirical evidence on the effect of persuasive communication directed to consumers, voters, donors and investors.

i) the states of the world are equally likely,

ii) $q \in (\frac{1}{2}, 1)$.

These assumptions are without loss of generality and are common in a variety of models concerning collective decision making. In the "Jury" literature the parameter q represents the threshold of reasonable doubt. The reason is the following. A decision maker would prefer alternative A over B if and only if

$$-q \times \Pr\{\omega = \beta\} \geq -(1-q) \times \Pr\{\omega = \alpha\} \iff \Pr\{\omega = \alpha\} \geq q,$$

that is to say, if and only if the likelihood of being in state α is sufficiently solid. These assumptions directly implies that decision maker's default action is B. Suppose now that prior to making a decision, the decision maker observes a signal $s \in \{a, b\}$ correlated with the true state of the world and denote its precision in state α and β with π_a and π_b, respectively[2]. Then, decision maker selects alternative A if and only if

$$\Pr\{\omega = \alpha | s\} \equiv \mu_\alpha(s) \geq q \tag{1.1}$$

where $\mu_\omega(s)$ is defined as the posterior belief about state ω given the observation of signal s. In the following section we describe the collective decision problem and we relate it to the Condorcet Jury Theorem.

1.1.1 The Collective Decision Problem

We translate now the simple problem into a collective decision. Consider a committee $\mathcal{I}$ made of three identical individuals. Each individual simultaneously casts a private vote for one of the two alternatives and the committee implements the most voted alternative. As usual, prior to casting a vote, individuals observe the realization of a signal correlated with the true state of the world as described above.

It is important to stress the difference between *private* and *public* signals.

Definition 1.1. Public Signal
The signal s is public if and only if s is public information, that is

$$\forall i \in \mathcal{I},\ s_i = s.$$

Definition 1.2. Private Independent Signals
The signal s_i is private if and only if s_i is private information, that is

$$\forall i \in \mathcal{I},\ \Pr\{s_i = s|\omega\} = \pi(s|\omega).$$

[2]Formally, $\pi_a = \Pr\{s = a|\alpha\}$ and $\pi_b = \Pr\{s = b|\beta\}$. We will also use the notation $\pi(s|\omega)$ to indicate the probability with which s is observed in state ω.

Whereas the members of a publicly informed committee share the same information, a member of a privately informed committee only knows her type and she is uncertain about what information have the other individuals observed[3].

We start the analysis assuming that individuals are independently informed by a private signal. Hence, the collective decision problem is the standard problem of a "Condorcet Jury" and the model considered here is thus a simplified version of Austen-Smith and Banks (1996). In particular, majority rule defines a Bayesian game and we are interested in the Bayes Nash equilibria of the game. The seminal contribution of Austen-Smith and Banks has been to show that sincere voting does not generally constitute an equilibrium of the game. Precisely, sincere voting constitutes an equilibrium of the game if and only if some stringent conditions are met[4]. The sincere voting behavior evidently represents a reference point as it requires individual to sincerely vote according to their private information and traces the exogenous behavior considered originally by Condorcet[5]. Therefore, Austen-Smith and Banks point out that the voting behavior considered by Condorcet is not a rational equilibrium of the game. To explain this point we need to define voting strategies. A voting strategy for individual $i \in \mathcal{I}$ is a map $v_i : S \to X$ from the set of signals to the set of alternatives. A voting strategy is **informative** if

$$v(a) = A \text{ and } v(b) = B.$$

A voting strategy is **sincere** if $v(s_i) = x$ if and only if for all $x' \neq x \in X$

$$\mathbb{E}[u(x, \omega)|s_i] \geq \mathbb{E}[u(x', \omega)|s_i].$$

The sincere voting condition expresses a rational condition from the point of view of an individual that acts independently from the other members of the committee. Finally, we say that the voting strategy $v(s_i)$ is **rational** if and only if it constitutes a Bayesian Nash equilibrium of the game. Therefore, differently from the sincere voting condition, rationality is a property of a profile of voting strategies rather than an individual rationality condition. As Austen-Smith and Banks explain, individuals have noisy information about other individuals' private signals. In principle, this information can be used to assess the probability with which a single vote could change the outcome of the collective decision

[3]Individual's private information determines her type for her preferences vary endogenously according to the observation of her private signal $s_i \in \{a, b\}$. If $\pi_s > \frac{1}{2}$, then type $s_i = a$ leans toward alternative A whereas type $s_i = b$ leans toward B.

[4]These conditions will be presented shortly.

[5]The original formulation of the Theorem takes individual behavior as given and the Theorem is described from a statistical perspective and not from a game theoretical point of view.

as individuals anticipate that only in these pivotal events their vote may affect their payoff. Formally, $v(s_i)^*$ constitutes a BNE of the game if and only if for all $i \in \mathcal{I}$

$$\mathbb{E}[u(x^*, \omega)|Piv(v^*_{-i}), s_i] \geq \mathbb{E}[u(x', \omega)|Piv(v^*_{-i}), s_i] \tag{1.2}$$

where $v^*(s_i) = x^*$. Importantly, the probability of casting a pivotal vote depends on the voting strategy being played. According to the informative voting strategy, an individual is pivotal when the remaining individuals observed contradictory information, that is with probability

$$Piv(\alpha) = \binom{3}{2}\pi_a(1 - \pi_a)$$

in state α, and with probability

$$Piv(\beta) = \binom{3}{2}\pi_b(1 - \pi_b)$$

in state β. The formulae follow taking into account all possible combinations with which two over three members of the committee may end up observing opposite information in the respective state of the world. Since private signals are independent across individuals, the expected utilities deriving from voting informatively are

$$\mathbb{E}[u(A, \omega)|Piv(v_{-i}), a] = -q \times \mu_\beta(a) \times Piv(\beta),$$
$$\mathbb{E}[u(B, \omega)|Piv(v_{-i}), b] = -(1 - q) \times \mu_\alpha(b) \times Piv(\alpha).$$

From equilibrium condition (1.2), informative voting is rational if and only if

$$\frac{\mu_\alpha(a)}{\mu_\beta(a)}\frac{1-q}{q} \geq \frac{Piv(\beta)}{Piv(\alpha)} \geq \frac{\mu_\alpha(b)}{\mu_\beta(b)}\frac{1-q}{q}. \tag{1.3}$$

Austen-Smith and Banks (1996, Theorem 1), state that for $\pi_a = \pi_b$, *sincere voting is both informative and rational when majority rule is being used to aggregate individuals' votes and majority rule is the optimal method of aggregating individuals' information.* To grasp this result notice that whenever $\pi_a = \pi_b$ the probability of casting a pivotal vote is the same in both states. For this reason, individuals are indifferent with respect to their vote and informative voting is rational and sincere[6]. Therefore, the symmetric signal $\pi_a = \pi_b$ is a very special requirement for informative voting to be rational as it frustrates pivotal considerations. In general for $\pi_a \neq \pi_b$ informative voting does not constitute an equilibrium when votes are aggregated through simple majority as in this case. For instance, suppose that $q = 0.6$, $\pi_a = 0.8$ and $\pi_b = 0.6$. Informative

[6]Clearly, majority rule is optimal only if $\pi_a = \pi_b \geq q$.

voting is sincere since $\mu_\alpha(a) > q$ and $\mu_\beta(b)1 - q$, nevertheless it does not satisfy equilibrium condition (1.3). Individuals would have overwhelming evidence of being pivotal in state β which prevents informative voting being rational as can be seen from the ratio $\frac{Piv(\beta)}{Piv(\alpha)} = 1.5$. Therefore

$$\mathbb{E}[u(B,\omega)|Piv(v_{-i}), b] > \mathbb{E}[u(A,\omega)|Piv(v_{-i}), b]$$

whereas

$$\mathbb{E}[u(A,\omega)|Piv(v_{-i}), a] < \mathbb{E}[u(B,\omega)|Piv(v_{-i}), a]$$

which implies that everyone voting informatively does not constitute an equilibrium of the game. Which are the equilibria of the game then? It is forthright to verify that everyone voting for alternative B does constitute an equilibrium that dominates the equilibrium in which everyone votes for alternative A. However, these rational voting behaviors violate the behavioral assumption made by Condorcet. We report the Theorem[7].

Condorcet Jury Theorem. Consider two mutually exclusive alternatives A and B such that for all $i \in \mathcal{I}$ one alternative is unequivocally better although the identity of the best alternative is in fact unknown. Suppose that each $i \in \mathcal{I}$ independently votes for the correct alternative with probability $p > \frac{1}{2}$. Then:

i) The probability with which the majority selects the correct alternative exceeds p;

ii) As the size of the committee approaches infinity the probability with which the majority selects the correct alternative goes to one.

When informative voting is not rational, the assumptions of the Theorem may be violated and the majority would not be accurate even if the committee were large.

Nevertheless, there are equilibrium behaviors consistent with the Theorem that arise whenever individuals are allowed to randomize their vote. Indeed, Wit (1998) shows that there are mixed strategy equilibria that aggregate information[8] and are consistent with the Theorem. Consider the previous example. There is an equilibrium in mixed strategies so that individuals always vote for B whenever $s_i = b$ and they randomize between A and B whenever $s_i = a$. Denote this mixed strategy with $\sigma_B = \Pr\{x = B|b\} = 1$ and $\sigma_A = \Pr\{x = A|a\} \in (0, 1)$,

[7] For a detailed formulation of the Theorem see Black (1958); McLean and Hewitt (1994). Miller (1986) provides some extensions; among others Young (1988); Ladha (1992) use the Theorem for a positive argument in favor of majority decisions.

[8] We say that information aggregates whenever the outcome of the collective decision is the decision individuals would have made if they knew the true state of the world.

respectively. This mixed voting strategy must make individuals indifferent about A or B conditional on $s_i = a$, thus in equilibrium

$$\frac{\mu_\alpha(a)}{\mu_\beta(a)}\frac{1-q}{q} = \frac{\pi_b(1-\pi_b)}{(\sigma_A\pi_a)(1-\pi_a\sigma_A)}.$$

It can be easily verified that $\sigma_A = 0.956$ and $\sigma_B = 1$ is an equilibrium of the game for $q = 0.6$, $\pi_a = 0.8$ and $\pi_b = 0.6$. In general, Wit (1998, Theorem 1, 2) shows that these mixed strategy equilibria satisfy the Condorcet Jury Theorem. A more general result is derived by McLennan (1998) who shows that *when sincere voting has the desirable properties described by Condorcet, there are Nash equilibria that also have these properties.*

The existence of equilibria that satisfy the Condorcet Jury Theorem advances the idea that large committees not only use all the available information efficiently, but also that they are asymptotically almost surely correct. Therefore, despite the failure of informative voting behavior being rational, the Condorcet Jury Theorem does hold in equilibrium. Nevertheless, Miller (1986) underlines that the requirements of the Theorem are very specific and some of its assumptions may not be considered reasonable approximations in general. The Theorem requires individuals to recognize a common goal. However, during elections, voters may not share a common goal, i.e., a Pareto superior alternative might not exist, and it would be simply misleading to talk about the accuracy of majority decisions. In this work, we focus our attention exclusively on the role of information and individual strategic behavior and we turn now on the case in which information is publicly shared.

When the signal is publicly observed there are just two cases to consider. With probability $\frac{1}{2}\pi_a + \frac{1}{2}(1-\pi_b)$ all individuals observe the public signal $s = a$ whereas with probability $\frac{1}{2}\pi_b + \frac{1}{2}(1-\pi_a)$ all individuals observe the public signal $s = b$. As long as information is perfectly correlated, the informative voting strategy does not generate additional equilibrium information concerning the probability of casting a pivotal vote. Intuitively, all individuals possess the same information and no single vote could be pivotal. Therefore, it is natural to focus on *sincere* voting equilibria[9]. Importantly, notice that if individuals are identical and they observe a public signal prior to casting a vote, the size of the committee does not affect the collective decision. The asymptotic statement of the Condorcet Jury Theorem does not apply and the behavior of the majority is not distinguishable from the behavior of any member of the committee.

[9]It is easy to verify that the strategy "always vote for alternative A" constitutes an undominated equilibrium. Nevertheless, this outcome is unlikely because voters are coordinating on always choosing their worst option.

In the next section we consider the same decision problem although we introduce an additional player, the persuader, who is able to design the information content of the signal. Therefore, we transform the decision problem in a Bayesian Persuasion problem.

For a comprehensive literature review about the Condorcet Jury Theorem see Piketty (1999) and more recently Vlaseros (2014).

1.2 Persuading a Decision Maker

In the previous section we described a basic decision problem. Given a threshold of reasonable doubt, relation (1.1) expresses the idea that a decision maker would prefer action A if and only if the observed evidence of being in state A is sufficiently strong. Formally, the observation of a signal correlated with the true state of of the world allows the decision maker to update her beliefs according to Bayes rule:

$$\Pr\{\omega = \alpha | s\} \equiv \mu_\alpha(s) = \frac{\frac{1}{2}\pi(s|\alpha)}{\frac{1}{2}\pi(s|\alpha) + \frac{1}{2}\pi(s|\beta)}. \tag{1.4}$$

Clearly the posterior belief of being in state α depends on correlation between s and the true state of the world. In this section we study the following problem as described by Kamenica and Gentzkow (2011). Could a persuader induce the decision maker to select his favorite alternative more often than she naturally would? The answer is yes whenever the persuader controls decision maker's information environment. To stick with our example, suppose that now there is a persuader who can design the information content of the signal and that he prefers alternative A regardless from the true state of the world. Kamenica and Gentzkow (2011) make the following general observations:

i) Each realization $s \in \{a, b\}$ leads to a posterior belief,

ii) Each signal $\{\pi(s|\omega)\}_\omega$ leads to a distribution of posterior beliefs,

iii) A distribution of posterior induces a distribution of decision maker's action.

Persuader's problem is that of finding a signal that maximizes the probability with which the decision maker selects his favorite alternative. These observations allow them to simplify the problem in a significant way. In particular, consider the properties of an optimal signal. The optimal signal induces a distribution of actions that maximizes persuader's expected utility. Therefore, Kamenica and Gentzkow look for the distribution of posterior that induces such distribution of actions subject to the Bayes rationality condition, that is the expected posterior must equal to the prior. Hence they construct a *concavification* of persuader's expected utility which associates an optimal and rational distribution of posterior beliefs as a function of decision maker's prior belief. Their approach characterizes in a very simple way the optimal signal. Denote with $\tau(s)$ the probability with which signal s occurs. Then, the optimal signal satisfies:

$$\pi(s|\omega) = \frac{\mu_\omega(s)\tau(s)}{p(\omega)} \tag{1.5}$$

where with $p(\omega)$ we denote the prior probability of state ω. Importantly, the optimal signal must induce a Bayes plausible belief, that is:

$$\sum_{s\in S} \mu_\omega(s) \times \tau(s) = p(\omega) \tag{1.6}$$

which means that the expected posterior probability equals the prior. When the state space is binary the problem can be simply solved directly looking for those $\pi(s|\omega)$ that maximizes persuader's expected utility. Consider the decision problem described in the previous section. Recall that the persuader prefers action A regardless from the true state of the world. On the other hand, since $q > \frac{1}{2}$, decision maker's default action is B. It is intuitive to see that there does not exist a signal that induces the decision maker to always choose A. Persuader's best option is to design a signal that provides a recommended action, that is whenever $s = A$ ($s = b$) the decision maker plays A (B). As Kamenica and Gentzkow show, the existence of this signal is related to the *revelation principle* Myerson (1979). Recall that:

$$\tau(a) = \frac{1}{2}\pi_a + \frac{1}{2}(1 - \pi_b). \tag{1.7}$$

Thus, in equilibrium the persuader maximizes the probability with which a occurs, that is $\tau(a)$. However, there is a constraint that concerns the persuader[10]. In particular, the signal must be incentive compatible which requires

$$\mathbb{E}[u(A,\omega)|a] \geq \mathbb{E}[u(B,\omega)|a] \text{ and } \mathbb{E}[u(B,\omega)|b] \geq \mathbb{E}[u(A,\omega)|b].$$

Notice that this is exactly the sincere voting condition. Then, the persuader maximizes $\tau(a)$ subject to the sincere voting condition. It is a linear simple problem that could be solved noting that in equilibrium it must be the case that $\pi_a = 1$. The reason is straightforward since for $\pi_a = 1$ alternative A is always

[10]If we directly look for those $\pi(s|\omega)$ that maximize persuader's expected utility, the Bayes rationality condition is implicitly satisfied. Nonetheless, the suggested recommendation must be incentive compatible from decision maker's point of view.

implemented in state A. Observe that for $\pi_a = 1$ the condition $\mathbb{E}[u(B,\omega)|b] \geq \mathbb{E}[u(A,\omega)|b]$ is always satisfied because $\mu_\beta(b) = 1$. On the contrary, $\tau(a)$ is strictly decreasing in π_b. Observing that in equilibrium it must be the case that $\pi_a = 1$, the condition $\mathbb{E}[u(A,\omega)|a] \geq \mathbb{E}[u(B,\omega)|a]$ translates into

$$-q \times \mu_\beta(a) = -q\frac{1-\pi_b}{1+1-\pi_b} \geq -(1-q) \times \mu_\alpha(a) = -(1-q)\frac{1}{1+1-\pi_b}.$$

Rearranging the inequality we get

$$\pi_b \geq 1 - \frac{1-q}{q}$$

which must hold with equality in equilibrium. Notice that at $\pi^* = (1, 1-\frac{1-q}{q})$ we have $\mathbb{E}[u(A,\omega)|a] = \mathbb{E}[u(B,\omega)|a]$ and $\mathbb{E}[u(B,\omega)|b] > \mathbb{E}[u(A,\omega)|b]$. To verify that this is the unique equilibrium signal, for $\pi_a < 1$ or $\pi_b > 1 - \frac{1-q}{q}$ the probability with which a occurs, and thus alternative A is selected, decreases. On the other hand, $\pi_b < 1 - \frac{1-q}{q}$ violates the sincere voting condition for $\pi_a = 1$. In equilibrium we have that $\tau(a) = \frac{1}{2}\left[1 + \frac{1-q}{q}\right]$. It can be verified that π satisfies (1.5).

The optimal signal is characterized by the fact that the decision maker is sure of her action when she observes $s = b$. This happens precisely because the signal is always truthful in state α which means that b can only be observed in state β. Second, the optimal signal makes the decision maker indifferent when she observes $s = a$. Intuitively, if she were strictly better off selecting A, the persuader could slightly decrease her confidence by increasing the probability with which his signal results in the recommendation a. Finally, from the previous point, the decision maker does not gain from being persuaded. She gets the same expected utility she would get by always choosing alternative B.

The model can be applied to a variety of situations and we report Kamenica and Gentzkow (2011)'s example to grasp the variety of situations that could be captured by the Bayesian persuasion framework. A Judge must decide whether to convict or to acquit a defendant. The Judge believes that the defendant is innocent 70% of the times. On the other hand, the prosecutor wants to convict the defendant regardless from the fact that he might be innocent. Kamenica and Gentzkow interpret the signal as an investigation that must be truthfully reported to the Judge. That is to say, the prosecutor designs π that generates signal s with probability $\tau(s)$. Thus, we can think of the realized s as a witness testimony, the result of a legal report and so on. The prosecutor can not lie about the realization of the signal. The prior belief is sufficient to determine the optimal signal. From the concavification of prosecutor's expected utility, they find according to (1.5) the following optimal signal:

$$\text{Prob}\{guilty|GUILTY\} = 1 \text{ and } \text{Prob}\{innocent|INNOCENT\} = \frac{4}{7}.$$

That is, the Judge observes the signal "the defendant is guilty" with probability 1 one when the defendant is effectively guilty and he observes the signal "the defendant is innocent" just with probability $\frac{4}{7}$ when the defendant is innocent. This signal induces a distribution of posterior that is Bayes plausible and hence incentive compatible. Finally, at the optimal signal the Judge convicts the defendant 60% of the times although she knows that he is guilty just 30% of the times.

Before introducing the collective decision, it is worth to stress that Kamenica and Gentzkow's approach can be extended quite easily to a multiple receivers setting if each receiver only cares about her own action and the persuader supplies a public signal. On the contrary, the concavification approach does not extend easily in those situations in which receivers care about each other's action and the persuader sends private signals. In the next section we also deal with that scenario although we stress that the results can not be obtained constructing the concavification of persuader's expected utility.

1.2.1 Persuading Decision Makers

Whenever the persuader releases information in pursuance of persuading a group of individuals, there are two important aspects that should be considered. First, the persuader could supply public or private information. Second, how is the collective decision made? The way in which the collective decision is implemented clearly affects the design of the optimal signal. Intuitively, we may think think that persuading a group to reach an unanimous consent is harder than obtaining a majority consent. As a matter of fact, this intuition is proven to be correct as Alonso and Câmara (2015) and Wang (2015) show. We could also ask which is the simpler way to persuade a group so as to reach the desired collective decision. Shall the persuader privately persuade each individual or shall he publicly persuade the committee? This issue has been explored mainly by Wang (2015) and Taneva (2015). In this section we work on our simple example to introduce the main results derived in the aforementioned articles.

To begin with, we consider the case in which the persuader designs a public signal. If the members of the committee are identical, they all share the same threshold q, persuader's problem is straightforward. If the collective decision relies on simple majority, the persuader has to convince at least two over three individuals. Nonetheless, as long as the signal is publicly observed and all individuals are identical, it is as if the persuader is persuading a single individual. In such a case, there would be no difference between simple majority or any other k-rule up to unanimity. The reason is simple. The realization of the signal is publicly observed and, in equilibrium, individuals vote sincerely. Alonso and Câmara (2015) on the contrary assume that individuals differ in their preferences. As they show, this introduces a new rationale for persuasion. The persuader is able to identify a *winning coalition* that responds sincerely to his signal whereas the remaining voters would always neglect public information. Whenever individuals share *common value* preferences, the problem is simpler for the persuader could directly target a representative voter who splits the electorate in two coalitions. The first coalition includes the representative voter and obeys the public signal. The second coalition acts regardless from the public signal. Nevertheless, the first coalition includes the representative voter and consists exactly of the majority of voters.

Individuals have common value preferences if they all agree that alternative A is the best alternative in state α and alternative B is the best alternative in state β. However, individuals may differ in their preference intensity q_i. We assume that $\frac{1}{2} < q_1 < q_2 < q_3 < 1$ so that, although individuals share a common goal leaning toward the same alternative, the third individual is the *hardest to persuade* for she requires the stronger evidence in order to support alternative A. Recall that $q > \frac{1}{2}$ implies that, by default, all individuals would prefer alternative B. If the collective decision is implemented according to majority rule, it is straightforward to see that the second individual is the representative voter. That is to say, in equilibrium the persuader designs a signal that persuades individual 1 and 2 whereas the third individual is never willing to support alternative A. To understand this equilibrium behavior, consider the optimal signal when the decision is made by just one decision maker, that is:

$$\pi_a^* = 1 \text{ and } \pi_b^* = 1 - \frac{1-q}{q}.$$

Clearly, π_b^* is increasing in q. The reason is the following: the larger the threshold of reasonable doubt, the more the evidence the individual requires to support alternative A. Thus, as q increases the persuader must supply a signal less biased in favor of recommendation A otherwise the individual would not respond sincerely. Therefore, as long as the voting rule is simple majority, the optimal signal persuades individual 1 and 2. On the contrary, unanimous verdicts are harder to reach. According to unanimity, the persuader must also persuade the third individual who is the hardest to persuade. Indeed, it is easily verified that whereas the persuader prefers majority rule, the committee prefers unanimity or to delegate the decision to the third individual.

The persuader might easily persuade a committee to select his favorite alternative if he can supply a public signal. What happens when he can not supply a public signal? Let us assume now that whereas the persuader still designs π, each individual privately observes the realization $s_i \in \{a, b\}$. We can think of this scenario in two different ways. The persuader individually approaches the members of the committee who are "persuaded" only if they observe persuader's favorite realization. Or, the persuader chooses the distribution of types in the committee. As a matter of fact, the choice of π_a and π_b determines how private information is distributed in the committee.

As individuals are privately informed, rational voting behavior is affected by pivotal considerations. Suppose first that $q_i = q > \frac{1}{2}$ for all $i \in \mathcal{I}$. It is interesting to realize that the optimal public signal derived above is never optimal whenever individuals are privately informed. As it is clear from (1.3), equilibrium voting behavior is determined by the ratio

$$\frac{Piv(\beta)}{Piv(\alpha)} = \frac{\pi_b(1-\pi_b)}{\pi_a(1-\pi_a)}.$$

At the optimal public signal $\pi_a = 1$. This provides overwhelming evidence of casting a pivotal vote in state β. For in state α all individuals observe $s = a$, the only case in which a type a voter could be pivotal is when $\omega = \beta$. Therefore, this signal places null probability of being pivotal in state α and prevents informative voting being an equilibrium. The persuader could induce individuals to vote informatively supplying a symmetric signal satisfying $\pi_a = \pi_b \geq q$. Notice that now the probability with which alternative A is selected, $W_A(\pi)$, according to the informative voting behavior is

$$W_A(\pi) = \frac{1}{2}\left[\pi_a^3 + \binom{3}{2}\pi_a^2(1-\pi_a)\right] + \frac{1}{2}\left[(1-\pi_b)^3 + \binom{3}{2}(1-\pi_b)^3\pi_b\right]. \quad (1.8)$$

The meaning of this expression is straightforward. In state ω alternative A is selected whenever all individuals observe $s = a$ or when a combination of two over three individuals observes $s = a$ and one individual has observed $s = b$. When $\pi_a = \pi_b$, alternative A is selected 50% of the times. Recall that if the persuader would not engage in information control, alternative A would never win. Thus, a symmetric signal significantly increases persuader's expected payoff. Nevertheless, the persuader can do better than that. If we allow individuals to randomize their vote, the persuader could design an asymmetric signal $\pi_a > \pi_b$ so that $\sigma_A \in (0, 1)$ and $\sigma_B = 1$. The optimal signal guarantees that the probability with which a single vote is in favor of alternative A in state α, $v_A(\alpha)$, is higher than the probability with which a single vote favors alternative B in state β, $v_B(\beta)$, that is

$$v_A(\alpha) = \sigma_A \pi_a > v_B(\beta) = \pi_b.$$

Equation (1.8) could be rewritten in terms of voting probabilities as follows:

$$W_A(\pi) = \frac{1}{2}\left[v_A(\alpha)^3 + \binom{3}{2} v_A(\alpha)^2(1 - v_A(\alpha))\right] + \frac{1}{2}\left[(1 - v_B(\beta))^3 + \binom{3}{2}(1 - v_B(\beta))^3 v_B(\beta)\right]$$

which is clearly increasing in $v_A(\alpha)$ and decreasing in $v_B(\beta)$ and justifies the structure of the signal. Intuitively, for $\pi_a > \pi_b$, individuals understand that they are more likely to individually observe $s_i = a$. If they were voting informatively, alternative A would result to be the winning alternative too often relatively to their preferences. The equilibrium behavior is straightforward since $\sigma_A < 1$ reduces the probability of a single vote supporting alternative A. However, as long as $v_A(\alpha) \geq v_B(\beta)$, persuader is, at least, weakly better off.

Importantly, observe that the nature of the signal is affected by the size of the committee. If there were more than three voters, the probability of casting a pivotal vote in state ω would change and the persuader would adjust the information content of the signal. Nevertheless, recall that this kind of voting behavior satisfies the Condorcet Jury Theorem. That is to say, although for a finite number of voters the persuader can induce the majority to select alternative A most of the times, as long as votes are independent, a very large committee would still select the correct alternative most of the times.

Wang (2015) and Taneva (2015) show that the persuader is weakly better off under a public signal. Nevertheless, also in the private case the persuader may exploit preference diversity in order to increase his chances of manipulating the collective decision. Suppose now that $q_1 = 0.6$, $q_2 = 0.7$ and $q_3 = 0.8$. The optimal public signal that maximizes the probability with which majority selects alternative A is characterized by $\pi_a^* = 1$ and $\pi_b^* = \frac{4}{7}$. Nevertheless, a privately informed committee would not vote informatively according to this signal because of pivotal considerations. However, the persuader can exploit individuals' differences so as to induce a profitable asymmetric profile. In particular, he designs π so that individuals 1 and 2 vote informatively whereas the third voter always support alternative B. To see why this voting profile is rational and profitable, consider the event in which individual 1 or 2 is pivotal. Without loss of generality, consider individual 1. She is pivotal when individual 2 observes signal a. Indeed, the third individuals always vote for alternative B, thus this is the only event in which a tie could occur. The equilibrium condition for individuals 1 and 2 becomes:

$$\frac{\pi_a}{1-\pi_b}\frac{1-q_i}{q_i} \geq \frac{1-\pi_b}{\pi_a}, \quad i = 1, 2.$$

If the persuader makes the second voter indifferent, then the first individual is strictly better off voting informatively[11]. In equilibrium the persuader would choose $\pi_a^* = 1$ and $\pi_b^* = \sqrt{\frac{q_2}{1-q_2}}$. This signal is obtained simply solving the equilibrium condition for the second individual. Although individuals are privately informed, the persuader is able to set $\pi_a^* = 1$ which implies that alternative A always win in state α.

In this section we questioned one assumption of the Condorcet Jury Theorem and we showed how its positive results are weakened if we consider information endogenously supplied rather than exogenously given. We relied on a simple example to show how the insights of the Bayesian Persuasion model apply to collective decision settings. We stressed the importance of the nature of the signal. Whenever the persuader is able to supply a public signal, individuals vote sincerely and in equilibrium they share they same posterior belief. Thus, the persuader simply targets the representative voter which is identified according to the voting rule. In our example, we ordered individuals according to their threshold of reasonable doubt so that the third voter was the hardest to persuade. If the collective decision is made through majority rule, the median or representative voter is the second individual. Importantly, following Alonso and Câmara (2015) we underlined that in such a situation, unanimity would serve better the purpose of the committee rather than majority rule. Whenever the persuader can not supply a public signal, voting behavior dramatically changes. Individuals condition their vote on the event of being pivotal which reduces the extent with which the persuader can manipulate the outcome of the collective decision. Nevertheless, also in this case, preferences diversity may allow the persuader to target a coalition of voters increasing his capabilities of manipulating the collective decision.

1.3 Conclusion and Foreshadowing

The Condorcet Jury Theorems promotes the efficiency of majority decisions considering information neutrally supplied. However, information is rarely neutral. In this chapter we considered a simple collective decision problem and we introduced a new player who acts as a Bayesian persuader. We showed that the

[11] Clearly, if the second voter is indifferent, the third voter is not willing to vote informatively since $q_3 > q_2$.

optimistic conclusions advanced by the Condorcet Jury Theorem may not be founded for a non neutral information design. In particular, the Theorem completely underestimates the importance of public information that nevertheless completely sways majorities outcomes. We relied on a simple example to show the working of the Bayesian Persuasion model and how its insights applies to collective decisions in order to contextualize the following chapters in these two broad literatures.

In the next chapter we allow individuals to possess both private and public information. Private information is exogenous and reflects individuals' differences which arise endogenously in the model. Nevertheless, individuals also observe a strategically designed public signal. Kawamura and Vlaseros (2015) and Liu (2015) show that in presence of both kind of signals there are several equilibria although they argue that the equilibrium in which individuals neglect their private information is the "intuitive" equilibrium, in the sense that it is easier to reach since it requires a lower degree of coordination with respect to a mixed strategy equilibrium. Rather, we show that whenever the public signal is endogenously supplied, this intuitive equilibrium is indeed the unique equilibrium of the game. In addition, we show that in such a situation, unanimity serves better the purpose of a Jury deciding whether to convict or not a defendant.

In the third and the fourth chapters we shift our attention to collective decisions characterized by a large number of voters. Myerson (1998a) argues that in large games individuals may only have an imperfect description of the exact number of voters. For instance, during elections, it would be difficult for an individual to assess the exact number of voters showing up at the ballots. In the third chapter we asymptotically characterize the outcome of a collective decision in which information is strategically designed by an interested party. We characterize the asymptotic properties of the collective decision for privately informed voters and publicly informed voters. In addition, we also consider an imperfect public signal in the sense that, due to the presence of a small fraction of irrational voters whose behavior is not manipulable by information, although rational voters vote informatively, votes are less than perfectly correlated. The main goal of this chapter is to show that interested party's gain from transforming an imperfect public signal to a perfect public signal depends on the size of the electorate. For a large electorate, an imperfect public signal implicitly allows voters to form beliefs about the probability of casting a pivotal vote. As the signal tends to become perfectly correlated and thus perfectly public, voters tend to acquire overwhelming information about casting a pivotal vote that undermines interested party's goal. On the other hand, for a smaller electorate, as the signal tends to become perfectly correlated pivotal considerations lean to disappear allowing the interested party to perfectly control voters' ballots.

Finally, in the fourth chapter we study a large election characterized by the presence of partisan and rational voters. Rational voters could abstain and we show how a persuader could strategically exploit abstention to his advantage when he has the support of the majority of partisan voters. Swing voters are fooled in the sense that information is designed in such a way to convince them to stay away from the ballots. In order to induce rational voters to cast a vote for a particular alternative, persuader needs to supply a signal whose information content is sufficiently precise and not extremely biased towards his favorite alternative. On the other hand, if persuader wants voters to abstain, he can reduce the informativeness of his signal. Thus, rational voters are induced to abstain with high frequency leaving the decision to partisan voters. Importantly, differently from Feddersen and Pesendorfer (1996), we show that under the influence of an information controller, voluntary voting behavior could be strategically manipulated preventing information to aggregate.

CHAPTER 2

Voting with Public Evidence

Endogenous Information and Persuasion

Introduction

Individuals involved in collective decisions usually rely on information in order to make the most accurate decision. Individuals belonging to the decision board may rely on their private information but also on public information. Public information may be released by experts summoned by the committee itself though public information is pervasive and could be related to many different sources.

Recently, Kawamura and Vlaseros (2015) and Liu (2015) analyzed collective decisions characterized by the presence of both kind of information. In particular, they study the outcome of a voting game in which individuals, in addition to their private information, observe some public information. As Austen-Smith and Banks (1996), they show that there is a prominent equilibrium characterized by a voting strategy that perfectly correlates the votes with the realization of the public signal, regardless from voters' private information. Kawamura and Vlaseros (2015) argue that this equilibrium is *intuitive* and support their claim with empirical evidence. They show that, in a controlled laboratory experiment, individuals were more prone to coordinate their vote on the public signal rather than showing behaviors that required the use of both kind of information and randomization. Although Kawamura and Vlaseros (2015) demonstrate the existence of complex rational voting behaviors that satisfy the Condorcet Jury Theorem, they did not find empirical evidence supporting these behaviors. This may raise several concerns about the widely used assumption about privately informed voters[1] as it neglects the importance of public information and its

[1] For instance, the strategic Condorcet Jury Theorem literature initiated by Austen-Smith

effect on voting behavior.

As in Kawamura and Vlaseros (2015) and Liu (2015), we study a collective decision problem. Individuals are members of a committee who must vote for one over two alternatives. Voters are privately informed but they also observe a public signal. In this chapter we endogenize the public signal and we allow its information content to be strategically designed by an expert that might benefit from sharing some information with the committee. Therefore, our work is closely related to Kamenica and Gentzkow (2011) and Alonso and Câmara (2015) that characterize the structure of the optimal signal in a sender-receiver(s) model[2]. The main result of this chapter is to show that, whenever the public signal is endogenously supplied, the set of equilibria found in Kawamura and Vlaseros (2015) and Liu (2015) collapses to a unique equilibrium that reflects the properties derived by Kamenica and Gentzkow (2011). Our model is very close to Alonso and Câmara (2015) that extend the Bayesian persuasion framework to a multiple receivers setting. In their model voters' preferences are ex ante known by the information controller and common knowledge. This information is crucial to the information controller who is thus able to split the electorate in different coalitions that would respond informatively to the supplied signal. In our model, voters share the same preferences. However, different preferences are generated endogenously within the model as in Austen-Smith and Banks (1996). The fact that this heterogeneity is information-based crucially distinguishes us from Alonso and Câmara for the expert does not know in advance the composition of the collective. After having established our main result, that is, after that we rule out all equlibiria in which individuals rely on their private information, we apply our model to a specific issue firstly discussed by Feddersen and Pesendorfer (1998). They show that privately informed jurors may convict innocent defendants with high probability whenever the verdict is delivered according to unanimity rule. They suggest that unanimous rule should be abandoned in favor of less stringent rules that nonetheless improves the quality of the verdict. Coughlan (2000) defends unanimous verdicts observing that communication inside the Jury and the possibility of a mistrial actually protect innocent defendants. We place our model on this delicate issue for the following reason. In both models jurors are privately informed by an exogenous source. Nonetheless, during a trial information reflects the attempts of the prosecutor to persuade the Jury to convict the defendant[3]. Furthermore, during a trial all

and Banks (1996) relies on the assumption that prior to casting a vote, voters are partially informed by an independent private signal.

[2] Alonso and Câmara (2015) refer to the sender as the information controller.

[3] Indeed, in Kamenica and Gentzkow (2011)'s leading example, a prosecutor designs an

the evidence is discussed in front of the Jury. This allows us to forward the idea that in such a circumstance, jurors deal with public evidence and we conclude that i) unanimous verdicts are no worse than majority verdicts and that ii) the possibility of a mistrial may significantly increase the quality of the verdict.

2.1 The Model

2.1.1 Setup

A collective decision $d \in \mathcal{D} = \{A, B\}$ has to be implemented by a committee $\mathcal{I} = \{1, 2, \dots, n\}$ made of n individuals. We assume that n is odd. Each individual $i \in \mathcal{I}$ simultaneously casts a private vote $x \in X = \{A, B\}$ for one of the two alternatives. The collective decision is determined by *majority rule*. There is a binary state of the world $\Omega = \{\alpha, \beta\}$ and each state is equally likely. Individuals' utility $u_i : \mathcal{D} \times \Omega \to \mathbb{R}$ takes the following form:

$$u_i(d, \omega) = \begin{cases} 0 & \text{if } d = A, \omega = \alpha, \text{ or } d = B, \omega = \beta \\ -q & \text{if } d = A, \omega = \beta, \\ -(1-q) & \text{if } d = B, \omega = \alpha. \end{cases}$$

and $q \in (\frac{1}{2}, 1)$ for all $i \in \mathcal{I}$. Individuals want to match the collective decision with the state of the world. However, without any further information they all prefer alternative B. In the "Jury" literature q represents the threshold of reasonable doubt. A generic individual i that assigns the posterior belief $\mu_i(\alpha)$ to the event $\omega = \alpha$ would prefer alternative A over B *if and only if*:

$$-q\ (1 - \mu_i(\alpha)) \geq -(1-q)\mu_i(\alpha) \iff \mu_i(\alpha) \geq q,$$

that is to say, if and only if the evidence of being in state α is sufficiently strong.

2.1.2 Information

Before casting a vote, individuals independently receive a private signal $s_i \in S = \{a, b\}$ drawn according to the conditional probability distribution satisfying:

$$\Pr\{s_i = a|\alpha\} = \Pr\{s_i = b|\beta\} = p \in (\frac{1}{2}, 1).$$

In addition, all individuals commonly observe a public signal $s \in S = \{a, b\}$ which is independent from the private signal. The public signal is characterized

investigation so as to persuade the Judge to convict a defendant who is more likely to be innocent.

by the following precisions:

$$\Pr\{s = a|\alpha\} = \pi_a,$$
$$\Pr\{s = b|\beta\} = \pi_b.$$

The private signal characterizes individuals private information. Although the members of the committee share the same preferences, they may end up having different information because of what they learn from their private signals. On the other hand, the public signal represents verifiable evidence advanced by an expert to the committee who strictly prefers the approval of an alternative. Thus, the public signal is endogenously determined and we denote with $\pi = (\pi_a, \pi_b)$ the vector of conditional probabilities, i.e., the precision of the public signal.

2.1.3 Beliefs

We denote with $\mu_i(\omega, s_i, s)$ the posterior belief of a generic individual she assigns to state $\Omega = \omega$ conditional on signals s_i and s. Posterior beliefs are formed according through Bayes rule:

$$\mu_i(\alpha, a, a) = \frac{p\pi_a}{p\pi_a + (1-p)(1-\pi_b)} \qquad \mu_i(\beta, a, a) = \frac{(1-p)(1-\pi_b)}{p\pi_a + (1-p)(1-\pi_b)}, \quad (2.1)$$

$$\mu_i(\alpha, a, b) = \frac{p(1-\pi_a)}{p(1-\pi_a) + (1-p)\pi_b} \qquad \mu_i(\beta, a, b) = \frac{(1-p)\pi_b}{p(1-\pi_a) + (1-p)\pi_b}, \quad (2.2)$$

$$\mu_i(\alpha, b, a) = \frac{(1-p)\pi_a}{(1-p)\pi_a + p(1-\pi_b)} \qquad \mu_i(\beta, b, a) = \frac{p(1-\pi_b)}{(1-p)\pi_a + p(1-\pi_b)}, \quad (2.3)$$

$$\mu_i(\alpha, b, b) = \frac{(1-p)(1-\pi_a)}{(1-p)(1-\pi_a) + p\pi_b} \qquad \mu_i(\beta, b, b) = \frac{p\pi_b}{(1-p)(1-\pi_a) + p\pi_b}. \quad (2.4)$$

Notice that posterior beliefs are endogenously determined by the choice of π made by the expert.

2.1.4 Expert's Public Signal and Timing

The public signal π is designed by an expert who strictly prefers the approval of alternative A irrespectively from the true state of the world:

$$u^E(A, \omega) = 1 > u^E(B, \omega) = 0, \quad \forall\, \omega \in \Omega.$$

Thus, the expert designs π in order to maximize the expected probability with which alternative A is selected[4]:

$$\pi = \text{argmax } \Pr\{d = A|\mathbf{v}\} \tag{2.5}$$

where $\mathbf{v}$ denotes the profile of voting strategies which depends on π. Voting strategies are described in the following subsection. On the contrary, from the point of view of the expert, a strategy is simply defined by a pair of conditional probability distributions $(\pi_a, \pi_b) \in [0,1]^2$ that determines the probability with which s results in a or b:

$$\Pr\{s = a\} = \frac{1}{2}\pi_a + \frac{1}{2}(1 - \pi_b),$$

$$\Pr\{s = b\} = \frac{1}{2}\pi_b + \frac{1}{2}(1 - \pi_a).$$

We assume that the the expert truthfully reveals signal s to the committee. Under this assumption the signal consists of hard verifiable evidence. Kamenica and Gentzkow (2011) argue that there are many contexts in which commitment is legally mandated. For instance, the Due Process Clause of the Fourteenth Amendment requires the Prosecutor to disclose all material evidence although it may hurt his case against the defendant. Information gathering in organizations often involve commitment that may be formalized through contracts whereas there are many settings in which the decision to conduct tests or studies is publicly observable, and the results are verifiable ex post. Kamenica and Gentzkow (2011) and Alonso and Câmara (2015) extensively discuss this assumption.

The timing of the game is summarized as follows:

i) The expert chooses π;

ii) Nature determines the state of the world;

iii) Each individual observes her private and public signal and votes are cast;

iv) Majority decision is implemented and payoffs are realized.

2.1.5 Voting Strategies and Equilibrium

The members of the committee cast a vote $x \in \{A, B\}$ after the observation of their private and the public signal. A voting strategy is a map $v_i : S \times S \to X$ that associates a vote to any pair of signals. We restrict our attention on symmetric pure strategies. As Kawamura and Vlaseros (2015), we define responsive strategies, i.e., those strategies that depend on s_i and s as follows.

[4]From the normalization it follows that $\mathbb{E}[u^E(d,\omega)] = \Pr\{d = A\}$.

Definition 2.1. A voting strategy v_i is *individually informative* if

$$\text{for all } s_i, s \in S,\ v_i(s_i, s) = s_i.$$

According to the individually informative voting strategy v_i, individual i votes for what her private signal indicates irrespectively from the public signal.

Definition 2.2. A voting strategy v_i is *obedient* if

$$\text{for all } s_i, s \in S,\ v_i(s_i, a) = A \text{ and } v_i(s_i, b) = B.$$

According to the obedient strategy v_i, individual i votes for what the public signal indicates irrespectively from her private signal. In this sense, she obeys to expert's information.

Furthermore, as long as the content of the signals may be discordant, there is another pure strategy of interest.

Definition 2.3. A voting strategy v_i is *A-prudent* if

$$v_i = \begin{cases} A & \text{if } s_i = a \text{ and } s = a, \\ B & \text{otherwise.} \end{cases}$$

The *B-prudent* voting strategy is defined analogously. A voting strategy is A-prudent if voter i casts a vote for alternative A only if her signals agree on A. Otherwise, one signal in favor of B is sufficient to induce her to vote for B.

We follow Kamenica and Gentzkow (2011) and we use the Sender-preferred subgame perfect equilibrium: given expert's choice of π and signals' realizations s_i and s, for all $i \in \mathcal{I}$,

$$x^* = \text{argmax}_{x \in \{A,B\}}\, \mathbb{E}[u_i(x, \omega | s_i, s, v_{-i}, \pi)], \tag{2.6}$$

where v_{-i} denotes the profile of voting strategies without component i. If some individual is indifferent between the two alternatives at a given belief, we assume she votes for A. Sender preferred equilibrium ensures that an optimal signal exists and it simplifies the analysis without altering the functioning of the model.

Definition 2.4. Equilibrium is a pair $(\pi^*, \mathbf{v}^*)$ where π^* satisfies (2.5) and induces $\mathbf{v}^*$ such that, for all members of the committee, the equilibrium voting strategy prescribes action x_i^* satisfying (2.6).

It is worth to remark that the information content of the signal as characterized by π_a and π_b induces a set of posterior beliefs described by (2.1)-(2.4). At each $\mu_i(\omega, s_i, s)$ the equilibrium voting strategy profile must specify an optimal action $x_i^* \in X$. Therefore, for all individuals in the committee, the individual voting strategy must be consistent with the posterior beliefs induced by π^*.

Importantly, differently from Kamenica and Gentzkow (2011), in our setting the rationality condition imposed on beliefs is implicitly satisfies. Whereas Kamenica and Gentzkow (2011) look for a distribution of decision maker's posterior beliefs that maximizes persuader's payoff, here persuader directly chooses the conditional distribution π that maximizes his payoffs. Because beliefs are rationally formed through Bayes rule, posterior beliefs satisfy the Bayes rationality condition. On the contrary, the Bayes rationality condition imposed in Kamenica and Gentzkow (2011) is necessary as not all distributions of posterior beliefs are rational.

2.2 Solving the Model

The model has potentially many equilibria. Nevertheless, we focus on those voting behaviors that seem most plausible as described in the previous section.

Although individuals share the same preference parameter q, they may end up having different interim preferences over A and B that stem from the observation of their private signal. Thus, as long as individuals may not share the same posterior belief, they also may end up casting different votes.

The voting strategy being played crucially determines the probability with which alternative A is implemented. A natural benchmark is given by the individually informative voting strategy. Denote with $W_d(\omega|\cdot)$ the probability with which decision $d \in \{A, B\}$ is selected in state ω.

Corollary 2.1. If, for all $i \in \mathcal{I}$, v_i is individually informative, then:

$$W_A(\alpha|\mathbf{v}) = W_B(\beta|\mathbf{v}) = \sum_{k=\frac{n+1}{2}}^{n} \binom{n}{k} p^k (1-p)^{(n-k)} \equiv \bar{W}. \tag{2.7}$$

Equation (2.7) says that the probability with which the correct alternative is selected in equilibrium is the same in both states. This follows from the fact that the private signal is symmetric. It can be easily shown that for $p \geq q$, *sincere voting* would be a Bayes Nash equilibrium without the public signal.

Lemma 2.1. *Suppose that, for all $i \in \mathcal{I}$, v_i is individually informative. Then,*

$$\Pr\{d = A|\mathbf{v}\} = \frac{1}{2}. \tag{2.8}$$

Proof. Expert's expected utility is the probability with which alternative A is selected. Since for all individual v_i is individually informative, $W_A(\alpha) = W_B(\beta)$.

Thus:

$$\Pr\{d = A|\mathbf{v}\} = \frac{1}{2}\left[\bar{W} + 1 - \bar{W}\right] = \frac{1}{2}.$$

□

Notice that if π induces the individually informative strategy, then the probability with which alternative A is selected does not depend on π.

Lemma 2.2. *Suppose that, for all $i \in \mathcal{I}$, v_i is obedient. Then,*

$$\Pr\{d = A|\mathbf{v}\} = \Pr\{s = a\}. \tag{2.9}$$

Proof. If all members of the committee endorse the obedient strategy, alternative B is selected whenever $s = b$ and alternative A is selected whenever $s = a$, then:

$$\begin{aligned}\Pr\{d = A|\mathbf{v}\} &= \frac{1}{2}\left[1 \times \pi_a + 0 \times (1 - \pi_a)\right] + \frac{1}{2}\left[1 \times (1 - \pi_b) + 0 \times \pi_b\right] \\ &= \frac{1}{2}[\pi_a + (1 - \pi_b)] \\ &= \Pr\{s = a\}.\end{aligned}$$

□

Notice that if $\Pr\{s = a\} > \frac{1}{2}$ then the expert strictly prefers to induce the obedient strategy over the individually informative strategy.

Lemma 2.3. *Suppose that, for all $i \in \mathcal{I}$, v_i is A-prudent. Then,*

$$\Pr\{d = A|\mathbf{v}\} = \frac{1}{2}\left[\pi_a \bar{W} + (1 - \pi_b)(1 - \bar{W})\right]; \tag{2.10}$$

Suppose that, for all $i \in \mathcal{I}$, v_i is B-prudent. Then,

$$\Pr\{d = A|\mathbf{v}\} = \frac{1}{2}\left[\pi_a + (1 - \pi_a)\bar{W} + (1 - \pi_b) + \pi_b(1 - \bar{W})\right]. \tag{2.11}$$

Proof. Consider the A-prudent strategy. If all members of the committee endorse the A-prudent strategy, then alternative A is selected in state A only if the realization of the public signal is a and at least $\frac{n+1}{2}$ individuals received the private signal $s_i = a$. The reasoning in state B is analogous, and it directly proves equation (2.10). The proof for the B-prudent strategy is analogous. □

For future reference denote with $\mathbf{v}^o$ the voting profile in which all voters endorse the obedient strategy and with $\mathbf{v}^B$ the profile in which all voters endorse the B-prudent strategy. To solve the model we proceed in two steps. First, we derive the optimal signal consistent with each voting strategy. Precisely, the signal must maximize the probability with which alternative A is implemented and the voting strategy must be rational, i.e., the voting strategy must satisfy (2.6). Second, we order expert's preferences over voting strategies and uniquely characterize the equilibrium.

We begin with the obedient strategy. To simplify the notation let $\bar{q} \equiv \frac{1-q}{q}$ and $\bar{p} \equiv \frac{1-p}{p}$. Importantly, under the obedient voting strategy it is straightforward to observe that there are no pivotal considerations. Consider the following signal: $\pi_a = 1$ and $\pi_b = 0$ which implies that $\Pr\{s = a\} = 1$. Although this is clearly a BNE equilibrium of the voting game, no voters has a unilateral profitable deviation, we require the obedient voting strategy to be individually rational.

Proposition 2.1. *The unique optimal signal π^o consistent with $\mathbf{v}^o$ such that the obedient voting strategy is individually rational is given by*

$$\pi_a^o = 1 \text{ and } \pi_b^o = 1 - \bar{q}\bar{p}. \tag{2.12}$$

Proof. From lemma 2.2, the expert maximizes the probability with which individuals publicly observe $s = a$. Condition (2.6) requires that, at the beliefs induced by π, the voting strategy must specify an optimal conditional on the voting strategy being played by the other voters. As there are no pivotal considerations, we consider those equilibria in which the obedient voting strategy is individually rational or *sincere*. This condition is equivalent to the following set of inequalities:

$$\mathbb{E}[u_i(A|s_i, a)] \geq \mathbb{E}[u_i(B|s_i, a)]$$

and

$$\mathbb{E}[u_i(B|s_i, b)] > \mathbb{E}[u_i(A|s_i, b)].$$

The dependence on π is understood. Notice that if the first inequality is satisfied for $s_i = b$, then it is also satisfied for $s_i = a$. Consider the first inequality conditional on $s_i = b$:

$$\begin{aligned}\mathbb{E}[u_i(A|b, a)] \geq \mathbb{E}[u_i(B|b, a)] &\iff -q\mu_i(\beta, b, a) \geq -(1-q)\mu_i(\alpha, b, a) \\ &\iff \bar{q}\bar{p}\frac{\pi_a}{1-\pi_b} \geq 1.\end{aligned}$$

Since $\Pr\{s = a\}$ increases linearly in π_a, the expert sets $\pi_a = 1$ and π_b solves $-q\mu_i(\beta, b, a) = -(1-q)\mu_i(\alpha, b, a)$, that is $\pi_b = 1 - \bar{q}\bar{p}$. Notice that at $\pi_a = 1$, $\mu_i(\alpha, s_i, b) = 0$, thus, conditional on $s = b$ it is rational to vote for alternative B regardless from the private signal and $\mathbf{v}^o$ is consistent with the beliefs induced by π. Second, for $\pi_a < 1$ or for $\pi_b > 1 - \bar{q}\bar{p}$ the expert is decreasing the probability of a. Therefore,

$$\pi^o = (\pi_a^o = 1, \pi_b^o = 1 - \bar{q}\bar{p})$$

is the unique optimal signal consistent with the obedient strategy and this completes the proof. □

Proposition 2.1 specifies expert's most preferred signal that induces the obedient strategy when we restrict our attention to those equilibria that are individually rational. Clearly, at π^o the B-prudent strategy is not enforceable in equilibrium. Intuitively, conditional on the public signal being b, all individuals strictly prefer to cast a vote for alternative B irrespectively from their private information. Nevertheless, the expert might be able to design π so that the B-prudent strategy is enforceable in equilibrium. If this is the case, he might deviate from π^o whenever the probability of $d = A$ is higher if individuals play v^B.

Proposition 2.2. *The B-prudent strategy can be enforced if and only if $p \geq q$. The unique optimal signal consistent with $\mathbf{v}^B$ is given by:*

$$\pi_a^B = \frac{\bar{q} - \bar{p}}{\bar{q}(1 - \bar{p}^2)} \text{ and } \pi_b^B = \frac{1 - \bar{q}\bar{p}}{1 - \bar{p}^2}. \tag{2.13}$$

Before proving the statement, it is worth to underline that whenever individuals are playing the x-prudent strategy, strategic considerations come into play. Nonetheless, the symmetric assumption about the private signal and the fact that n is odd simplifies the analysis. To see why, suppose that individuals are playing the B-voting strategy and that individual i observes $s_i = b$ and $s = b$. The strategy prescribes her to cast a vote for alternative B. However, her vote is pivotal only if $\frac{n-1}{2}$ individuals have observed $s_i = a$, that is, with probability

$$piv(\omega|\mathbf{v}_{-i}^B) = \binom{n}{\frac{n-1}{2}} [p(1-p)]^{\frac{n-1}{2}}$$

which is the same in both states by symmetry. Therefore, in the expected gain deriving from voting for A rather than B, the probability of casting a pivotal votes simply cancels out.

Proof. From lemma 2.3, the expert maximizes the probability with which the public signal realizes a. Condition (2.6) requires that, at the beliefs induced by π, the voting strategy must specify an optimal action. If individuals are playing v^B, the set of inequalities that must be satisfied is

$$\mathbb{E}[u_i(A|s_i, a, \mathbf{v}_{-i}^B)] \geq \mathbb{E}[u_i(B|s_i, a, \mathbf{v}_{-i}^B)],$$

$$\mathbb{E}[u_i(A|a, b, \mathbf{v}_{-i}^B)] \geq \mathbb{E}[u_i(B|a, b, \mathbf{v}_{-i}^B)],$$

and

$$\mathbb{E}[u_i(B|b, b, \mathbf{v}_{-i}^B)] > \mathbb{E}[u_i(A|b, b, \mathbf{v}_{-i}^B)].$$

The problem can be simplified noting that, from proposition 2.1, it must be the case that

$$\mathbb{E}[u_i(A|b, a, \mathbf{v}^B_{-i})] = \mathbb{E}[u_i(B|b, a, \mathbf{v}^B_{-i})],$$

that is

$$\bar{q}\bar{p}\frac{\pi_a}{1-\pi_b} = 1. \tag{2.14}$$

However, differently from the obedient strategy, conditional on the public signal being b, all those voters who privately observe a must weakly prefer to cast a vote for alternative A, that is

$$\frac{\bar{q}}{\bar{p}}\frac{1-\pi_a}{\pi_b} \geq 1. \tag{2.15}$$

Suppose that the inequality holds strictly. Then the expert can slightly increase π_a or slightly decrease π_b so that the probability with which the public signal realizes a increases. This, means that also condition (2.15) must hold with equality. Thus, we have a linear system of two linear equations in two unknowns. Solving the system, the unique solution is

$$\pi^B_a = \frac{\bar{q}-\bar{p}}{\bar{q}(1-\bar{p}^2)} \text{ and } \pi^B_b = \frac{1-\bar{q}\bar{p}}{1-\bar{p}^2}.$$

It is easily verified that $\mathbb{E}[u_i(A|a, b, \mathbf{v}^B_{-i})] = \mathbb{E}[u_i(B|a, b, \mathbf{v}^B_{-i})]$ implies

$$\mathbb{E}[u_i(B|b, b, \mathbf{v}^B_{-i})] > \mathbb{E}[u_i(B|b, b, \mathbf{v}^B_{-i})].$$

Therefore, the profile $\mathbf{v}^B$ induced by π^B is rational since it satisfies condition (2.6). Second, notice that for $\bar{p} = \bar{q}$ we have $\pi^B = (0, 1)$. On the other hand, for $p < q$ the solution requires $\pi_a < 0$ which is not possible. This means that $\mathbf{v}^B$ is enforceable if and only if $p \geq q$ and completes the proof. □

The B-prudent strategy might seem appealing to the expert. Intuitively, individuals are always willing to vote for alternative A. The only instance in which they cast a vote for alternative B is whenever the public and the private signal agree on B. Nonetheless, in order to enforce this strategy the expert must reduce the likelihood with which the public signal realizes a. Particularly, notice that:

$$\pi^B_a = \frac{\bar{q}-\bar{p}}{\bar{q}(1-\bar{p}^2)} < \pi^B_b = \frac{1-\bar{q}\bar{p}}{1-\bar{p}^2}.$$

On the contrary, it is easy to see that the expert has no interests in inducing the A-prudent strategy. From (2.10) it follows that the expert can induce the committee to select alternative A at most with probability $\frac{1}{2}$ when $\pi_a = 1$ and $\pi_b = 0$. Therefore, in equilibrium the expert induces the members of the committee to endorse the obedient or the B-prudent strategy.

The next proposition characterizes the conditions under which persuader prefers to induce the obedient strategy in equilibrium rather than the B-prudent strategy through the choice of π.

Proposition 2.3. *In equilibrium the expert induces the obedient voting strategy rather than the B-prudent strategy if and only if*

$$1 - \pi_b^o \geq \pi_a^B \left(1 - \bar{W}\right) + \bar{W} \left(1 - \pi_b^B\right). \tag{2.16}$$

Proof. In equilibrium the expert designs π in order to induce the obedient strategy if and only if

$$\Pr\{d = A|\mathbf{v}^o\} \geq \Pr\{d = A|\mathbf{v}^B\}$$

that is, if and only if

$$\Pr\{s = a|\pi^o\} \geq \frac{1}{2}\left[\pi_a^B + (1 - \pi_a^B)\bar{W} + (1 - \pi_b^B) + \pi_b^B(1 - \bar{W})\right]$$
$$\bar{q}\bar{p} \geq \pi_a^B \left(1 - \bar{W}\right) + \bar{W}\left(1 - \pi_b^B\right)$$

which is exactly condition (2.16). □

Condition (2.16) is a function of the precision of the private signal. After some algebra we can express the condition more conveniently as:

$$1 - \bar{W} \leq \frac{\bar{q}\bar{p}^2}{\bar{q} - \bar{p}}. \tag{2.17}$$

Both sides of the inequality are decreasing in p and both approach 0 as $p \to 1$. On the contrary, when p is close to q, whereas the LHS is strictly less than 1 (it is a probability), the RHS has a large value since the denominator is close to 0. In addition, as the committee gets larger, the probability $\bar{W}$ approaches 1 since $p > q > \frac{1}{2}$. This suggests that the B-prudent strategy is played in equilibrium only if the committee is small. In particular, we have the following corollary.

Corollary 2.2. As the number of the members of the committee increases, the unique equilibrium signal is $\pi^* = \pi^o$ and $\mathbf{v} = \mathbf{v}^o$.

Proof. Simply note that

$$\lim_{n\to\infty}\left\{1 - \bar{W} \leq \frac{\bar{q}\bar{p}^2}{\bar{q} - \bar{p}}\right\} = 0 \leq \frac{\bar{q}\bar{p}^2}{\bar{q} - \bar{p}}$$

which means that the expert is always better off when he induces the obedient strategy. □

Example. Suppose $n = 3$, $q = 0.6$ and $p = 0.8$. The probability $1 - \bar{W}$ at $p = 0.8$ is 0.104. On the other hand $\frac{\bar{q}\bar{p}^2}{\bar{q}-\bar{p}} = 0.1$. Thus, since $0.104 > 0.1$ in equilibrium the expert induces the B-prudent strategy, that is

$$\pi_a^B = \frac{\bar{q} - \bar{p}}{\bar{q}(1 - \bar{p}^2)} = 0.66 < \pi_b^B = \frac{1 - \bar{q}\bar{p}}{1 - \bar{p}^2} = 0.88$$

and $\Pr\{d = A|\mathbf{v}^B\} = 0.58$. It can also be verified that for $p \leq 0.7945$ in equilibrium $\pi^* = \pi^o$. Finally, for $n = 5$ there is no $q \in (0.5, 1)$ such that in equilibrium the expert induces the B-prudent strategy.

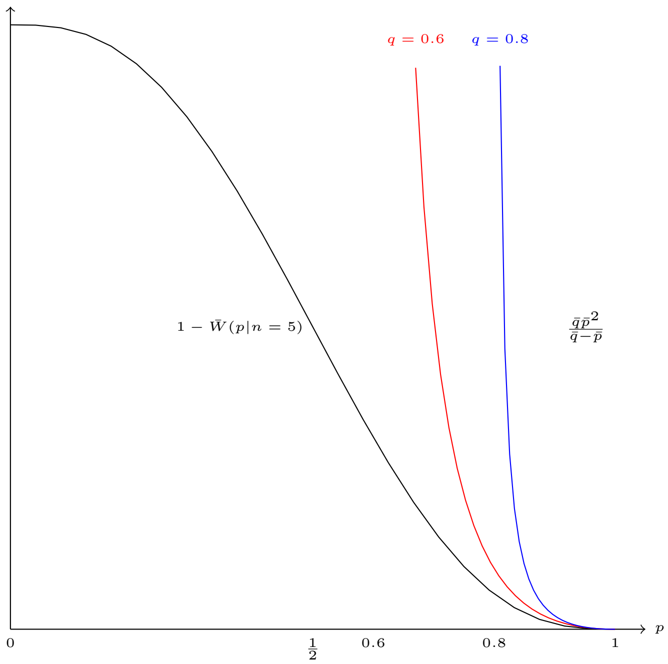

Figure 2.1: Equilibrium condition as a function of p. For $n \geq 5$ there is no $q > \frac{1}{2}$ such that the B-prudent strategy is induced in equilibrium.

The B-prudent strategy is not as appealing as the obedient strategy. Under obedience, individuals exactly follows what the evidence supplied by the expert tells them to do. On the contrary, the B-prudent strategy also requires the use of private information. Under this strategy, there is just only one event in which a generic voter casts a vote for alternative B, that is whenever her signals agree on B. In order to convince the members of the committee to follow this voting strategy, the expert must increase the probability with which the public signal is b. Because the private signal is informative, in state β private information "aggregates" when $s = b$ in the sense that, as $n \to \infty$, the probability with which the committee selects alternative B in state β conditional on b approaches 1. Although this remains true in state α conditional on a, as long as $\Pr\{s = a\} < \Pr\{s = b\}$, alternative B is more likely to be selected. However, when the committee is made just of three individuals, although the private signal is informative it is not sufficient to correctly "aggregate" the information in both states. The expert exploits then this weakness deviating from the obedient strategy.

Nonetheless, the occurrence of the B-prudent strategy in equilibrium is more a pathological case rather than the norm. Furthermore, the conditions under which the obedient strategy is enforceable follows the same logic of Kamenica and Gentzkow (2011). In particular:

i) Individuals are sure of their action when they observe b,

ii) Individuals are indifferent when they observe a.

Point i) follows from the fact that $\pi_a = 1 \implies \mu_i(\beta, s_i, b) = 1$. On the other hand, point ii) follows from this simple intuition. If individuals are not indifferent when $s = a$, then the expert can either increase π_a or decrease π_b in order to raise the probability with which the public signal realizes in a. Finally, he can do that until individuals are exactly indifferent between casting a vote for A or B. The equilibrium condition implies that, whenever indifferent, individuals select expert's most preferred action rather than using a mixed strategy. The sender-preferred equilibrium hence just simply ensures that expert's utility is upper semi-continuous in π. Otherwise, in order to avoid a mixed strategy profile the expert could slightly decrease π_a so that $\mathbb{E}[u_i(A|\cdot)] > \mathbb{E}[u_i(B|\cdot)]$. This also shows that the restriction on pure voting strategies is without loss of generality if the public signal is endogenously supplied by a biased expert. A more formal argument follows from the observation that conditional on $s = a$, individuals are indifferent between voting for A or B. Therefore, under π^o, individuals' expected payoff is the same payoff they would get by always voting for B. A mixed strategy would be played only if voters could increase their payoff with respect to case in which they always vote for alternative B. However, the expert just prevent this occurrence.

Finally, it is worth to stress the difference between our equilibrium and Alonso and Câmara (2015). In our model, individuals differ because of their private information. In Alonso and Câmara (2015) individuals differ because of their preferences which are common knowledge and ex-ante known. This completely changes the way in which the expert adjusts his signal to the committee. First, differences in private information affects individuals' belief which are the key to persuasion for the expert. Secondly, private information is stochastic.

On the contrary, in Alonso and Câmara (2015) the composition of the committee is known in advance. For this reason the expert knows exactly which types are sufficient to persuade. For the same reason, they find that unanimous rule induce the expert to supply a more informative signal since he must convince the *hardest to persuade voter*. In our model, the hardest to persuade voter is endogenously determined in equilibrium through the observation of the private signal. In particular, those individuals who observe $s_i = b$ are the hardest to persuade. Although majority rule is used to aggregate individuals' vote, the expert always persuade type-b individuals since he prefers to induce the obedient voting strategy rather than other strategies which rely on private information.

2.3 Application: Trial and Mistrial

Kawamura and Vlaseros (2015) and Liu (2015) show that the members of a committee may easily coordinate their decisions on a sufficiently informative public signal neglecting their own private information. In the previous section we showed that whenever the public signal is strategically engineered by a biased expert, in equilibrium the outcome of the collective decision exactly reflects the recommendation implied by the public signal. In other words, in equilibrium the voting strategy is *obedient*.

Starting from that observation, in this section we study the problem introduced by Feddersen and Pesendorfer (1998) and Coughlan (2000). That is, are unanimous verdicts capable of protecting innocent defendants during criminal trials? Whereas Feddersen and Pesendorfer argue that actually, because of private information, unanimous verdicts increase the probability of convicting innocent defendants, Coughlan argue that, on the contrary, unanimous verdicts increase the accuracy of the verdict whenever we take into account the possibility that a trial may be declared void.

We reinterpret the model in what follows. The committee $\mathcal{I}$ represents a Jury who must decide whether to convict or to acquit the defendant. We denote with G (*guilty*) and I (*innocent*) the states of the world. The set of possible outcomes is $\mathcal{D} = \{A, C, M\}$ where A stands for *acquit*, C for *convict* and M denotes that a *mistrial* has occurred. We assume that the approval of decision A or B requires unanimity[5]. Therefore, if a single individual expresses a preference which differs

[5] This extension departs from the previous model in two ways. First, we focus on unanimous verdicts rather than majority verdicts and, second, we assume that whenever the Jury fails to reach an unanimous consensus, the trial is declared void. Thus, we introduce a new outcome, a mistrial, which affects the payoff of the Jury and of the prosecutor. The latter represents the biased expert.

from the rest of the Jury, then a mistrial occurs. Preferences are specified as in the previous section. In addition we assume that $u_i(M, \omega) = -m$ in both states. From the point of view of the Jury, the cost of a mistrial is less than the cost deriving from convicting an innocent: $m < q$. On the contrary, we assume that the expert (the prosecutor from now on), gets a payoff of 0 if a mistrial occurs. This is without loss of generality since the prosecutor wants to maximize the probability with which the defendant is convicted. The timing of the game and the information structure is the same as presented in the previous section. For the sake of interpretation, we denote the signal space with $S = \{a, c\}$.

The motivation leading this application is the following. During a trial the prosecutor may be inclined to force a guilty verdict irrespectively from the innocence of the defendant. Second, the evidence he supplies to the Jury is publicly observed. This suggests that, as we showed before, the Jury will neglect their private information and rely just on what the prosecutor brings to the court as evidence. Therefore, we contrast Feddersen and Pesendorfer (1998) results which strictly rely on private information and we study up to what extent the possibility of a mistrial improves the collective decision.

We assume that p and n are such that the prosecutor designs π in order to induce the obedient voting strategy. Hence, without loss of generality we neglect the private signal. From the previous section, the optimal signal inducing the obedient strategy is:

$$\pi_c^o = 1 \text{ and } \pi_a^o = 1 - \bar{q}\bar{p}.$$

Whenever $s = c$ the defendant is convicted and whenever $s = a$ the defendant is acquitted. Furthermore, we have to take into account the additional effect of a mistrial when the verdict requires unanimity. Suppose that $\mathbf{v}_{-i} = \mathbf{v}^o$, that is, all jurors but i are endorsing the obedient voting strategy. Juror i has two possibilities. She can obey the public signal or she can induce a mistrial with probability 1. Since the verdict requires unanimity it is sufficient that one juror deviates from the prescribed vote and a mistrial is declared.

The set of inequalities that ensure the obedient strategy being consistent with the posterior induced by π in the presence of a mistrial are:

$$\mathbb{E}[u_i(C|c, \mathbf{v}_{-i}^o)] \geq \mathbb{E}[u_i(A|c, \mathbf{v}_{-i}^o)] = -m$$

and

$$\mathbb{E}[u_i(A|a, \mathbf{v}_{-i}^o)] > \mathbb{E}[u_i(C|a, \mathbf{v}_{-i}^o)] = -m.$$

Suppose that jurors are endorsing the obedient strategy. Equilibrium requires that, given π^o, for all jurors $i \in \mathcal{I}$ these constraints are satisfied. The first inequality becomes:

$$\mathbb{E}[u_i(C|c, \mathbf{v}^o_{-i})] \geq -m$$

if and only if

$$\begin{aligned} -q\mu_i(I,c) &\geq -m \\ \frac{m}{q} &\geq \frac{p(1-\pi_a)}{p(1-\pi_a)+(1-p)\pi_c}. \end{aligned}$$

From proposition 2.1 we know that the optimal π inducing the obedient voting strategy requires $\pi_c = 1$, hence we have:

$$\begin{aligned} \pi_a &\geq 1 - \frac{1-p}{p}\frac{m}{q-m} \\ \pi_a^o &= 1 - \frac{1-p}{p}\frac{m}{q-m}. \end{aligned}$$

To see whether the possibility of a mistrial increases the probability with which the Jury takes the correct decision, we have to compare the precision of the public signal in state I, that is when the defendant is innocent. If

$$1 - \frac{1-p}{p}\frac{m}{q-m} > 1 - \frac{1-p}{p}\frac{1-q}{q} \tag{2.18}$$

then the probability with which the Jury observes the correct signal when the defendant is innocent is higher and the probability of a correct decision has increased.

Proposition 2.4. *The possibility of a mistrial increases the accuracy of the Jury* if and only if

$$m < (1-q)q.$$

Proof.

$$1 - \frac{1-p}{p}\frac{m}{q-m} > 1 - \frac{1-p}{p}\frac{1-q}{q}$$

if and only if

$$m < (1-q)q.$$

□

The intuition is straightforward. Suppose that $m = 0$. Then, the Jury would not induce a mistrial if and only if π^o is perfectly informative, that is, if $m = 0$ then $\pi^o = (1, 1)$. If a mistrial has no cost for the Jury, then jurors would not convict or acquit the defendant unless the available evidence solves all uncertainty. On the other hand, if the cost of a mistrial is high, this would not affect the outcome of the game. In particular, a necessary condition for a mistrial to increase the probability of a correct verdict is $m < 1 - q$, that is, a mistrial must cost less than acquitting a guilty defendant. In the Jury literature preferences are usually described by the threshold of reasonable doubt, with $q > \frac{1}{2}$ which express the concerns of convicting innocent defendants. However, if the felony pending on the defendant is particularly grave, the Jury might be also concerned in acquitting guilty defendants as well. Whenever $q = \frac{1}{2}$, a mistrial enhances the probability of a correct verdict only if $m < \frac{1}{4}$ which is the maximum value m could take such that the possibility of a mistrial affects the equilibrium signal.

Feddersen and Pesendorfer (1998) show that if the members of a Jury are privately informed, then unanimous verdict may actually increase the probability o convicting the innocent. On the contrary, Coughlan (2000) argues that communication and mistrials may on the other hand bring unanimous verdict to dominate other decision rules. Both analysis starts from the assumption that jurors only hold private information. This is a sensible assumption. In particular, it neglects the fact that either the defense or the prosecutor do have clear incentives in acquitting or convicting the defendant, irrespectively from the fact that he may be innocent or not. In this application, the possibility of mistrial may increase the quality of the verdict of the Jury. The reason is the following. If the cost of a mistrial is sufficiently low, the Jury requires a more informative signal in order to take a decision. If the public signal is not precise enough, a mistrial occurs and the interpretation is straightforward: the supplied evidence is not sufficient to convict or acquit the defendant. Whereas in our model the public signal stems from the attempts of a prosecutor to persuade the Jury, in Coughlan communication is modeled as public information that emerges through discussion. Finally, in our analysis the efficiency of the Jury's decision does not depend on the aggregation rule. Indeed, we would obtain the same result under simple majority. Whereas Alonso and Câmara (2015) show that under unanimity the expert must supply a more informative signal, in our model the signal would remain constant. This is because jurors differ in their private information and not in their thresholds of reasonable doubt. Nonetheless, as explained, the prosecutor might be worse off by the possibility of a mistrial. It is also worth to stress this result in contrast with Feddersen and Pesendorfer (1998). In their model, under unanimity, private information adversely affect the Jury and, as the number of jurors increases the probability with which an innocent defendant is convicted increases. On the contrary, when the information is public jurors are indifferent with respect to their decision. Unanimity may serve better the purpose of the Jury if jurors differ in their preferences (as in

Alonso and Câmara (2015)) or if communication is introduced (as in Coughlan (2000)).

Finally, we advance the idea that the toy model developed in this section could be extended to all those situations in which the lack of an agreement between members of the committee may lead to the implementation of a third option meanwhile a new decision has to be taken.

2.4 Communication

A natural extension of the model is to allow individuals to communicate their private information. There are two possible cases to consider. In the first case, private information is sincerely revealed inside the committee whereas the expert is not allowed to observe this communication stage. In the second case, private information is publicly revealed and also the expert observes this information. The second case is analogous to the situation inquired by Alonso and Câmara (2015) with the exception that some individual may prefer to retain her private information, especially if we consider a larger space of signal. This follows from the fact that private information generates different preferences over alternatives at the interim stage that could be exploited by the expert as Alonso and Câmara demonstrate. This extension may allow us to consider the value of private information and it is left for future research.

2.5 Conclusion

In this work we studied how an expert can strategically engineer the content of a public signal in order to persuade a collective of privately informed individuals to select his favorite alternative most of the times. We showed that under mild conditions the expert designs the public signal in order to induce the committee to coordinate on what the signal suggests, that is to say, in equilibrium the expert induces the obedient voting strategy. Under this voting strategy, the members of the committee only rely on the public information supplied by the expert. Nevertheless, when the committee is very small, the expert might be able to induce a voting strategy in which the voting decision also depends on the private signal. However, this is more a pathological case since for a committee of size 5 or more this strategy is no longer enforceable. This also confirms Kawamura and Vlaseros (2015) and Liu (2015) intuition's that a committee is more likely to coordinate on the public signal rather than on individuals' private information. On the light of these results, we studied the outcome of a trial in which the public signal represents the evidence advanced by a prosecutor who wants to convict the defendant. We showed that, in such a case, the possibility of a mistrial might increase the accuracy of the verdict whereas unanimity does not perform worser than other less stringent rules.

In many circumstances the public signal can be thought as information supplied by an expert who has particular interests and, in such situations, the beneficial role of private information disappears since information is specifically designed in such a way that individuals only care about the public evidence. In particular, the interests of the expert reflect the design of the signal which prevents the expression of other mixed equilibria that rely both on private and public information. Finally, although in equilibrium individuals neglect their private information, the public signal internalizes the informativeness of the private signal. As the precision of the private signal increases, the expert reduces the bias of his public signal towards his favorite recommendation in favor of the correct voting suggestion.

CHAPTER 3

The Condorcet Jury Theorem and Information Control

Introduction

The Condorcet Jury Theorem states that majorities are more likely than any single individual to select the best alterantive when the best alternative is in fact uncertain. For this reason, the Theorem has been used as a positive argument for decision making by majority rule (e.g., Miller (1986); Young (1988); Ladha (1992)) suggesting that large democracies tend to aggregate information. Not surprisingly, the main assumptions of the Theorem and its applicability have been widely questioned and discussed. Nevertheless, the Theorem is silent about the source of information. In other words, whereas the Theorem, especially its strategic version (e.g., Austen-Smith and Banks (1996); Wit (1998)) highlights the role of information, it considers information neutrally supplied. However, there are many situations in which we should expect information to be strategically designed rather than neutrally supplied. The most common example in which the Theorem is used to show the superiority of majority decisions is that of a criminal trial. However, during a trial, the evidence observed by jurors is the result of the efforts made by the prosecutor and the defense in order to convict or to acquit the defendant[1]. Nonetheless, a criminal trial is not an exceptional instance in which information might not be neutral. McCloskey and Klamer (1995) argue that an important share of economic resources is devoted to in-

[1] In Kamenica and Gentzkow (2011)'s leading example, there is a judge who decides whether to acquit or to convict the defendant and all the available information consists of an investigation designed by a prosecutor that aims to maximize the probability of convicting the defendant.

fluence the decision of many decision makers, DellaVigna and Gentzkow (2009) study empirically the extent with which strategic information is used to manipulate individuals' decisions. The neglect of this aspect of information might prevent us to consider potential situations in which the optimistic conclusion of the theorem fail to hold[2].

In this chapter we are concerned with large majority decisions in which all the available information is supplied by a *Persuader* whose aim is to persuade the majority to select his favorite alternative most of the times. In particular, we consider a large election between two alternatives in which the number of voters, rather than being known in advance, is uncertain and Poisson distributed (Myerson, 1998a,b, 2000). Furthermore, we also characterize two different ways in which information could be supplied and we also consider the presence of different types of voters. This allow us to distinguish three different *societies*. In the *independent society* individuals are informed by a *conditionally independent private signal*. Thus, the independent society represent the classic Condorcet Jury although the conditional distribution from which individuals draw their private information is strategically designed by Persuader. Then we consider the *controlled society*. In this society Persuader supplies a *public signal* which implies that all individuals observe the same information. Finally, we consider the *imperfectly controlled society*. In this society although the signal is publicly supplied, a small fraction of voters is characterized by an independent (or irrational) behavior that can not be manipulated through the design of the information content of the public signal. In this sense the society is imperfectly controlled. We characterize the outcome of the game in the three different society as the expected number of voters goes to infinity and we are concerned with the question "is majority decision efficient as the Theorem predicts?" Not surprisingly, in the independent society the Theorem works as expected regardless from Persuader's attempt to manipulate majority outcome. Accordingly, in the controlled society the opposite occurs. As information is publicly supplied, in equilibrium votes are perfectly correlated with the realization of the public signal that is determined by Persuader's strategic behavior. Persuader's payoff is maximum in the controlled society that could be compared to Ladha (1992)'s description of an autocratic society:

> In contrast, autocratic societies restrict the dissemination of information

[2] Miller (1986) argues that a natural extension of the Theorem is one in which voters' level of information is the result of a political campaign strategy. Also he concludes asserting that: "it is inequalities or biases (of particular sort) in the information levels in the electorate, and *not generally low levels of information*, that threaten the "success" of the electoral process". Therefore, we should not expect the theorem to work if information is strategically designed.

> and the airing of various views or interpretations. Citizens thus lack access to different points of view and are usually limited to the experiences of their immediate surroundings. Votes in such societies, when they occur, have little meaning: all surroundings. All Voters must vote with the autocrat. The votes will be nearly perfectly correlated; majority-rule voting may be no better than any individual's vote.

In the imperfectly controlled society votes are nearly perfectly correlated. Nevertheless, contrary to Ladha's prediction, majority rule voting beats the accuracy of any individual's vote. Because of population uncertainty, although rational voters are willing to obey to the public signal, the noise introduced by the presence of an irrational voting behavior with whom few voters are characterized, reinstates strategic considerations in the model. Nevertheless, the working of the Theorem is compromised and information aggregates just in Persuader's favorite state, that is the state in which his favorite alternative is recognized by society as the better one.

Finally, we suggest that the imperfectly controlled society represents a perturbed version, in the sense of Selten (1975), of the controlled society, that is voters are subject to a small probability of casting a mistaken ballot that prevents votes from being perfectly correlated. We show that as the tremble goes to 0, or in other words as the fraction of irrational voters goes to 0, the outcome of the imperfectly controlled society does not approach the controlled society. In the limit, although the fraction of mistaken votes is going to zero, voters infer overwhelming evidence of being pivotal in Persuader's worst state. In order to manipulate voters' behavior, Persuader's signal becomes perfectly informative as trembles go to zero. Therefore, in the limit, rather than approaching the controlled society, the outcome of the imperfectly controlled society approaches the same outcome we would observe if voters had perfect information about the underlying state of Nature.

3.1 The Model

3.1.1 Voters

Voters are members of a "Jury" who must express their opinion about the question "what is the true state of Nature?" by casting a vote. There are two equally likely states of Nature, $\omega \in \Omega = \{\alpha, \beta\}$, each voter can vote for A or B and the action set is denoted by $C = \{A, B\}$. Several interpretations are available. Since we focus on majority decisions it is natural to interpret the model in a political perspective such as an election or a referendum. Thus, the most voted alternative wins the election and the payoff deriving from the implementation of alternative c is realized. We denote with $u(c, \omega)$ the utility deriving from the victory of alternative c in state ω.

Assumption 3.1. Common value.
For all voters

$$u(A, \alpha) = u(B, \beta) = 0$$

$$u(A, \beta) = -q, \; u(B, \alpha) = -(1-q).$$

Assumption 3.1 guarantees that voters agree on the Pareto superior alternative which is assumed to exist. The parameter q is usually described in the Jury literature as the threshold of reasonable doubt[3]. Here it describes society's preference. If $q > \frac{1}{2}$ voters agree that a mistake in state β is worse than a mistake in state α.

Assumption 3.2. Default action.
Without any additional information, voters' default action is to vote for alternative B.

Assumption 3.2 says that, without additional information, alternative B always wins the election and provides us a natural benchmark.

The Condorcet Jury Theorem accents the importance of a large number of voters. Myerson (1998a) argues that in such a case it would be difficult to assess the exact number of voters. We embrace the idea and we assume that the number of voters is described by a Poisson random variable N with mean k. Then, the probability with which there are exactly n voters is given by

$$\Pr\{N = n|k\} = e^{-k}\frac{k^n}{n!}$$

which is constant across states of Nature. We denote with $n_A(\alpha)$ and $n_B(\beta)$ the number of votes for the correct alternative in state α and β, respectively. Finally, in the same manner, let $v_A(\alpha)$ and $v_B(\beta)$ be the probabilities with which a randomly drawn voter votes for the correct alternative. Myerson (1998a) showed that $n_A(\omega)$ and $n_B(\omega)$ are independent Poisson random variables with mean $kv_A(\omega)$ and $kv_B(\omega)$.

Prior to casting a vote, voters observe a signal $s \in S = \{a, b\}$ drawn from the family of conditional distributions

$$\mathcal{F} = \{\pi(s|\omega)\}_{\omega \in \Omega}.$$

[3]See for instance Feddersen and Pesendorfer (1998); Coughlan (2000).

Then, conditional on $s \in S$, voters form the posterior belief $\mu_\omega(s)$ according to Bayes rule. As Myerson (1998a) explain, in a Poisson game voters have additional information concerning the probability of being recruited as a voter. However, as the expected number of voters is constant across states, this information simplifies out in the Bayesian reasoning. Therefore, since the states of Nature are equally likely:

$$\mu_\omega(s) = \frac{\pi(s|\omega)}{\sum_{y \in \Omega} \pi(s|y)}.$$

A voting strategy describes the probability with which a recruited voter casts a ballot for alternative A or B conditional on the observation of her signal. Let denote with σ_c these probabilities so that

$$\sigma_A : a \to [0,1] = \Pr(x = A|a)$$
$$\sigma_B : b \to [0,1] = \Pr(x = B|b).$$

Clearly voting probabilities are pinned down by the voting strategy being played. We say that the voting strategy $\sigma = (\sigma_A, \sigma_B)$ is informative if $\sigma_A = \sigma_B = 1$ for votes reflect the suggestion provided by the signal.

3.1.2 Persuader

Persuader designs the information content of the signal observed by voters. Let π_s denotes the precision of the signal so that

$$\pi_a = \Pr(s = a|\alpha)$$
$$\pi_b = \Pr(s = b|\beta).$$

We assume that Persuader's preferences do not match society's interests. Denote with $u^P(c,\omega)$ persuader's utility.

Assumption 3.3. Conflict of Interest.

$$u^P(c,\omega) = \begin{cases} 1 \text{ if } & c = A \\ 0 \text{ if } & c = B. \end{cases}$$

Assumption 3.3 says Persuader's utility does not depend on the state of Nature. In particular, he strictly prefers alternative A to be the winning alternative regardless from the true state of Nature. Therefore, persuader's expected payoff is simply the probability with which alternative A wins the election, that is

$$\mathbb{E}[u^P(c,\omega)] = W_A \equiv \frac{1}{2}\Pr\{n_A(\alpha) - n_B(\alpha) \geq 0\} + \frac{1}{2}[1 - \Pr\{n_B(\beta) - n_A(\beta) \geq 0\}].$$

3.1.3 Timing and Equilibrium

The game proceeds as follows.

i) Persuader designs π;

ii) Nature selects the state and the number of voters;

iii) Signals are observed and votes are cast;

iv) The most voted alternative wins the election and payoffs are realized, ties are broken in favor of the correct alternative[4].

In equilibrium no voter has a profitable unilateral deviation and Persuader's signal maximizes the probability with which alternative A wins the election.

Definition 3.1. Equilibrium.
Equilibrium is defined as a pair (σ^*, π^*) such that

i) For all voters recruited by Nature, beliefs are formed according to Bayes rule and

$$\mathbb{E}[u(c,\omega)|piv(\sigma^*), \sigma^*, \pi] \geq \mathbb{E}[u(c,\omega)|piv(\sigma^*), \sigma', \pi]. \tag{3.1}$$

ii) The signal maximizes the probability with which alternative A wins the election

$$\pi^* = \text{argmax } W_A\left(\sigma^*(\pi)\right). \tag{3.2}$$

Condition (3.1) says that, given the voting strategy and the choice of π, σ^* must be optimal in the sense that no voter has a profitable deviation at her belief taking into account that she might cast a pivotal vote[5]. Whereas condition (3.1) focuses on voters rational behavior, condition (3.2) says that the optimal signal maximizes Persuader's expected utility given that voters are best replying to their information. Together, conditions (3.1) and (3.2) express the idea that no player in the game has a unilateral profitable deviation. Finally, we follow Kamenica and Gentzkow (2011) and we require that, whenever voters are indifferent at a given belief, that is to say whenever at $\mu_\omega(s,\pi)$ they are indifferent between, at least, two voting strategies say σ^* and σ', then in equilibrium, voters play the voting strategy σ^* that prescribes them Persuader's favorite action

[4]Our results do not depend on the tie rule. We could use alternative rules at the expense of longer computations.

[5]Consider a randomly drawn voter. She understands that her vote might affect her payoff only in a pivotal event. That is, all the events in which the number of votes for alternative A and B differ by just 1 or 0 votes.

at $S = s$. This assumption implies that $W_A(\cdot)$ is upper semicontinuous in π which in turn ensures the existence of an optimal signal. Also, this assumption implicitly rules out mixed strategy equilibria that might be played whenever voters are indifferent at a given belief and it allows us to focus on informative voting equilibria. Informative voting requires voters to perfectly correlate their vote with the public signal. Intuitively, considering that the information content of the signal is strategically designed by a Persuader, this equilibrium outcome might be easily said to display persuasion and that is why it is of particular interest here.

3.1.4 Societies and Commitment

Before proceeding further, it is worth to define the concept of "society" in this model.

Definition 3.2. A society is characterized by the following primitives:

- Types of voter recruited by Nature;
- Information.

In this model, the choice of the society in which the game is played is exogenously determined.

Independent Society: in the independent society randomly drawn voters are identical and privately and independently informed as in the classic Condorcet Jury. Therefore, in this society, Persuader sends to each recruited voter a signal $s \in \{a, b\}$ drawn from π.

Controlled Society: in the controlled society randomly drawn voters are identical an publicly informed. In this society all voters share the same information s publicly drawn from π.

Imperfectly Controlled Society: in the imperfectly controlled society all voters share the same public information s as in the controlled society. Nevertheless, with some small probability $\varepsilon \in (0, \frac{1}{2})$, Nature draws an irrational voter whose voting behavior is predetermined and thus not controllable.

Implicit in this formulation is the commitment assumption, see Kamenica and Gentzkow (2011). The commitment assumption says that signal s is observed with probability

$$\Pr(S = s) = \sum_{\omega \in \Omega} \Pr(\Omega = \omega)\pi(s|\omega),$$

that is to say, the signal is always truthfully revealed or, in other words, Persuader is not allowed to lie.

3.2 Independent Society

The independent society represents the classical Condorcet Jury arranged in a Poisson game. According to Ladha (1992) and, more recently, Feddersen and Pesendorfer (1998), private information reflects the idea that voters may interpret the same evidence differently because of their *model of reality*. In the independent society each recruited voter is assigned a type $s \in \{a, b\}$ according to the endogenous distribution π designed by the Persuader. In this society the information content of the signal affects voters' private interpretation of the signal. Therefore, the choice of π determines the type-distribution of recruited voters[6].

To begin with, we describe voting probabilities in the independent society. Consider the voting strategy $\sigma = (\sigma_A, \sigma_B)$. If voters are endorsing the voting strategy σ, then the probabilities with which a randomly drawn voter cast a ballot for alternative c in state ω are

$$v_A(\alpha) = \pi_a \sigma_A + (1 - \pi_a)(1 - \sigma_B)$$
$$v_B(\beta) = \pi_b \sigma_B + (1 - \pi_b)(1 - \sigma_A).$$

To understand these voting probabilities consider first $v_A(\alpha)$. In state α a randomly drawn voter observes a with probability π_a and thus votes for A with probability σ_A. On the other hand she observes b with probability $1 - \pi_a$ and thus votes for A with probability $1 - \sigma_B$. The reasoning for $v_B(\beta)$ is analogous. In addition, for voting is compulsory, $v_A(\alpha) + v_B(\alpha) = 1$ and the same is true in state β. Voting probabilities determine the number of votes for alternative A and B and therefore they pin down pivotal events. Recall that a vote may change the outcome of the election only if there is a tie or one alternative is beyond the other by just one vote. Let $T_m(\omega)$ denotes the probability with which alternative A beats alternative B by exactly m votes. Since $n_A(\omega)$ and $n_B(\omega)$ are independent Poisson random variables we have

$$\begin{aligned} T_m(\omega) &= \Pr\{n_A(\alpha) - n_B(\alpha) = m\} \\ &= e^{-k} \left(\frac{v_A(\alpha)}{v_B(\alpha)} \right)^{\frac{m}{2}} I_{|m|} \left(2k\sqrt{v_A(\alpha) v_B(\alpha)} \right) \end{aligned} \tag{3.3}$$

where $I_{|m|}(\cdot)$ is the modified Bessel function of the first kind[7]. Therefore, with

[6] Recall however that all voters recruited by Nature are *identical*, they differ because of the private information designed by the Persuader.

[7] See Abramowitz et al. (1966).

a slight abuse of notation

$$piv(\alpha) = T_0(\alpha) + T_{-1}(\alpha)$$
$$piv(\beta) = T_0(\beta) + T_{+1}(\beta).$$

Intuitively, a vote in state α is pivotal only if it breaks a tie, $m = 0$, or alternative A is beyond by just one vote, $m = -1$. The reasoning is state β is analogous. In order to simplify the analysis, we also introduce the following notation. Denote with $U(c|s)$ the conditional expected utility deriving from casting a ballot for alternative $c \in \{A, B\}$ conditional on signal $s \in \{a, b\}$ given the profile of strategies (π, σ).

Austen-Smith and Banks (1996) demonstrated that the sincere informative voting behavior ($\sigma = (1, 1)$) constitutes a rational profile if and only if majority voting is the optimal method of aggregating individual's information and $\pi_a = \pi_b$. Not surprisingly, a similar result holds in the Poisson framework.

Lemma 3.1. *Independent society; If $\pi_a = \pi_b \geq q$, then informative voting is rational.*

Proof. If voting is informative then $\sigma_A = 1$ and $v_A(\alpha) = \pi_a$. Analogously, $\sigma_B = 1$ and $v_B(\beta) = \pi_b$. Rationality condition (3.1) requires

$$U\left(A|a, piv(\sigma), \pi\right) \geq U\left(B|a, piv(\sigma), \pi\right)$$
$$U\left(B|b, piv(\sigma), \pi\right) > U\left(A|b, piv(\sigma), \pi\right).$$

Expanding the conditional expected utilities we obtain

$$-q\mu_\beta(a)piv(\beta) \geq -(1-q)\mu_\alpha(a)piv(\alpha)$$

$$-(1-q)\mu_\alpha(b)piv(\alpha) \geq -q\mu_\beta(b)piv(\beta)$$

and combining the inequalities

$$\frac{\pi_a}{1-\pi_b}\frac{1-q}{q} \geq \frac{piv(\beta)}{piv(\alpha)} \geq \frac{1-\pi_a}{\pi_b}\frac{1-q}{q}. \tag{3.4}$$

In the following we will refer to condition (3.4) as the *informative voting* condition. Suppose now that $\pi_a = \pi_b \geq q$. Then $v_A(\alpha) = v_B(\beta)$. As a consequence, from expression (3.3) it follows that $T_0(\alpha) = T_0(\beta)$. Furthermore, because $v_A(\alpha) = v_B(\beta)$,

$$T_{-1}(\alpha) = \Pr\{n_A(\alpha) - n_B(\alpha) = -1\}$$
$$= \Pr\{n_B(\beta) - n_A(\beta) = -1\} = T_{+1}(\beta).$$

That is to say, at $\pi_a = \pi_b$ the probability of casting a pivotal vote is the same across state and $piv(\alpha) = piv(\beta)$. Therefore, pivotal considerations cancel out and informative voting is rational if it is sincere. Finally, this implies that informative voting is a best reply to any π satisfying $\pi_a = \pi_b \geq q$. □

Lemma (3.1) says that if Persuader supplies a symmetric signal satisfying $\pi_a = \pi_b \geq q$ then votes perfectly reflect the private recommendation embedded by the signal. Myerson (1998a) shows that there are mixed strategy equilibria that arise whenever $\pi_a \neq \pi_b$ that satisfies the Condorcet Jury Theorem. Although the Persuader may gain from mixed strategy equilibria, we are concerned with voting behaviors characterized by a large number of voters. The following proposition states that, as the expected number of voters approaches infinity, that is as $k \to \infty$, the unique equilibrium signals satisfy $\pi_a \geq \pi_b = q$. Before describing the proposition, it is worth to recall Myerson (1998a)'s approximation formulae. In particular, as $k \to \infty$ we have that, for k large enough

$$T_m(\omega) \approx \left(\frac{v_A(\omega)}{v_B(\omega)}\right)^{\frac{m}{2}} \frac{e^{-k\left[\sqrt{v_A(\omega)}-\sqrt{v_B(\omega)}\right]^2}}{\sqrt{k}\sqrt{4\pi\sqrt{v_A(\omega)}\sqrt{v_B(\omega)}}} \tag{3.5}$$

that allows us to make the following approximation

$$\frac{piv(\beta)}{piv(\alpha)} \approx \exp\left(k\left[\left(\sqrt{v_A(\alpha)} - \sqrt{v_B(\alpha)}\right)^2 - \left(\sqrt{v_B(\beta)} - \sqrt{v_A(\beta)}\right)^2\right]\right). \tag{3.6}$$

The approximation (3.6) contains the leading term of the ratio that strictly depends on the expected number of voters k. Intuitively, the behavior of the ratio depends on the difference between $v_A(\alpha)$ and $v_B(\beta)$.

Proposition 3.1. *Independent society; As the expected number of voters approaches infinity, voters play the informative voting strategy if and only if π^* satisfies $\pi_a^* = \pi_b^* \geq q$.*

Proof. The equilibrium voting strategy depends on the ratio $\frac{piv(\beta)}{piv(\alpha)}$. From approximation (3.6) it is direct to conclude that

$$\lim_{k\to\infty} \frac{piv(\beta)}{piv(\alpha)} = 1.$$

Whenever $\lim_{k\to\infty} \frac{piv(\beta)}{piv(\alpha)}$ diverges to infinity or approaches zero, voters' unique best response is to always elect alternative B or A. On the contrary, from lemma (3.1) we know that informative voting is rational for $\pi_a = \pi_b \geq q$ which implies that voting probabilities are constant, $v_A(\alpha) = v_B(\beta)$ and $\lim_{k\to\infty} \frac{piv(\beta)}{piv(\alpha)} = 1$. □

Proposition 3.1 directly implies that the informative voting equilibrium behavior in the limit satisfies the Condorcet Jury Theorem.

Proposition 3.2. ***Condorcet Jury Theorem, Independent Society.*** *Consider the signal consistent with the informative voting behavior. As the expected number of voters approaches infinity, the probability with which majority selects the correct alternative approaches one in both states. Formally, as* $k \to \infty$

$$\Pr\{n_A(\alpha) - n_B(\alpha) \geq 0\} \to 1$$
$$\Pr\{n_B(\beta) - n_A(\beta) \geq 0\} \to 1.$$

The proof follows (Myerson, 1998a, Theorem 2).

Proof. From proposition 3.1 in the limit π^* satisfies $\pi_a = \pi_b \geq q$ and σ^* is informative. Since $q > \frac{1}{2}$ we have that $v_A(\alpha) = v_B(\beta) > \frac{1}{2}$. Without loss of generality, assume that $\pi^* = (q, q)$. This shows that in the limit the majority is expected to be correct in both states for

$$\lim_{k \to \infty} \frac{n_A(\alpha)}{n_B(\alpha)} = \lim_{k \to \infty} \frac{n_B(\beta)}{n_A(\beta)} = \frac{q}{1-q} > 1.$$

On the other hand, since $n_c(\omega)$ are independently Poisson distributed, their difference is Skellam distributed. Thus, for $c \in \{A, B\}$ and $c' \neq c$:

$$\mathbb{E}\left\{n_c(\omega = c) - n_{c'}(\omega = c)\right\} = k[v_c(\omega = c) - v_{c'}(\omega = c)]$$
$$\mathrm{Var}\left\{n_c(\omega = c) - n_{c'}(\omega = c)\right\} = k.$$

Therefore the expected value of these differences divided by their standard deviation goes to infinity as $k \to \infty$ and the probability of a majority error goes to zero in each state. □

A direct implication of proposition 3.2 is that, regardless from Persuader's attempt to manipulate the outcome of the election, information aggregates and alternative A wins the election when alternative A is in fact the best alternative, that is with probability $\frac{1}{2}$ for the states of Nature are equally likely. Therefore, in the independent society:

$$\lim_{k \to \infty} W_A^{IS}(\sigma^*(\pi^*), k) = \frac{1}{2}$$

and Persuader's gain deriving from designing the information content of the signal is exactly $\frac{1}{2}$.

3.3 Controlled Society

The controlled society represents the antithetic version of the classical Condorcet Jury for the signal consists of public information. Meaning that all voters observe the same information and thus all recruited voters are characterized by the same information type. Thus, after having observed the public signal, voters' uncertainty only concerns the state of Nature and the number of recruited voters. Differently from the independent society, there is no uncertainty about what information could another randomly drawn voter have observed. For this reason, it is convenient to condition voting probabilities on the observation of the public signal. Consider the voting strategy σ, then, for all $\omega \in \{\alpha, \beta\}$

$$v_A(\omega|a) = \sigma_A$$
$$v_B(\omega|b) = \sigma_B.$$

It is then clear how the public signal affects voting behavior. Whenever voters endorse the informative voting strategy, as the following lemma shows, a voter could be pivotal only if, other than herself, Nature recruited no voters.

Lemma 3.2. *If voting is informative, then*

$$piv(\alpha) = piv(\beta) = e^{-k}.$$

Proof. Consider what happens when $s = a$. Recall that informative voting implies that $\sigma_A = \sigma_B = 1$. Therefore, $v_A(\alpha|a) \times v_B(\alpha|a) = 0$. In order to compute the probability of the events in which the alternatives receive the same number of votes or differ by just one vote, we need to consider the conditional voting probabilities, thus

$$\begin{aligned} T_0(\alpha) &= e^{-k} I_0\left(2k\sqrt{v_A(\alpha|a)v_B(\alpha|a)}\right) \\ &= e^{-k} I_0(0) = e^{-k} \end{aligned}$$

since $I_0(0) = 1$ by the properties of $I_{|m|}(\cdot)$. Notice that e^{-k} is the probability with which Nature draws zero voters. Analogously, consider $T_{-1}(\alpha)$. Intuitively, if $s = a$ is public information and σ is informative, then the probability with which alternative A is beyond by one vote conditional on $s = a$ is zero. Indeed

$$T_{-1}(\alpha) = e^{-k}\left(\frac{v_B(\alpha|a)}{v_A(\alpha|a)}\right)^{\frac{1}{2}} I_1\left(2k\sqrt{v_A(\alpha|a)v_B(\alpha|a)}\right) = 0$$

since $I_1(0) = 0$. By symmetry, $T_0(\beta) = e^{-k}$ and $T_{+1}(\beta) = 0$. Analogously, the same reasoning holds for $s = b$. □

Although voters are publicly informed, because they are engaged in a Poisson game they could still cast a pivotal vote if they endorse the informative voting strategy. Whenever a voter is the only voter recruited by Nature, this voter is by all means pivotal. Nevertheless, for voters are equally likely to be pivotal in both states of Nature, pivotal considerations cancel out and informative voting is rational if it is sincere[8]. Before characterizing the optimal signal and equilibrium voting behavior, we characterize Persuader's payoff in the controlled society.

Lemma 3.3. *Controlled society; If voters are voting informatively, then the probability with which alternative A wins the election is the probability with which the public recommendation is a and it does not depend on the expected number of voters. Formally,*

$$\begin{aligned} W_A^{CS}(\sigma(\pi), k) &= \Pr(s = a) \\ &= \frac{1}{2}\pi_a + \frac{1}{2}(1 - \pi_b). \end{aligned}$$

Proof. Consider $\sigma_A = \sigma_B = 1$. If $s = a$ all voters vote for alternative A, conversely if $s = b$ then all voters vote for alternative B. Then, regardless from the expected number of voters,

$$\Pr(s = a) = \frac{1}{2}\pi_a + \frac{1}{2}(1 - \pi_b)$$

since the states of Nature are equally likely. □

Lemma 3.3 is straightforward and allows us to simplify Persuader's problem. First, as long as the signal consists of public information, differently from the Independent society, Persuader is not concerned by the expected number of voters. Nevertheless, a comment is in order. With positive probability, Nature could recruit 0 voters, and, in that event, Persuader's payoff would be $\frac{1}{2}$. Nevertheless, as our focus is on large election, we can neglect this event. Indeed, for $k = 100$ the probability with which Nature recruits 0 voter in state ω is $e^{-100} \approx 3.7 \times 10^{-44}$. Second, from lemma 3.2, it follows that pivotal considerations cancel out. Finally, as voters are identical, the Persuader supplies the optimal signal he would supply if he were persuading one single voter. The equilibrium signal shares the same properties of the Bayesian persuasion model (Kamenica and Gentzkow, 2011).

[8] Following Austen-Smith and Banks (1996), we say that σ is sincere if it maximizes the expected utility of a randomly drawn voter that behaves as if she were playing alone, i.e., neglecting pivotal considerations.

Proposition 3.3. *Controlled society; In equilibrium voting is informative and the optimal signal satisfies*

$$\pi_a^* = 1, \pi_b^* = 1 - \frac{1-q}{q} \tag{3.7}$$

regardless from the expected number of voters.

Proof. By lemma 3.2 we know that $piv(\alpha) = piv(\beta)$ if $\sigma = (1,1)$. Then, pivotal considerations simplify out in the informative voting condition and we can simply rephrase the informative voting condition in the controlled society as the sincere voting condition

$$U(A|a,\pi) \geq U(B|a,\pi) \text{ and } U(B|b,\pi) > U(A|b,\pi).$$

We show that π^* satisfies the informative voting condition and that the Persuader can not increase his payoff by deviating from π^*. First, at $\pi_a^* = 1$ we have that $\mu_\beta(b) = 1 \implies U(B|b,\pi) > U(A|b,\pi)$. On the other hand, at $\pi_a^* = 1$ we have that

$$\frac{1}{1-\pi_b}\frac{1-q}{q} \geq 1.$$

Therefore, π_b^* solves $U(A|a,\pi) = U(B|a,\pi)$ for $\pi_a^* = 1$. This means that, at π^*, informative voting is rational. Finally, it is straightforward to see that Persuader has no profitable deviation. Recall that $W_A^{CS}(\sigma(\pi)) = \Pr(s = a)$. Then,

$$W_A^{CS}(\sigma^*(\pi^*)) > W_A^{CS}(\sigma^*(\pi'))$$

where π' is such that either $\pi_a < 1$ or $\pi_b > \pi_b^*$. On the other hand, for $\pi_b < \pi_b^*$, informative voting would no longer be rational since it would make $U(B|a,\pi) > U(A|a,\pi)$. This shows that $\pi^* = \operatorname{argmax} W_A^{CS}(\sigma^*(\pi))$. □

Two comments are in order. The first is that at π^* voters get the same payoff they would get under their default behavior, that is always voting for B. According to the informative voting strategy, the only mistake voters could make is to elect alternative A in state β. This happens whenever $s = a$ occurs in state β. Therefore voters' expected loss deriving by playing the informative voting strategy is $-q\frac{1}{2}(1-\pi_b^*) = -\frac{1}{2}(1-q)$. On the other hand, recall that at π^*, conditional on $s = a$, voters are indifferent between always voting for B and voting informatively. However, if voters were playing their default behavior, the only mistake they could make would be to elect alternative B in state α. This would surely happen and the expected loss deriving from the default behavior would be $-\frac{1}{2}(1-q)$ which is exactly the same expected loss deriving by playing the informative voting strategy in equilibrium. Second, this implies that, also in this case, the focus on pure strategy is without loss of generality. To prevent a mixed strategy profile, Persuader could slightly decrease π_a^* by an infinitesimal amount so that $U(A|a,\pi) > U(B|b,\pi)$ and his payoff would change by a negligible amount. In addition, even if voters were randomizing between alternative A and B conditional on $s = a$, in a large election Persuader's payoff would still be close to $\Pr(s = a)$. This follows from the fact that a mixed strategy profile would satisfy $\sigma_A \in (\frac{1}{2}, 1)$ which implies $v_A(\alpha|a) > \frac{1}{2}$. Then, by the same reasoning described in Proposition 3.2, in a large election the probability with which A gets elected conditional on $s = a$ would approach 1.

In the controlled society voters' concerns about the probability of casting a pivotal vote are irrelevant in equilibrium for the signal consists of public information. This directly means that Persuader behaves as if he were persuading a single voter and the behavior of the majority is not distinguishable from the behavior of a single voter.

Proposition 3.4. ***Persuasive Outcomes, Controlled Society.***
In the controlled society alternative A always wins when alternative A is the best alternative. Alternative B wins when alternative B is the best alternative with probability $\Pr(s = b)$.

Proof. In equilibrium $\pi_a^* = 1$ implies that in state α the only possible realization of the signal is a. Therefore, in state α alternative A wins with probability 1. In state β, alternative B wins the election only if $s = b$. This happens with probability $\pi_b^* = \Pr(s = b)$ since $\pi_a^* = 1$. □

Proposition 3.4 underlines the fact that, in the controlled society, voters' behavior is completely determined by Persuader's signal. This means that, rather than the majority, all voters are completely swayed and the realization of the public signal completely determines the winning alternative. Finally, in the controlled society Persuader's gain deriving from designing the information content of the signal is exactly $\Pr(s = a)$.

3.4 Imperfectly Controlled Society

In the imperfectly controlled society Persuader's signal consists of public information. Nevertheless, we assume that Nature could recruit two types of voters. With probability $1-\varepsilon$ Nature recruits a rational voter. On the other hand, with probability ε Nature recruits an irrational voter. We call such a voter type-ε voter. Irrational voters always vote against the public information. This means that conditional on the observation of $s = a$, an irrational voter would vote for alternative B and vice versa for $s = b$. Under this assumption, differently from the controlled society, although the signal is publicly supplied, informative voting would not yield perfectly correlated votes. In this sense, the society is imperfectly controlled. We assume that $\varepsilon \in (0, \frac{1}{2})$. In particular, we expect ε to be small so that the electorate is mainly characterized by rational voters. There are several ways in which such an irrational behavior could be justified. For instance, this irrational behavior could arise from the misinterpretation of the ballot card. In 1974, Italy hold an abrogative referendum that asked voters to repeal a previous law legally enforcing divorce. The wording of the referendum were thus considered by some confusing for a "yes" vote would had expressed the individual will to abolish divorce rather than maintaining it. Some voters may be confused by the wording of a referendum or the symbols in a ballot and that could lead them to cast a"wrong" vote. Exactly for that reason, the EU Commission asked Prime Minister D. Cameron to review the wording of the referendum concerning the permanence of Britain in the European Union[9]. Nevertheless, we can also think about ε-types as independent voters whose behavior is not manipulable.

In the imperfectly controlled society voting probabilities are affected by the presence of irrational voters even if rational voters endorse the informative voting strategy. Indeed, suppose that σ is informative. Then, in order to compute voting probabilities we must take into account that a recruited voter could be either rational or irrational. As in the controlled society, it is convenient to condition these probabilities on the public signal. Then, for all $\omega \in \{\alpha, \beta\}$

$$v_A(\omega|a) = 1 - \varepsilon$$
$$v_B(\omega|b) = 1 - \varepsilon.$$

The intuition beyond these voting probabilities is straightforward. Consider $s = a$ and a randomly drawn voter. With probability $1-\varepsilon$ she is a rational voter and she votes for alternative A with probability 1 because of the informative voting strategy. On the other hand, with probability ε, Nature has recruited an irrational voter. Irrational voters are characterized by the fact that they vote against the public information. Thus, conditional on $s = a$ the probability that an ε-type voter votes for alternative A is zero.

The next lemma characterizes Persuader's payoff conditional on rational voters playing the informative voting strategy. To ease the reading let us introduce

[9]http://www.theguardian.com/politics/2015/sep/01/eu-referendum-cameron-urged-to-change-wording-of-preferred-question

the following notation:

$$w_A(a) \equiv \Pr\{n_A(\omega) - n_B(\omega) \geq 0|a\}$$
$$w_A(b) \equiv (1 - \Pr\{n_B(\omega) - n_A(\omega) \geq 0|b\})$$

that are the probabilities with which alternative A wins the election conditional on $s \in \{a, b\}$.

Lemma 3.4. *Imperfectly controlled society; Suppose that σ is informative. Then the probability with which alternative A wins the election depend on the expected number of voters and*

$$W_A^{ICS}(\sigma(\pi), k) = \frac{1}{2}[\pi_a w_A(a) + (1 - \pi_a)w_A(b)] + \frac{1}{2}[\pi_b w_A(b) + (1 - \pi_b)w_A(a)].$$

Proof. If σ is informative then rational voters vote for A whenever $s = a$ and they vote for B whenever $s = b$. Nevertheless, as $v_c(\omega|s) < 1$ for all $\omega \in \{\alpha, \beta\}$ votes are not perfectly correlated. Consider what happens when $s = b$. Although the majority of voters would vote for B, we have that

$$w_A(b) \equiv 1 - \Pr\{n_B(\omega) - n_A(\omega) \geq 0|b\} > 0$$

for

$$w_A(b) = \sum_{m=0}^{\infty} T_m(\omega|b) > 0$$

since the number of voters is Poisson distributed and $v_A(\omega|b) = \varepsilon > 0$. This follows from the fact that, regardless from s, Nature could draw a majority of irrational voters. The expression for $W_A^{ICS}(\sigma(\pi), k)$ follows directly. Finally, since the terms $w_A(s)$ depend on the expected number of voters, see (3.3), the probability with which alternative A wins the election depends on the expected number of voters. □

The fact that Nature could recruit irrational voters directly impacts Persuader's payoff. In particular, since

$$v_A(\omega|a) = 1 - \varepsilon > \frac{1}{2} > v_A(\omega|b) = \varepsilon$$

it is easy to conclude that, if voters are voting informatively, $w_A(a) > w_A(b)$. Intuitively, it is more likely that alternative A wins the election when $s = a$ rather than $s = b$ since the majority of voters is expected to be rational.

The presence of irrational voters also affect the behavior of rational voters. A randomly drawn rational voter anticipates that, even if rational voters are voting informatively, Nature could have recruited enough irrational voters so that her vote could change the outcome of the election. Let us denote with $\tau(s)$ the ratio of pivotal probabilities conditional on signal s. Since our focus on large electorates, that is when k is large, from (3.5)

$$\tau(a) = \frac{piv(\beta)}{piv(\alpha)} \approx \frac{T_0(\beta)\left[1+\sqrt{\frac{v_A(\beta|a)}{v_B(\beta|a)}}\right]}{T_0(\alpha)\left[1+\sqrt{\frac{v_B(\alpha|a)}{v_A(\alpha|a)}}\right]}.$$

Furthermore from (3.5) it also follows that $T_0(\alpha) = T_0(\beta)$ since the expected number of voters is independent from ω. Then

$$\tau(a) \approx \sqrt{\frac{v_A(\alpha|a)}{v_B(\beta|a)}} = \sqrt{\frac{1-\varepsilon}{\varepsilon}} \tag{3.8}$$

$$\tau(b) \approx \sqrt{\frac{v_A(\alpha|b)}{v_B(\beta|b)}} = \sqrt{\frac{\varepsilon}{1-\varepsilon}}. \tag{3.9}$$

Expression (3.8) and (3.9) says that, for k large enough, the probability of casting a pivotal vote depends exclusively on the rate at which Nature draws an irrational voter. Although Persuader's, by assumption, is not allowed to control ε, in order to induce the informative voting strategy his signal must internalize these pivotal considerations.

Proposition 3.5. *Imperfectly controlled society; For k large enough, In equilibrium voting is informative and the optimal signal satisfies*

$$\pi_a^* = 1, \pi_b^* = 1 - \frac{1-q}{q}\frac{1}{\tau(a)}.$$

Proof. Consider k large enough so that approximations (3.8) and (3.9) hold. The informative voting condition requires

$$U\left(A|a, piv(\sigma), \pi\right) \geq U\left(B|a, piv(\sigma), \pi\right)$$
$$U\left(B|b, piv(\sigma), \pi\right) \geq U\left(A|b, piv(\sigma), \pi\right).$$

Expanding the conditional expected utilities we obtain

$$\frac{\pi_a}{1-\pi_b}\frac{1-q}{q} \geq \tau(a)$$

$$\frac{1-\pi_a}{\pi_b}\frac{1-q}{q} < \tau(b).$$

Now we show that under π^* informative voting is rational. To begin with notice that for $\varepsilon \in (0, \frac{1}{2})$ the ratio $\tau(b)$ is strictly higher than 0. Then, at $\pi_a^* = 1$ we have $\tau(b) > 0$. Thus, conditional on $s = b$ voting for alternative B is rational.

On the other hand, π_b^* solves $U(A|a, piv(\sigma), \pi) = U(B|a, piv(\sigma), \pi)$ for $\pi_a^* = 1$. Also, we have that for $\varepsilon \in (0, \frac{1}{2})$ the ratio $\frac{1}{\tau(a)}$ belongs to the interval $(0, 1)$ which allows us to conclude that $\pi_b^* \in (\frac{1}{2}, 1)$. Therefore, $\pi^* \in [0, 1]^2$ is a feasible signal such that informative voting is rational. Finally, we have to show that Persuader has no profitable deviation. Since $w_A(a) > w_A(b)$ and the fact that $w_A(s)$ does not depend on π, a deviation from π^* is not profitable as for $\pi^b < \pi_b^*$ informative voting would no longer be rational whereas $\pi_b > \pi_b^*$ increases the probability with which rational voters vote fore alternative B. □

The optimal signal in the imperfectly controlled society shares the same characteristics of the optimal signal the Persuader would supply in the controlled society. Nevertheless, there is a crucial difference. In the imperfectly controlled society the optimal signal internalizes pivotal considerations. In particular, observe that $\tau(a) > 1$ which means that, conditional on $s = a$ voters understand that they are more likely to be pivotal in state β. Conditional on $s = a$ most of the voters are voting for alternative A which implies that the event "alternative A is winning by one vote" is more likely than the event "alternative A is loosing by one vote". However, the latter event happens to be a pivotal event in state α whereas the former constitutes a pivotal event in state β and that is the reason why $\tau(a) > 1$.

Differently from the independent society, in the imperfectly controlled society the likelihood of pivotal events do not depend on the precision of the signal. Indeed, conditional on the voting strategy being played, the probability of these events is entirely determined by the rate ε. This allow the Persuader to manipulate voters' belief through the design of the information content of the signal. Indeed, the equilibrium condition $U(A|a, piv(\sigma), \pi) = U(B|a, piv(\sigma), \pi)$ is equivalent to

$$q\mu_\alpha(a)piv(\alpha) = (1 - q)\mu_\beta(a)piv(\beta).$$

Therefore, through the choice of the signal the Persuaders induces those posterior beliefs that equalize voters' expected loss conditional on being pivotal. The following proposition characterizes the limiting outcome of the game.

Proposition 3.6. ***Weak Condorcet Jury Theorem, Imperfectly Controlled Society.***
As the expected number of voters approaches infinity:

i) The probability with which A wins the election approaches $\Pr(s = a)$*;*

ii) The probability with which the majority selects the correct alternative goes to one in state α and to π_b^* in state β.

Proof. As $\varepsilon \in (0, \frac{1}{2})$, we have that $v_A(\alpha|a) > \frac{1}{2}$ and $v_B(\beta|b) > \frac{1}{2}$. This shows that conditional on $s \in \{a, b\}$ the majority is expected to elect the alternative suggested by the public signal since for all $\omega \in \{\alpha, \beta\}$

$$\lim_{k\to\infty} \frac{n_A(\omega|a)}{n_B(\omega|a)} = \lim_{k\to\infty} \frac{n_B(\omega|b)}{n_A(\omega|b)} = \frac{1-\varepsilon}{\varepsilon} > 1.$$

From proposition 3.2 it follows that as $k \to \infty$

$$w_A(a) \equiv \Pr\{n_A(\omega) - n_B(\omega) \geq 0|a\} \to 1$$
$$w_A(b) \equiv 1 - \Pr\{n_B(\omega) - n_A(\omega) \geq 0|b\} \to 0.$$

Therefore

$$W_A^{ICS}(\sigma(\pi), k) \to \Pr(s = a)$$

which implies that alternative B wins the election with $\Pr(s = b) = \frac{1}{2}\pi_b^*$. □

Proposition 3.6 characterize the limit outcome of the game in the imperfectly controlled society. Intuitively, as $k \to \infty$, the noise introduced by irrational voters disappear and the majority of voters is completely swayed by the public signal. Nevertheless, in order to induce the informative voting strategy, the optimal signal must internalize voters' pivotal consideration and this limits the extent with which Persuader can manipulate the outcome of the election. Indeed, let us compare Persuader's payoff in the controlled and imperfectly controlled society in the limit. Since in both societies in the limit $\pi_a^* = 1$, it is sufficient to compare the precision of the signal in state β. Recall that π_b^* denotes the equilibrium probability with which b is observed in state β and therefore Persuader's payoff is decreasing in π_b. Then

$$\pi_b^{*CS} = 1 - \frac{1-q}{q} > \pi_b^{*ICS} = 1 - \frac{1-q}{q}\sqrt{\frac{\varepsilon}{1-\varepsilon}}$$

since, for $\varepsilon \in (0, \frac{1}{2})$, $\sqrt{\frac{\varepsilon}{1-\varepsilon}} < 1$. Therefore, Persuader's gain is maximum in the controlled society. Another important conclusion could be drawn from this comparison. In the controlled society Persuader behaves as if he were persuading a single voter. This is no longer true in the imperfectly controlled society. The previous inequality underlines that the optimal signal in the controlled society would not make informative voting rational in the imperfectly controlled society. Nevertheless, if the Persuader were to persuade a single rational voter in the imperfectly controlled society, he would supply the same signal he supplies in the controlled society. However, as we demonstrated, Persuader's signal must internalize pivotal considerations in order to persuade a majority, therefore, in the imperfectly controlled society, a majority of voters is *harder to persuade* than a single voter.

3.5 Convergence in Societies

In this section we are concerned with the following issue. Consider voting probabilities in the imperfectly controlled society and suppose, without loss of generality, that $s = a$:

$$v_A(\alpha|a) = 1 - \varepsilon.$$

If there were no irrational voters, then $v_A(\alpha|a) = 1$, that is to say as $\varepsilon \to 0$ voting probabilities in the imperfectly controlled society approach voting probabilities in the controlled society. Nevertheless, this does not necessarily mean that as $\varepsilon \to 0$ the imperfectly controlled society approaches the controlled society. We say that the imperfectly controlled society approaches the controlled society if there exists a sequence of equilibria that approach the equilibrium play in the controlled society. The following definition states this concept formally.

Definition 3.3. We say that the imperfectly controlled society approaches the controlled society if there exists a sequence of ICS-games $\{\mathrm{ICS}_{\varepsilon^n}\}_{n=1}^{\infty}$ that converges to the CS, in the sense that

$$\lim_{n\to\infty} \varepsilon^n = 0$$

for which there is some associated sequence of equilibria $\{(\pi^n, \sigma^n)\}_{n=1}^{\infty}$ that converges to (π^{CS}, σ^{CS}) i.e., such that

$$\lim_{n\to\infty} (\pi^n, \sigma^n) = (\pi^{CS}, \sigma^{CS})$$

where (π^{CS}, σ^{CS}) denotes the equilibrium played in the controlled society as the expected number of voters approaches infinity.

Definition 3.3 underlines that as $\varepsilon \to 0$ the equilibrium in the imperfectly controlled society must approach the equilibrium in the controlled society in order to state that the former converges to the latter. The definition can be compared to the notion of trembling hand perfection. In the controlled society in equilibrium voters play the informative voting strategy, that is $\sigma_A = 1$ and $\sigma_B = 1$. This means that, the equilibrium voting strategy places null probability on the action "vote for alternative B" conditional on $s = a$. The concept of trembling hand perfection identifies equilibria that are robust to the possibility that, with some positive probability, players make mistake. Suppose that, in the CS, voters are subject to a small probability of making a mistake, that is to vote for alternative B conditional on signal a and to vote for alternative A conditional on signal b. Let ε denotes such small probabilities. Then we obtain $v_A(\alpha|a) = 1 - \varepsilon$ and $v_B(\beta|b) = 1 - \varepsilon$, that is the voting probabilities in the imperfectly controlled society. Importantly, notice also that our definition of convergence focuses on trembles made by voters whereas Persuader's signal is unaffected by those trembles.

The next proposition shows that the imperfectly controlled society does not converge to the controlled society.

Proposition 3.7. *The imperfectly controlled society does not converge to the controlled society as $\varepsilon \to 0$.*

Proof. Consider k large enough so that, for any large k the equilibrium profile in the imperfectly controlled society satisfies

$$(\pi, \sigma)^* = \left(\left(\pi_a = 1,\, \pi_b(\varepsilon) = 1 - \frac{1-q}{q}\sqrt{\frac{\varepsilon}{1-\varepsilon}}\right), \sigma = (1,1)\right).$$

Proposition 3.5 states that this is the optimal play for all $\varepsilon \in (0, \frac{1}{2})$. Consider now a sequence of games in the ICS indexed by n that converges to the CS in the sense that $\varepsilon^n \to 0$. Then for all $n = 1, 2, \ldots$, the signal satisfying

$$\pi_a = 1, \pi_b(\varepsilon^n) = 1 - \frac{1-q}{q}\sqrt{\frac{\varepsilon^n}{1-\varepsilon^n}}$$

induces an equilibrium in the imperfectly controlled society (see Proposition 3.5). For only $\pi_b(\varepsilon^n)$ depends on ε, observe that

$$\begin{aligned}\lim_{n\to\infty} \pi_b(\varepsilon(n)) &= \lim_{n\to\infty} \left(1 - \frac{1-q}{q}\sqrt{\frac{\varepsilon^n}{1-\varepsilon^n}}\right) \\ &= 1 - \frac{1-q}{q} \times 0 = 1\end{aligned}$$

since $\varepsilon < 1$, therefore we conclude that

$$\lim_{n\to\infty} (\pi^n, \sigma^n) = (\bar{\pi} = (1,1), \sigma = (1,1)).$$

As n approaches infinity, or as ε approaches 0, Persuader's signal converges toward the perfect informative signal $\pi_a = 1$ and $\pi_b = 1$ whereas voters' strategy is constant and informative. Nevertheless we have that

$$\lim_{n\to\infty} \pi^n \neq \pi^{CS},$$

Therefore, we conclude that the imperfectly controlled society **does not** converge to the controlled society. □

Before commenting the result, the following observation comes straightforwardly.

Corollary 3.1. As $n \to \infty$, that is as the probability with which Nature recruits an irrational voter goes to 0, in large elections, the outcome of the imperfectly controlled society approaches the outcome reached in the independent society:

$$\lim_{n\to\infty} (\pi^n, \sigma^n) = (\bar{\pi} = (1,1), \sigma = (1,1))$$

implies that, for all $s \in \{a, b\}$

$$\Pr\{n_A(\alpha) - n_B(\alpha) \geq 0\} \to 1$$
$$\Pr\{n_B(\beta) - n_A(\beta) \geq 0\} \to 1$$

for, in the limit, signal a is observed with probability 1 in state α and signal b is observed with probability 1 in state β.

Proposition 3.7 states that the imperfectly controlled society does not approach the imperfectly controlled society. This might seem odd at first. Intuitively, as $\varepsilon \to 0$ Nature just recruits rational voters or votes become perfectly correlated with the suggestion embedded by the public signal. Nevertheless, the intuition is proven to be wrong because it neglects rational voters' strategic considerations. The crucial difference between the two controlled society is that in the imperfectly controlled one, informative voting is tailored on voters' beliefs about casting a pivotal vote. Recall that, in large elections, conditional on signal a the ratio

$$\frac{piv(\beta)}{piv(\alpha)} \approx \frac{T_{+1}(\beta)}{T_{-1}(\alpha)} \approx \sqrt{\frac{1-\varepsilon}{\varepsilon}}$$

essentially determines voters' behavior. What happens when ε goes to 0? The limit clearly diverges to infinity and the reason is the following. Conditional on a, when ε is close to zero, almost everyone is voting for alternative A and this generates overwhelming evidence of being pivotal in state β, that is when alternative A is winning in the wrong state of the world. In order to accommodate this beliefs, Persuader's only option is to decrease the probability with which signal a occurs in state β. That is, $\pi_b \to 1$ as ε goes to 0. Therefore, as ε vanishes, Persuader supplies the perfect informative signal that induces voters to be correct in both states of Nature.

Finally, consider Persuader's preferences over ε in the imperfectly controlled society. Suppose that Persuader could slightly reduce or decrease the presence of irrational behavior and thus of those voters who he can not control. From the previous proposition it follows that Persuader would be better off by increasing the presence of irrational voters observing that

$$\frac{\partial \pi_b^*}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(1 - \frac{1-q}{q}\sqrt{\frac{\varepsilon}{1-\varepsilon}}\right) < 0.$$

The intuition relies again on rational voters' strategic considerations. In particular, the presence of this irrational behavior decreases the correlation between votes and, as a consequence, the evidence of being pivotal in one state rather than the other one becomes less stronger. Eventually, at $\varepsilon = \frac{1}{2}$, the probability of casting a pivotal vote is the same in both state and and the informative voting strategy would exactly counter balance the presence of such irrational behavior.

3.6 Related Literature

This paper is mainly related to the literature on Bayesian Persuasion initiated by Kamenica and Gentzkow (2011) They show how a Sender can persuade a Receiver to change her action by designing what she can learn from a public signal which is truthfully revealed to her. Alonso and Câmara (2015) extend the model to a collective decision setting and they characterize how the optimal signal is affected by the aggregation rule. In both models the optimal signal obeys the same intuition and the public signal makes voters indifferent conditional on the realization that suggests to vote for Persuader's favorite alternative. Two other related works come from Wang (2015) and Taneva (2015) who consider different information structures. In general, they show that Persuader's favorite environment is that in which his information is publicly supplied. We distinguish our work in two respects. First, we consider an election affected by population uncertainty. Second, as the afore mentioned works, we consider two different information structures. In addition, we also consider an intermediate situation in which the informative voting strategy does not produce perfectly correlated votes because of the presence of irrational voters. Another related article comes by Ekmekci and Lauermann (2015) who study how a Persuader could manipulate the outcome of the election planning the number of recruited voters in each state of the world rather than the content of a public signal. Our model lacks these considerations for our focus is on information and, by assumption, the expected number of voters is constant across states.

3.7 Conclusion

The Condorcet Jury Theorem has been used as a positive argument in favor of majority decisions as it suggests that large majorities are almost surely correct. Nevertheless, the Theorem is silent about the nature of information. Information is considered exogenous and sufficiently precise. In this chapter we considered information endogenously supplied by a Persuader who wants to manipulate the outcome of the election. We considered private and public signals. Whenever voters are informed by an independent private signal the Condorcet Jury Theorem holds regardless from Persuader's attempts to manipulate to collective decision. On the contrary, whenever Persuader supplies a public signal he perfectly controls voters' action. In particular, voters coordinate their vote on the suggestion embedded by the public signal. Then, we introduce the presence of irrational voters, that is to say, voters whose behavior could not be manipulated. This reintroduces in the model strategic voting considerations that limit the extent with which Persuader's can manipulate the outcome of the election. Finally we showed that, in large election, as the presence of irrational voters vanishes the equilibrium behavior does not approach the equilibrium behavior when there are no irrational voters. The reason lies on pivotal considerations that, rather than disappear, generate overwhelming evidence of being pivotal in the state in which the correct alternative is the one Persuader likes the less. This suggests that the sincere behavior derived under a public signal is not robust to trembles or to sharp departures from rationality.

CHAPTER 4

Persuasion in Large Elections

In March 2015, during the primary election for the Democratic Party in Campania, Roberto Saviano, a notorious Italian journalist, firmly suggested voters to stay away from the ballots[1]. He accused those primary elections to favor clientelism inside region Campania. His appeal was to not legitimate through a democratic process the documented mechanism of logrolling inside the territory. Although Saviano's attempt was not to support any particular candidate, his message may be put on the context of a more general decline of turnout rates over the years[2]. The failure to mobilize voters represent a false step for modern democracies because it leaves important decisions be taken by a small number of voters who might not carefully represent society's interests at all. In this chapter, we raise the following question: *who benefits from abstention?* Or, more precisely, *can information be designed in such a way to reduce voters' turnout on purpose?*

The aim of this chapter is to develop a theoretical framework to study the relationship between the supply of public information and voters behavior. We study a two candidates common value large election in which all the available information is designed by a persuader who strictly prefers the winner of the election to be one specific candidate. We allow voters to abstain and we also assume that the electorate is mainly made of rational voters although there is a small fraction of partisan voters who may support one candidate or the other. Whereas Feddersen and Pesendorfer (1996) characterize the relationship between voluntary voting and partisan voters, we study how the presence of

[1] http://www.repubblica.it/politica/2015/03/02/news/saviano-primarie-in-campania-dico-no-basta-clientele-108522144/.

[2] For instance, Gerber and Green (2000) argue that this decline may be in part explained by a change in the electoral campaigning strategies.

partisan voters affects the supply of public information and thus voting behavior.

Our main results can be summarized as follows. Suppose that voters are biased in favors of candidate B and that, before casting a vote, they observe a signal which informs them about the quality of candidates A and B. If voters were freely allowed to look up for their information, they will look for information that conforms to their bias as Calvert (1985), more recently Suen (2004) and Oliveros and Várdy (2014) show. However, if the supply of information is given, the slant of information simply determines voters behavior. Suppose that voters observe the realization of a binary signal $s \in \{a, b\}$ where the signal s implies a recommended action, that is, vote for A if a and vote for B if b. We can measure the slant, or the bias, of information as follows. If $\Pr\{s = a\} > \Pr\{s = b\}$, then information is biased in favor of candidate A. If the source of information is extremely biased in favor of candidate A, rational voters would never rely on this information and they would always elect candidate B. Nevertheless, as $\Pr\{s = a\}$ decreases, voters may change their behavior. In particular, they would support candidate B when the signal suggests them to do so and they might abstain when the signal suggests to vote for candidate A. Finally, if $\text{Prob}\{s = a\}$ decreases even more, voters may start considering trusting the source of information and their vote would precisely reflect the implied recommendations. This voting behavior is summarized in figure 4.1. Consider now

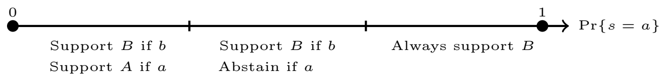

Figure 4.1: Voting behavior and slant.

$\Pr\{s = a\}$ representing the slant of the electoral campaign of candidate A. Is there an optimal way to design the slant so that candidate A is the winner of the election most of the times? Clearly, when $\Pr\{s = a\}$ is close to one, candidate A loses the consensus of rational voters which we assume to constitute the majority of voters. Thus, the optimal slant would concede that candidate A is not always the best candidate, so $\Pr\{s = a\} < 1$. However, we can say more about the optimal slant if we consider the role of partisan voters. Let θ_A and θ_B denote the number of partisans for candidate A and B. If $\theta_A < \theta_B$ the electoral campaign must convince rational voters to support candidate A. Without their support, candidate A is not able to win the election. On the contrary, for $\theta_A > \theta_B$ the optimal slant may favor candidate A most of the times. Indeed, candidate A does not need the support of rational voters. If they abstain, candidate A may simply rely on his partisanship base to win the election. In general, there exists an optimal way to design the slant of the electoral campaign and we show how it relies on the presence of partisan voters and why it may induce rational voters to stay aside from the ballots.

The remainder of the chapter is organized as follows. From section 4.1 to 4.4 we formalize the model and we characterize the equilibria of the game. In section 4.2 we relate our paper with the recent literature whereas in section 4.5 we discuss the theoretical framework of the model. Conclusions follow.

4.1 The Model

We study a two candidates common value election characterized by a Poisson distributed number of voters. Voting is voluntary, that is voters have the option to abstain. There are two kinds of voters, partisans and non partisans. Non partisan voters, or rational voters[3], constitute the majority and want to take the correct decision. On the other hand, partisan voters always prefer one candidate over the other. Prior to casting a ballot, voters receive a public signal correlated with the state of the world. The main novelty here is that we allow the signal to be endogenously and strategically designed by a Persuader who strictly favors one candidate over the other. We can think of the Persuader designing the *slant* of the electoral campaign for his favorite candidate. After the observation of the public signal, voters simultaneously cast a ballot for one of the two candidates or they abstain. The winner of the election is selected through majority rule and payoffs are realized. The timing of the game is as follows:

1. The Persuader chooses the slant of the electoral campaign;
2. Nature selects the state of the world;
3. The public signal is drawn;
4. Nature selects the number of voters;
5. Voters observe the public signal;
6. Ballots are cast and payoffs are realized. Ties are broken in favor of the correct alternative.

Formally, $C = \{A, B\}$ denotes the set of candidates and $\Omega = \{\alpha, \beta\}$ the states of the world which we assume to be equally likely. The number of voters n is drawn from a Poisson distribution (Myerson (1998a, 2000)) with mean k

[3] From now on simply voters.

which is independent from the state of the world. Thus, the probability with which Nature draws n voter is

$$\Pr\{N = n|\omega\} = e^{-k}\frac{k^n}{n!}.$$

The fact that k is constant across states implies that being recruited by Nature as a voter does not convey any information about the state of the world. The probability that a voter is a partisan is $p \in (0, \frac{1}{2})$ and independent across voters. With probability $\theta_A \in (0, 1)$ a partisan voters prefers candidate A and with probability $\theta_B = 1 - \theta_A$ she prefers candidate B.

Before casting a ballot, voters observe a public signal $s \in S = \{a, b\}$. The probability with which the signal recommend a or b is fully characterized by the slant $\pi = (\pi_a, \pi_b) \in [0, 1]^2$ where $\pi_a = \Pr\{s = a|\alpha\}$ and π_b is defined analogously[4]. Conditional on signal s voters form the posterior belief

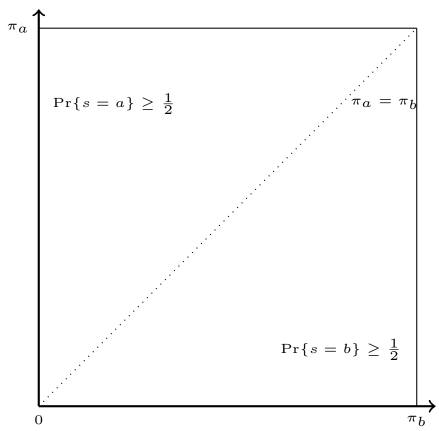

Figure 4.2: Slant of the electoral campaign. Above the line $\pi_a = \pi_b$ the public signal supports most of the times candidate A.

$$\mu_\omega(s) = \frac{\pi(s|\omega)}{\sum_{\omega' \in \Omega} \pi(s|\omega')}$$

since voters gather no additional information for being recruited in the game and each state is equally likely. Voters can cast a ballot for one of the two candidates ore they can abstain. The action set is denoted by $X = \{A, B, \phi\}$ where ϕ denotes abstention. Voters want to match the winner of the election with the

[4]We can think of s as a recommended action.

state of the world and they are characterized by the following preferences:

$$u(c,\omega) = \begin{cases} 0 & \text{if } c = A, \omega = \alpha \text{ or } c = B, \omega = \beta, \\ -q & \text{if } c = A, \omega = \beta, \\ -(1-q) & \text{if } c = B, \omega = \alpha. \end{cases}$$

We assume $q > \frac{1}{2}$ which means that a mistake in state β is more costly than a mistake in state α. Thus, without any other further information voters would prefer candidate B. On the contrary, the Persuader strictly prefers candidate A to be the winner of the election regardless from the true state of the world:

$$u^P(c,\omega) = \begin{cases} 1 \text{ if } c = A \\ 0 \text{ otherwise} \end{cases} \qquad \omega \in \{\alpha, \beta\}.$$

Notice that voters and persuader's preferences are not aligned.

A voting strategy $v : S \to X$ maps signals $\{a, b\}$ into a vote or a blank ballot. Following Oliveros and Várdy (2014), we refer to particular voting strategies as follows:

$$v(s) = AB \text{ iff } \begin{cases} v(a) = A, \\ v(b) = B. \end{cases}$$

Thus, the space of voting strategies is

$$\mathcal{V} = \{AA, A\phi, AB, \phi B, BB, \phi\phi\}.$$

Each voting strategy determines the probability $\tau_c(\omega|\cdot)$ with which a randomly drawn voter casts a ballot for alternative c in state ω. We call these probabilities *voting probabilities*. As long as voters observe the realization of a public signal, voters' information is perfectly correlated. Therefore, it is convenient to express these voting probabilities conditional on the public draw. So we have

$$\tau_A(\omega|s,v) = p\theta_A + (1-p) \cdot \Pr\{x = A|v,s\}$$
$$\tau_B(\omega|s,v) = p\theta_B + (1-p) \cdot \Pr\{x = B|v,s\}.$$

To understand these voting probabilities consider $\tau_A(\omega|s,v)$. With probability p Nature draws a partisan voter who votes for candidate A with probability θ_A. With probability $1-p$ Nature draws a rational voter who votes for candidate A with probability 1 if and only if her strategy prescribes $v(s) = A$. Importantly, observe that $\tau_c(A|s,v) = \tau_c(B|s,v)$. The fact that voting probabilities are constant across states comes from the fact that the probability with which Nature recruits a particular type of voter is independent from the true state of the world. Let us denote with $n_c(\omega|\cdot)$ the total number of votes for candidate c in state ω.

Recall that the number of voters is a random variable itself. Denote with $W_c(\omega)$ the probability with which candidate $c \in \{A, B\}$ wins the election. Then

$$W_A(\alpha|\cdot) = \Pr\{n_A(\alpha|\cdot) - n_B(\alpha|\cdot) \geq 0\}$$
$$W_B(\beta|\cdot) = \Pr\{n_B(\beta|\cdot) - n_A(\beta|\cdot) \geq 0\}.$$

An equilibrium is a pair (π^*, v^*) made of an optimal slant and an optimal voting strategy such that no player in the game has a profitable unilateral deviation. Formally, (π^*, v^*) is an equilibrium of the game if and only if for all voters, for all $v \in \mathcal{V}$ and for all $\pi \in [0,1]^2$

i) $\mathbb{E}[u(v^*|Piv(v^*), \pi^*)] \geq \mathbb{E}[u(v^*|Piv(v^*), \pi^*)]$,

ii) $\mathbb{E}[W_A(\pi^*, v^*)] \geq \mathbb{E}[W_A(\pi', v^*)]$.

We restrict the analysis to symmetric pure voting strategies and we assume that voters' indifference is solved in favor of the Persuader. This is equivalent to the *sender-preferred* equilibrium refinement in Kamenica and Gentzkow (2011). Condition i) says that v^* is a best reply to π^* and that voters are rational, they use all the available information and condition their votes on being pivotal. Condition ii) says that π^* is a best reply to v^* and that the Persuader maximizes the expected probability with which candidate A wins the election.

4.1.1 Pivotal Events and Voting Behavior

A rational voter anticipates that her vote is relevant to her payoff only when it changes the outcome of the election. A vote may change the outcome of the election only if there is a tie or one candidate is winning by just one vote. These events are called pivotal and a rational voter conditions her decision on being pivotal.

Conditional on signal s, voters are recruited in a Poisson game. Since the number of voters is Poisson distributed, the number of votes for candidate c conditional on signal s are independent Poisson random variables[5] with mean $k\tau_c(\omega|s, v)$. Denote with $T_m(\omega|s, v)$ the probability with which candidate A is winning by m votes conditional on signal s and voting strategy v. Because $n_c(\omega|s, v)$ are independent, their difference follows a Skellam distribution:

$$T_m(\omega|s, v) \equiv \text{Prob}\{n_A(\omega|s, v) - n_B(\omega|s, v) = m\}$$
$$= e^{-k[\tau_A(\omega|s,v)+\tau_B(\omega|s,v)]} \left(\frac{\tau_A(\omega|s, v)}{\tau_B(\omega|s, v)}\right)^{\frac{m}{2}} I_{|m|} \left(2k\sqrt{\tau_A(\omega|s, v)\tau_B(\omega|s, v)}\right)$$

[5]This follows from the decomposition property of the Poisson distribution, see Myerson (1998a,b).

where $I_z(x)$ is the modified Bessel function of the first kind[6]. Notice that, because voting is voluntary, $\tau_A(\omega|s,v) + \tau_B(\omega|s,v) < 1$ if $v(s) = \phi$ for some $s \in \{a,b\}$.

From equilibrium condition i), a voting strategy must be rational. A necessary condition for $v \in \mathcal{V}$ to be rational is that, conditional on $s \in \{a,b\}$, no voter has a profitable unilateral deviation. Formally, v is rational if only if, for all voters, for all $s \in \{a,b\}$ and for all $v' \in \mathcal{V}$, given π,

$$\mathbb{E}[u(v|Piv(v),\pi,s)] \geq \mathbb{E}[u(v'|Piv(v),\pi,s)]. \tag{4.1}$$

We denote with $Piv_x(\omega|v,s)$ the probability with which vote x is pivotal given that voters are playing v conditional on signal s. Then we have

$$Piv_A(\beta|s,v) = T_0(\beta|s,v) + T_{+1}(\beta|s,v),$$
$$Piv_B(\alpha|s,v) = T_0(\alpha|s,v) + T_{-1}(\alpha|s,v).$$

The explanation is the following. A vote for $x = A$ can only be pivotal in state β, that is in the state and in the events in which the victory of candidate A would result in a mistake. Therefore, we can write the conditional expected utility deriving from casting a vote for candidate A or B conditional on signal s given that voters are playing v as

$$U(A|s,v) = -q\mu_\beta(s)Piv_A(\beta|s,v), \tag{4.2}$$
$$U(B|s,v) = -(1-q)\mu_\alpha(s)Piv_B(\alpha|s,v). \tag{4.3}$$

Analogously, the conditional expected utility deriving from casting a blank ballot is

$$U(\phi|s,v) = -q\mu_\beta(s)T_{+1}(\beta|s,v) - (1-q)\mu_\alpha(s)T_{-1}(\alpha|s,v). \tag{4.4}$$

In state β abstention or a blank ballot yields a negative payoff only if candidate A is winning whereas in state α abstention leads to a negative payoff only if candidate B is winning.

The following lemma follows directly from (4.2), (4.3) and (4.4).

Lemma 4.1. ***Optimal conditional voting strategy.***
The voting strategy $v \in \mathcal{V}$ is rational if an only if $v(s) = x \in \{A,B,\phi\}$ satisfies the following conditions.

$$v(s) = A \textit{ if and only if } U(A|s,v) \geq U(B|s,v) \textit{ and } U(A|s,v) \geq U(\phi|s,v)$$

that is, if and only if

$$\frac{\pi(s|\alpha)}{\pi(s|\beta)}\frac{1-q}{q} \geq \frac{Piv_A(\beta|s,v)}{Piv_B(\alpha|s,v)} \textit{ and } \frac{\pi(s|\alpha)}{\pi(s|\beta)}\frac{1-q}{q} \geq \frac{T_0(\beta|s,v)}{T_{-1}(\alpha|s,v)}.$$

[6] See Abramowitz et al. (1966).

$$v(s) = B \textit{ if and only if } U(B|s,v) > U(A|s,v) \textit{ and } U(B|s,v) > U(\phi|s,v)$$

that is, if and only if

$$\frac{\pi(s|\alpha)}{\pi(s|\beta)}\frac{1-q}{q} < \frac{Piv_A(\beta|s,v)}{Piv_B(\alpha|s,v)} \textit{ and } \frac{\pi(s|\alpha)}{\pi(s|\beta)}\frac{1-q}{q} < \frac{T_1(\beta|s,v)}{T_0(\alpha|s,v)}.$$

$$v(s) = \phi \textit{ if and only if } U(\phi|s,v) \geq U(A|s,v) \textit{ and } U(\phi|s,v) \geq U(B|s,v)$$

that is, if and only if

$$\frac{\pi(s|\alpha)}{\pi(s|\beta)}\frac{1-q}{q} \leq \frac{T_0(\beta|s,v)}{T_{-1}(\alpha|s,v)} \textit{ and } \frac{\pi(s|\alpha)}{\pi(s|\beta)}\frac{1-q}{q} \geq \frac{T_1(\beta|s,v)}{T_0(\alpha|s,v)}.$$

Notice that conditional on $s = a, b$, the optimal action $x \in \{A, B, \phi\}$ depends on π and the voting strategy because the likelihood of casting a pivotal vote is determined by which $v \in \mathcal{V}$ voters are actually playing.

Finally, before concluding this section, we follow Oliveros and Várdy (2014); Feddersen and Pesendorfer (1999) and we use the following approximation. For $m \in \{-1, 0, 1\}$

$$T_m(\omega|s,v) \approx \left(\frac{\tau_A(\omega|s,v)}{\tau_B(\omega|s,v)}\right)^{\frac{m}{2}} \frac{e^{-k\left[\sqrt{\tau_A(\omega|s,v)}-\sqrt{\tau_B(\omega|s,v)}\right]^2}}{\sqrt{4\pi k\sqrt{\tau_A(\omega|s,v)\tau_B(\omega|s,v)}}} \tag{4.5}$$

since $\lim_{x\to\infty}\frac{\frac{e^x}{\sqrt{2\pi x}}}{I_z(x)} = 1$ for $z \in \{-1, 0, 1\}$. These formulas simplify the conditions in lemma 4.1. In particular, we have

$$\lim_{k\to\infty}\frac{Piv_A(\beta|s,v)}{Piv_B(\alpha|s,v)} = \lim_{k\to\infty}\frac{T_0(\beta|s,v)}{T_{-1}(\alpha|s,v)} = \lim_{k\to\infty}\frac{T_1(\beta|s,v)}{T_0(\alpha|s,v)} = \sqrt{\frac{\tau_A(\omega|s,v)}{\tau_B(\omega|s,v)}}. \tag{4.6}$$

Throughout the rest of the article, we assume that k is large enough so that the approximations hold. This is without loss of generality since, as Myerson (1998b) explains, for $n_c(\omega|s,v) > 7$, the error of these approximations is less then 1%.

4.1.2 Electoral Campaign

Persuader maximizes the expected probability with which candidate A wins the election, that is

$$\pi^* = \operatorname{argmax} \mathbb{E}[W_A(\pi, v^*(\pi))]$$

and

$$\mathbb{E}[W_A(\pi, v^*(\pi))] = \frac{1}{2}[W_A(\alpha|s,\pi,v^*(\pi))] + \frac{1}{2}[W_A(\beta|s,\pi,v^*(\pi))].$$

Notice that the voting strategy being played in equilibrium is a best response $v^*(\pi)$ to the slant chosen by the Persuader. The following proposition characterizes the limit probability with which candidate $c \in \{A, B\}$ wins the election given any voting strategy $v \in \mathcal{V}$.

Proposition 4.1. ***Limit winning probabilities.***
Given any $v \in \mathcal{V}$, if $\tau_c(\omega|s, v) > \tau_{c'}(\omega|s, v)$ and $c \neq c'$, then

$$\lim_{k\to\infty} \text{Prob}\{n_c(\omega|s, v) - n_{c'}(\omega|s, v) > 0\} = 1.$$

That is to say, conditional on signal s, if v is such that the expected number of votes for candidate c is higher than the expected number of votes for candidate $c' \neq c$, then candidate c wins the election almost surely conditional on signal s as $k \to \infty$.

Proof. The proof follows Myerson (1998a). Recall that $n_c(\omega|s, v)$ are independent Poisson random variables. Hence, their difference follows a Skellam distribution and so:

$$\mathbb{E}[n_c(\omega|s, v) - n_{c'}(\omega|s, v)] = k[\tau_c(\omega|s, v) - \tau_{c'}(\omega|s, v)]$$
$$\sqrt{Var}\,[n_c(\omega|s, v) - n_{c'}(\omega|s, v)] = \sqrt{k}.$$

Then we have

$$\lim_{k\to\infty} \frac{\mathbb{E}[n_c(\omega|s, v) - n_{c'}(\omega|s, v)]}{\sqrt{Var}\,[n_c(\omega|s, v) - n_{c'}(\omega|s, v)]} = \lim_{k\to\infty} \frac{k[\tau_c(\omega|s, v) - \tau_{c'}(\omega|s, v)]}{\sqrt{k}} = \infty$$

if $\tau_c(\omega|s, v) > \tau_{c'}(\omega|s, v)$ as claimed. □

In a large election as $k \to \infty$, we can characterize the probability with which candidate A is going to win the election for every $v \in \mathcal{V}$. The reason is the following. Each voting strategy induces a pair of voting probabilities. In turn, voting probabilities, thanks to proposition 4.1, simply determine whether candidate $c \in \{A, B\}$ wins or looses the election.

Lemma 4.2. ***Voting strategies and outcomes.*** *As $k \to \infty$:*
If $\theta_A > \theta_B$, then

$$\mathbb{E}[W_A(\pi, v)] = \begin{cases} 1 & \textit{if } v \in \{AA, A\phi, \phi\phi\}, \\ 0 & \textit{if } v = BB, \\ \Pr\{s = a\} & \textit{if } v \in \{AB, \phi B\}. \end{cases}$$

If $\theta_A < \theta_B$, then

$$\mathbb{E}[W_A(\pi, v)] = \begin{cases} 1 & \textit{if } v = AA, \\ 0 & \textit{if } v \in \{BB, \phi B, \phi\phi\}, \\ \Pr\{s = a\} & \textit{if } v \in \{AB, A\phi\}. \end{cases}$$

Proof. Appendix. □

Notice that lemma 4.2 characterizes the probability with which candidate A wins the election irrespectively from v being a best response. In the next section we shall consider the voting strategy being a best response to the slant π and hence we characterize the equilibrium strategies and the outcome of the game.

4.2 Equilibrium

The choice of the slant of the electoral campaign determines the probability with which voters observe signal a or b. In lemma 4.1 we pointed out that the optimal voting strategy is a best response to the slant, thus, $v^*(\pi)$ is not constant. Therefore, voting probabilities themselves depend on π and they are crucially determined by $v^*(\pi)$ as long as the majority is made of rational voters. Finally, from voting probabilities voters elicit additional information which concerns the probability of casting a pivotal vote.

To solve the model, we observe that for each $\pi \in [0,1]^2$ voters optimally respond by playing $v^*(\pi)$. Therefore, through the choice of the slant the Persuader determines voters' behavior. In turn, voting behavior pins down winning probabilities. Intuitively, we solve the game in a backward induction fashion. First, we find the signals that maximize the probability with which candidate A wins the election for any $v \in \mathcal{V}$ which is not strictly dominated. Second, we look for those π that induce the most profitable voting strategy.

Proposition 4.2. ***Equilibrium.***
For all $p \in (0, \frac{1}{2})$ and for all $q \in (\frac{1}{2}, 1)$, there exists and it is unique an optimal signal π^* such that $v^*(\pi^*)$ is rational and the persuader is maximizing the probability with which candidate A wins the election. Precisely,

$$\text{if } \theta_A \geq \theta_B\text{: } \pi^* = \left(\pi_a^* = 1, \pi_b^* = 1 - \tfrac{1-q}{q}\sqrt{\tfrac{\tau_B(\omega|a,\phi B)}{\tau_A(\omega|a,\phi B)}}\right) \text{ and } v^*(\pi^*) = \phi B;$$

$$\text{if } \theta_A < \theta_B\text{: } \pi^* = \left(\pi_a^* = 1, \pi_b^* = 1 - \tfrac{1-q}{q}\sqrt{\tfrac{\tau_B(\omega|a,AB)}{\tau_A(\omega|a,AB)}}\right) \text{ and } v^*(\pi^*) = AB.$$

Proof. Appendix. □

The equilibrium signal underlines that, regardless from θ_A, in state α the optimal slant is always perfectly informative. This means that $\mu_\beta(b) = 1$. As a result, conditional on $s = b$ voters are sure that $\omega = \beta$ and they strictly prefer to cast a ballot for candidate B. On the other hand, as we show in the appendix, conditional on $s = a$ the optimal signal makes voters indifferent. Importantly, the optimal signal shares the same properties of the optimal signal derived in Kamenica and Gentzkow (2011). Nevertheless, the optimal signal differs in what follows. The precision of the signal in persuader's worst state depends on the relative likelihood with which voters cast a pivotal vote. The reason is the following. Although voters' information is perfectly correlated because s is publicly observed, as long as the number of voter is uncertain and characterized by the presence of partisan voters, voters may affect the outcome of the election by casting a pivotal vote. Therefore, the optimal signal shall make voters indifferent conditional on casting a pivotal vote.

If type-A partisans outnumber type-B partisans, in equilibrium rational voters abstain whenever $s = a$. On the contrary, when the majority of partisans support candidate B in equilibrium rational voters never abstain. In the next section we discuss the role of abstention and partisan voters by studying the gain from the electoral campaign.

4.3 Value of the Electoral Campaign

The value of the electoral campaign depends on the conflict between voters and persuader's preferences. In state $\omega = \alpha$ their preferences are perfectly aligned. Moreover, at the optimal slant $\pi_a^* = 1$ which means that the electoral campaign is always truthful in state α. On the contrary, it is always the case that $\pi_b^* < 1$ and sometimes the persuader generates evidence in favor of candidate A in state $\omega = \beta$. This is because he wants candidate A to win the election regardless from the true state of the world. On the other hand, voters just want to match the winner of the election with the true state of the world. However, although each candidate is equally likely to be the best candidate, voters preferences are such that a mistake in state B is more costly than a mistake in state β, as we specified $q > \frac{1}{2}$. Therefore, without any information voters would always elect candidate B. A mistake in state α would be more costly than a mistake in state β and if voters were uniformed they would just always elect candidate A.

Definition 4.1. Value of the Electoral Campaign
The value of an electoral campaign characterized by the slant π is

$$\Delta(\pi) = \mathbb{E}[W_A(\pi, v^*(\pi))] - \mathbb{E}[W_A(v^*)]$$

where $v^* = BB$ as long as $q > \frac{1}{2}$.

Notice that, because of proposition 4.1, $\lim_{k\to\infty} \mathbb{E}[W_A(v^*)] = 0$ because the majority is made of rational voters. Therefore, in a large election the value of an optimal electoral campaign is simply $\Delta(\pi^*) = \mathbb{E}[W_A(\pi^*, v^*(\pi^*))]$.

How does the share of partisans affect the electoral campaign and its value? In proposition 4.2 we showed that there exists a unique equilibrium which depends on the value of θ_A. As $\theta_A \geq \theta_B$ the voting strategy switches from $v^*(\pi^*(\theta_A)) = AB$ to $v^*(\pi^*(\theta_A)) = \phi B$. In order to study the role of partisan voters we exploit the fact that in equilibrium $\pi_a^* = 1$ for all $\theta_A \in (0,1)$ whereas π_b^* depends continuously on θ_A, for all $\theta_A \in (0,1) \neq \frac{1}{2}$. Therefore, we can express the value of an optimal electoral campaign as a function of the share of partisans. Since in equilibrium $v^* \in \{\phi B, AB\}$, in large elections we have that, for all $\theta_A \in (0,1)$:

$$\mathbb{E}[W_A(v^*(\pi^*(\theta_A)))] = \Pr\{s = a|\pi^*(\theta_A)\}.$$

Then:

$$\Delta\left(\pi^*(\theta_A)\right) = \Pr\{s = a|\pi^*(\theta_A)\} = \begin{cases} \frac{1}{2}\left[1 + \frac{1-q}{q}\sqrt{\frac{\tau_B(\omega|a,AB)}{\tau_A(\omega|a,AB)}}\right] & \text{if } \theta_A < \frac{1}{2} \\ \frac{1}{2}\left[1 + \frac{1-q}{q}\sqrt{\frac{\tau_B(\omega|a,\phi B)}{\tau_A(\omega|a,\phi B)}}\right] & \text{if } \theta_A \geq \frac{1}{2}. \end{cases} \tag{4.7}$$

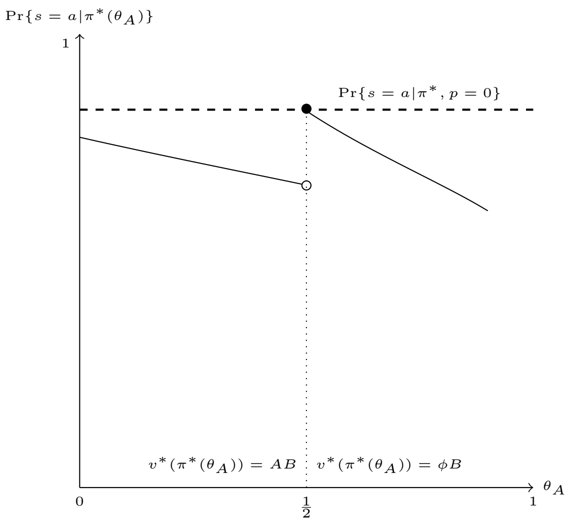

Figure 4.3: Value of an electoral campaign with optimal slant $\pi^*(\theta_A)$. $\Delta\left(\pi^*(\theta_A)\right)$ is upper semi-continuous in θ_A and locally decreasing in θ_A. At $\theta_A = \frac{1}{2}$, v^* switches from AB to ϕB and the function shows a jump. Whenever $p = 0$, in equilibrium $v^* = AB$ and the voting strategy is sincere. This is the maximum payoff the persuader can obtain as can be seen from the dashed line.

Proposition 4.3. ***Value of the electoral campaign and partisans*.**

1. *The value of an electoral campaign is locally decreasing in* θ_A *for all* $\theta_A \in (0,1) \neq \frac{1}{2}$.

2. *For some* $\theta'_A \geq \frac{1}{2}$, $\mathbb{E}[W_A(v^*(\pi^*(\theta'_A)))] > \mathbb{E}[W_A(v^*(\pi^*(\theta_A)))]$.

Point 1) says that $\frac{\partial \Delta(\pi^*(\theta_A))}{\partial \theta_A} < 0$. Nevertheless, point 2) says that for some $\theta'_A \geq \frac{1}{2}$, the value of an electoral campaign is higher for $\theta'_A > \theta_A \in (0, \frac{1}{2})$. This follows from the fact that $\Delta(\pi^*(\theta_A))$ has a jump at $\theta_A = \frac{1}{2}$. The proof is straightforward.

Proof. From proposition 4.2 we have

$$\pi^*_b(\theta_A) = \begin{cases} 1 - \frac{1-q}{q}\sqrt{\frac{\tau_B(\omega|a,AB)}{\tau_A(\omega|a,AB)}} & \text{if } \theta_A < \frac{1}{2} \\ 1 - \frac{1-q}{q}\sqrt{\frac{\tau_B(\omega|a,\phi B)}{\tau_A(\omega|a,\phi B)}} & \text{if } \theta_A \geq \frac{1}{2} \end{cases}$$

where $\sqrt{\frac{\tau_B(\omega|a,AB)}{\tau_A(\omega|a,AB)}} = \sqrt{\frac{p(1-\theta_A)}{p\theta_A+1-p}} < \sqrt{\frac{\tau_B(\omega|a,\phi B)}{\tau_A(\omega|a,\phi B)}} = \sqrt{\frac{1-\theta_A}{\theta_A}}$. Thus, there exists $\theta'_A \in [\frac{1}{2}, 1)$ such that $\mathbb{E}[W_A(v^*(\pi^*(\theta'_A)))] > \mathbb{E}[W_A(v^*(\pi^*(\theta_A)))]$ for any $\theta_A \in (0, \frac{1}{2})$. This proves point 2). To prove point 1), notice that

$$\frac{\partial \pi^*_b(\theta_A)}{\partial \theta_A} > 0 \;\; \forall \theta_A \in (0,1) : \theta_A \neq \frac{1}{2}.$$

□

Proposition 4.3 states that a marginal increase in the share of type-A partisans makes the persuader worse off. However, for some $\theta_A \geq \frac{1}{2}$ the persuader is better off. Although the statement might seem contradictory, the explanation is straightforward. Consider $\theta_A < \frac{1}{2}$. If rational voters abstain, candidate B is always going to win the election. Therefore, the persuader can either induce $v^* = AB$ or $v^* = A\phi$. As we showed in proposition 4.2, the persuader is better off whenever $v^* = AB$ if $\theta_A < \theta_B$. Intuitively, because $q > \frac{1}{2}$, voters are not willing to give up a vote for candidate B unless the slant reports $s = b$ most of the times. In such a case, candidate B would win the election most of the times. Rational voters would abstain conditional on $s = b$ whereas they are confident enough to cast a ballot for candidate A whenever $s = a$. Clearly, this is not optimal from persuader's point of view. If he increases the bias of the slant towards his candidate, rational voters strictly prefer to support candidate B conditional on $s = b$[7]. In equilibrium, $v^* = AB$ is enforced in such a way that conditional on $s = a$ voters are actually indifferent between the two candidates. Because v^* must be rational, the optimal signal takes into account

[7]Notice however that conditional on $s = b$ this would not change the outcome of the election. That is, irrespectively from the fact that $v(b) = \phi$ or B, conditional on $s = b$ candidate B wins the election.

voters' beliefs about casting a pivotal vote. As θ_A increases, the probability of casting a pivotal vote for candidate A decreases because most of the population is supporting candidate A. Thus, as θ_A increases, the persuader reduces the bias of his slant so to persuade voters to still support his candidate conditional on $s = a$. When $\theta_A \geq \frac{1}{2}$ the persuader has a different kind of leverage. In particular, he does not need rational voters to support him. Whenever rational voters abstain, candidate A is going to win the election. Recall that in equilibrium $\pi_a^* = 1$. This means that $s = b$ can not be observed in state A. This makes voters sure of their action when they observe $s = b$. Furthermore, he does not need to convince voters to vote for him. It is sufficient to convince them not to always support candidate B. In addition, it is less costly to convince them to abstain rather than casting a vote for A conditional on $s = a$. This is a consequence of the fact that in order to support candidate A, the persuader must supply enough evidence supporting his candidate. However, this would adversely affect his campaign. As long as $\pi_a^* = 1$, conditional on $s = a$ voters understand that the true state could be either A or B. The only way to supply evidence backing candidate A is to decrease the probability with which $s = a$ could be observed in state β. Overall, this reduces $\Pr\{s = a\}$. This is why at $\theta_A = \frac{1}{2}$ we observe a switch in the voting strategy. Nevertheless, as θ_A increases again, the persuader must reduce his bias towards a. Indeed, if voters abstain and $\theta_A > \theta_B$, a mistake is much more likely in state β rather than in state α. In particular, abstention would lead to a mistake only if candidate A were loosing by one in state α, which is much more less likely than candidate A winning by one, because $\theta_A > \theta_B$, and that event happens to be pivotal in state β.

The second question we are concerned with is whether the presence of partisan voters actually restrains or fosters elections' manipulation. Suppose that $p = 0$ so that there are not partisan voters. Then, it is easy to see that voters are no longer concerned with strategic considerations. All information is public which means that voters' actions are perfectly correlated and the probability of a casting a pivotal vote is zero whenever $k > 0$. Therefore, as in Alonso and Câmara (2015), in equilibrium voters vote sincerely, that is

$$v^*(\pi) = x \iff \mathbb{E}[u(x|s, v^*(\pi)), \pi] \geq \mathbb{E}[u(x'|s, v^*(\pi)), \pi] \;\; \forall x' \in \{A, B, \phi\}.$$

Corollary 4.1. Value of the electoral campaign. No partisans.
If $p = 0$ and k is constant across states, in equilibrium voting is sincere and

$$\pi_a^* = 1, \pi_b^* = 1 - \frac{1-q}{q}.$$

Moreover, $\Pr\{s = a|\pi^*, p = 0\}$ is the maximum payoff the persuader can achieve.

Proof. The optimal signal can be derived by means of proposition 4.2 or directly from Kamenica and Gentzkow (2011). Notice that at π^* we have $U(A|a, v^*(\pi)) = U(B|a, v^*(\pi))$ and $U(B|b, v^*(\pi)) > U(A|b, v^*(\pi))$. To see that this is the maximum payoff the persuader can achieve, it is sufficient to observe that $\pi_b^* = 1 - \frac{1-q}{q} \leq \pi_b^*(\theta_A)$, for all $\theta_A \in (0, 1)$. □

The fact that the persuader is better off without partisan voters is a direct consequence of Austen-Smith and Banks (1996). They show that whenever sincere voting is not rational is because voters are not taking into account all the available information. That is, they neglect the probability with which a vote may affect the outcome of the election and this information is generated in equilibrium through the choice of the voting strategy. However, if there are not partisan voters and the signal is publicly observed, information is perfectly correlated among voters and there is no additional equilibrium information voters can elicit to improve their decisions. On the other hand, when Nature draws a partisan voter with positive probability, for any voting strategy pivotal events are well defined and voters use this information so to decrease the probability of a mistake. Finally, observe that at $\theta_A = \theta_B$ the persuader achieves the same payoff he would have obtained for $p = 0$. This is because the probability of making a mistake when rational voters abstain depends just on θ_A. If $\theta_A = \theta_B$ the noise introduced by Nature drawing partisan voters is uninformative and it is easily verified that $\pi_b^*(\theta_A) = 1 - \frac{1-q}{q}$ at $\theta_A = \theta_B$.

4.4 Related Literature

Our work is mainly related to the growing body of literature on Bayesian Persuasion (Kamenica and Gentzkow, 2011). They consider a sender receiver framework and they show how a sender can change the actions of the receiver by designing what she can learn from a public signal. Alonso and Câmara (2015) and Wang (2015) extended the model to a multi receiver setting. Alonso and Câmara derive voters' preferences over voting rule in a persuasion framework whereas Wang studies the difference between private and public persuasion mechanism. He shows that the persuader is strictly better off when information is publicly shared. We depart from their works in several aspects. First, we assume a large population of voters whose size is uncertain. Second, although voters observe a public signal, population uncertainty and the presence of partisan voters introduce in the model strategic voting considerations. Finally, we study the role of abstention which, up to our knowledge, has never been considered before in a Bayesian persuasion setting. Thus, our work is also related to voluntary voting models as Feddersen and Pesendorfer (1996) and Krishna and Morgan (2012a). Another closely related paper is that of Ekmekci and Lauermann (2015). In particular, they study how election manipulation could be achieved by changing the number of voters in each state. From being recruited, voters elicit some information which suggests them the state of the world in which they are more likely to be drawn. This is how a persuader can gain from such a manipulation.

4.5 Discussion

In a seminal paper, Feddersen and Pesendorfer (1996) showed that under private information and common values, indifferent voters have an incentive to abstain although ex-ante they all prefer one candidate over the other. Moreover, they show how strategic voting along with the option to abstain may lead to superior outcomes in which information aggregates. More recently, in a related paper Krishna and Morgan (2012a) asked themselves whether voting should be a right or a duty. In general, they find that voluntary voting is welfare superior to compulsory voting which suggests voting should be a right. The intuition beyond these results is that indifferent voters prefers to let the outcome of the election be determined by those voters who are better informed. These works, and many other voting models, share the idea that:

- voters are privately informed;
- votes are independent across voters.

In our model information, i.e., the slant of the electoral campaign, is endogenously supplied in equilibrium and consists of a public signal rather than a private one. Therefore, voters' information is perfectly correlated as they are their votes. Even though rational voters may abstain, the positive outcomes derived in the afore mentioned papers fail to occur. This occurs for two main reasons. The first is that, as there is no private information, information aggregation problems do no arise. Second, the only source of information is strategically designed. Indeed, the slant of the electoral campaign is designed in such a way that, conditional on being pivotal, voters play persuader's favorite strategy. Whenever possible, the persuader exploits swing voters to increase the probability with which his favorite candidate wins the election. The failure of abstention to increase voters' welfare is a direct consequence of the fact that information is strategically designed by a player who does not share voters' preferences and that the signal is publicly observed.

The role of public signals and verifiable information.
In most of Condorcet related models, it is assumed that voters are privately informed. This reflects the fact that voters, or jurors, interpret the evidence according to their model of reality (Ladha, 1992) which results from their life experiences Feddersen and Pesendorfer (1998). Although this assumption may seem appropriate in a variety of settings, we argue on the contrary that a Persuader would prefer to share *public* and *verifiable* information.

Wang (2015) and Taneva (2015) show that, in a sender-multi receivers game, the persuader is better off if he relies on public information. That is, if the persuader could choose how to inform voters, he would choose to inform them with a public signal rather than a private one. Liu (2015) and Kawamura and Vlaseros (2015) show that privately informed voters might coordinate their vote on an exogenous public signal. That is, the presence of public information might lead voters to vote against their private information. Finally, empirical evidence supports the idea that the exposition to public biased information remarkably affects voting behavior[8].

The second issue concerns the reliability of the information publicly shared. Clearly, the diffusion of false evidence could jeopardize voters' perception about a candidate and the career of the candidate himself were the evidence discovered to be false. In general, the diffusion of *false* public information may dismantle the reputation of the object of the information or, even worse, it may lead to criminal charges[9]. In our model, the choice of the slant affects stochastically the realization of the public signal. Nonetheless, the public signal is always truthfully revealed although it may harm persuader's favorite candidate[10]. A deceitful issue is the difference between *wrong* and *false* information. During the Greek crisis, the IMF acted as an Expert whose advices were publicly known to the European community. IMF opinions had been followed publicly by the media, so that political discussions have referred to IMF *true* statements. Although the IMF conceded that "it made mistakes on Greece[11]", political actors decided on the ground of those advices. To translate what happened in the terms of our model, we could say that the IMF prepared a study π whose outcome had

[8] See for instance DellaVigna and Gentzkow (2009), Barone et al. (2015), Gentzkow et al. (2009).

[9] More recently, Volkswagen has been discovered declaring fake statistics about consumption and pollution levels of its cars. Hagens-Berman, one of the top litigation firm in the U.S., is expanding a class action against the German firm that could jeopardize Volkswagen's fate; http://www.hbsslaw.com/newsroom/Hagens-Berman-Expands-Class-Action-Suit-Against-Volkswagen-for-Emissions-Cheating-Software.

[10] Kamenica and Gentzkow (2011) discuss broadly this commitment assumption.

[11] http://www.wsj.com/articles/SB10001424127887324299104578527202781667088.

been s and it truthfully reported it to the interested parties. The study implied a recommended policy (austerity) associated to s so that we could say the IMF advanced evidence in favor of austerity measures. The implementation of these measures led to a non desired outcome. Consider the following study:

$$\Pr\{a|Austerity\} = 1, \Pr\{b|Stimulus\} < 1.$$

This study is biased in favor or signal a which suggests to implement measures of austerity. Since $\text{Prob}\{a|Stimulus\} > 0$, the advice "implement austerity measures" could be observed with positive probability when the correct policy to be implemented would be "stimulus". The recommendation came from a biased study which nevertheless was truthful. The key element of the argument is the following: the persuader may choose a biased source of evidence but he is not allowed to lie about what he discovers.

Ultimately, public verifiable evidence does not necessarily mean that it suggests good decisions all the times and this is exactly why persuasion may be effective at all.

Common value and preferences.
In our model we assumed that there exists a Pareto superior alternative over which voters agree upon and that rational voters are identical. It is well recognized that elections may be characterized by the absence of such a Pareto superior alternative whenever voters' interest are not aligned (Miller, 1986). However, our model could easily accommodate this situation. Suppose that with probability $1-p$ Nature draws a voter who wants to match the winner of the election with the state of the world whereas with probability p Nature draws a voter who wants to dis-match the winner of the election with the state of the world. Thus, although voters are characterized by a strong conflict of interest, Persuader's behavior is easy to characterize. In order to win the election, it is sufficient that he persuades the group who is most numerous and the model can be solved in the same way. Analogously, the assumption that rational voters are identical could be easily relaxed. Suppose now that all voters are rational but they differ in their preferences either because the share different "taste" parameter q or because, prior the public draw, they receive some private information. For some voters, their private information or their taste might be so strong that after observing signal s their attitude would not change. In this manner we could endogenously introduce partisan voters in our model. Although rational voters may still differ in their preference parameter, the persuader does not need to convince them all. Indeed, it is sufficient to persuade voter q_i from voter q_m so that the majority follows the action implied by the signal.

Costly electoral campaign and competition

Throughout the model we assumed that the persuader can choose any signal in $\pi \in [0,1]^2$. Although this assumption could clearly be relaxed without altering the main intuition of the paper, we assumed that any π has the same (null) cost. Kamenica and Gentzkow (2014) extended their basic model allowing the choice of π to be costly. Importantly, notice that the focus here is on the cost of π because the persuader always reports the true signal. A natural cost formulation for π is the entropy it induces on beliefs. Kamenica and Gentzkow show that the model can deal with such a cost formulation and it could be easily implemented in our model as long as the signal remains public. When one player can exclusively manipulate information, the choice of the slant is without loss of generality not costly. Nevertheless, if two or more persuaders were designing the electoral campaign of two or more candidates, differences in cost may determine the outcome of election. A natural extension of our model is to allow both candidates to design their own, possibly costly, electoral campaign. However, this is left for future research.

Who is the Persuader?

In our model we assumed that "the Persuader" is not the candidate himself. Whereas we can let without difficulties the Persuader be the candidate, it is worth to underline that electoral campaigns are mainly financed by private groups. For instance, in the 2012 Presidential Elections, Obama raised $632,177,423 against $389,088,268 raised by Romney. It is beyond the scope of this article to study the relationship between fundings and policies, however we can expect policies to be, at least partially, affected by those contributions. As table 4.1 shows, most of Romney's contribution came from the financial sector which might have affected voters' perception of the candidate after the 2008 crisis.

2012 Presidential Election

B. Obama		M. Romney	
University of California	$1,350,139	Goldman Sachs	$1,045,454
Microsoft Corp.	$815,645	Bank of America	$1,017,652
Google Inc.	$804,249	Morgan Stanley	$920,805
US Government	$736,722	JPMorgan Chase & Co	$835,596
Harvard University	$680,918	Wells Fargo	$693,576

Table 4.1: Top contributors, source: https://www.opensecrets.org/pres12/

The question "who is the Persuader" may be relevant also for other reasons. For instance, winning a referendum closely or winning it with a vast consensus may have an intrinsic political value. In general, the more the votes, the larger the seats in the Parliament. Therefore, the Candidate himself may have different preferences with respect the Persuader. The Persuader simply cares about who wins the election, the Candidate may care about how many votes is he able to get. Thus, whenever the Persuader and the Candidate coincide, the objective function might change leading to a different set of optimal signals.

4.6 Conclusion

In this paper we studied how the strategic design of information can manipulate the outcome of the election. Our results show that the optimal electoral campaign is affected by the presence of partisan voters. When the share of partisan voters that supports Persuader's favorite candidate is large, in equilibrium rational voters abstain. In order to win the election, Persuader relies only on partisan votes. He supplies enough information to convince rational voters to abstain rather than always support the opponent candidate. On the contrary, when partisan voters mainly support the opponent candidate, Persuader needs to convince rational voters to support him. This requires him to supply a more informative signal. Our findings rely on the assumption that information is publicly supplied. When voters receive private information, information tends to aggregate in large elections. On the contrary, the presence of public information reduces the information voters can elicit from being pivotal so that information never aggregates in both states.

4.7 Appendix

Proofs

This section contains the proofs omitted in the previous sections.

Proof of Lemma 4.2.

Proof. Suppose $\theta_A > \theta_B$. If voters play AA we have

$$\tau_A(\omega|a, AA) = \tau_A(\omega|b, AA) > \frac{1}{2} \implies \lim_{k\to\infty} \mathbb{E}[W_A(\omega, \pi, AA)] = 1$$

from proposition 4.1. If voters play $\phi\phi$ we have

$$\tau_A(\omega|a, \phi\phi) > \tau_B(\omega|b, \phi\phi) \implies \lim_{k\to\infty} \mathbb{E}[W_A(\omega, \pi, \phi\phi)] = 1.$$

If voters play $A\phi$ we have

$$\tau_A(\omega|a, A\phi) > \tau_A(\omega|b, A\phi) > \tau_B(\omega|b, A\phi) \implies \lim_{k\to\infty} \mathbb{E}[W_A(\omega, \pi, A\phi)] = 1$$

since $\tau_A(\omega|a, A\phi) = p\theta_A + 1 - p$ and $\tau_A(\omega|b, A\phi) = p\theta_A > p\theta_B = \tau_B(\omega|b, A\phi)$. If voters play AB we have

$$\tau_A(\omega|a, AB) > \tau_B(\omega|b, AB) > \frac{1}{2}$$

which implies

$$\lim_{k\to\infty} W_A(\omega|a, AB) = \lim_{k\to\infty} W_B(\omega|b, AB) = 1$$

and therefore

$$\lim_{k\to\infty} \mathbb{E}[W_A(\omega, \pi, AB)] = \frac{1}{2}\pi_a + \frac{1}{2}(1 - \pi_b) = \text{Prob}\{s = a\}.$$

The proof for $\theta_A < \theta_B$ is analogous. □

Proof of Proposition 4.2.

Proof. The proof is divided in two cases and assumes that $k \to \infty$ so that the results in proposition 4.1 hold. For ease of exposition, during the proof we let $Q \equiv \frac{1-q}{q}$ and $\rho(s, v) \equiv \lim_{k\to\infty} \frac{Piv_A(\omega|s,v)}{Piv_B(\omega|s,v)}$

Case 1: $\theta_A > \theta_B$.

From lemma 4.2 it follows directly that in equilibrium the optimal voting strategy must belong to $\{BB, AB, \phi B\}$. If it is not so, for any other strategy candidate A always wins the election. This is clearly not optimal since $q > \frac{1}{2}$. Intuitively, a pivotal voter would rather deviate. On the other hand, if $v = BB$ candidate A never wins the election. Therefore, the optimal signal must induce either $v^*(\pi) = AB$ or $v^*(\pi) = \phi B$, if possible.

From lemma 4.1, $v = \phi B$ if and only if

$$U(A|a, \phi) = U(B|a, \phi) \text{ and } U(B|b, B) > U(A|b, B).$$

Then we have:

$$\frac{\pi_a}{1 - \pi_b} Q = \rho(a, \phi) \text{ and } \frac{1 - \pi_a}{\pi_b} Q < \rho(b, B).$$

From lemma 4.2 we have that $\mathbb{E}[W_A(\pi, \phi B)] = \Pr\{s = a\}$. For $\pi_a = 1$ and $\pi_b = 1 - Q\frac{1}{\rho(a,\phi)}$, $v = \phi B$ is rational and $\Pr\{s = a|\pi\} > \Pr\{s = a|\pi'\}$ for any other $\pi' \neq \pi$ such that $v = \phi B$ is rational. Let

$$\pi_{\phi B} = \text{argmax} \Pr\{s = a|v^*(\pi) = \phi B\}.$$

So: $\pi_{\phi B} = \left(\pi_{a\phi B} = 1, \pi_{b\phi B} = 1 - Q\frac{1}{\rho(a,\phi)}\right)$.

From lemma 4.1, $v = AB$ if and only if

$$U(A|a, A) \geq U(B|a, A) \text{ and } U(B|b, B) > U(A|b, B).$$

Then we have:

$$\frac{\pi_a}{1-\pi_b}Q \geq \rho(a, A) \text{ and } \frac{1-\pi_a}{\pi_b}Q < \rho(b, B)$$

which are exactly the same conditions for $v = \phi B$ being played in equilibrium with the caveat that now pivotal events depend on the voting strategy $v = AB$. So: $\pi_{AB} = \left(\pi_{aAB} = 1, \pi_{bAB} = 1 - Q\frac{1}{\rho(a,A)}\right)$. Notice that $\pi_{bAB} \neq \pi_{b\phi B}$. In particular:

$$\pi_{bAB} = 1 - Q\sqrt{\frac{p\theta_B}{p\theta_A + 1 - p}} > 1 - Q\sqrt{\frac{\theta_B}{\theta_A}} = \pi_{b\phi B}.$$

Therefore,

$$\mathbb{E}[W_A(\pi_{\phi B}, \phi B)] > \mathbb{E}[W_A(\pi_{AB}, AB)]$$

since $\Pr\{s = a\}$ is strictly increasing in π_a and strictly decreasing in π_b. This shows that, for $\theta_A > \theta_B$, the pair $(\pi_{\phi B}, \phi B)$ uniquely satisfies equilibrium condition i) and ii).

Case 2: $\theta_A < \theta_B$.

The proof for case 2 is analogous. From lemma 4.2 it follows directly that in equilibrium the optimal voting strategy must belong to $\{BB, AB, A\phi\}$. Therefore, the optimal signal must induce either $v^*(\pi^*) = AB$ or $v^*(\pi^*) = A\phi$ if possible. From the previous case, we have $\pi_{AB} = \left(\pi_{aAB} = 1, \pi_{bAB} = 1 - Q\frac{1}{\rho(a,A)}\right)$. On the other hand, From lemma 4.1, $v = A\phi$ if and only if

$$U(A|a, A) \geq U(B|a, A) \text{ and } U(B|b, B) = U(A|b, B).$$

Then we have:

$$\frac{\pi_a}{1-\pi_b}Q \geq \rho(a, A) \text{ and } \frac{1-\pi_a}{\pi_b}Q = \rho(b, B)$$

The equality is uniquely satisfied for $\pi_a = 1 - \rho(b, \phi)\frac{1}{Q}\pi_b$. The condition $U(A|a, A) \geq U(B|a, A)$ becomes:

$$\frac{1 - \rho(b, \phi)\frac{1}{Q}\pi_b}{1 - \pi_b}Q \geq \rho(a, A).$$

Since $\Pr\{s = a\}$ is strictly decreasing in π_b the condition must hold with equality. Thus:

$$\pi_{A\phi} = \left(\pi_{aA\phi} = 1 - \rho(b, \phi)\frac{1}{Q}\pi_{bA\phi},\ \pi_{bA\phi} = \frac{\rho(a, A) - Q}{\rho(a, A) - \rho(b, \phi)}\right).$$

Notice that $A\phi$ is be played in equilibrium only if $Q > \rho(b, \phi)$. Otherwise we would have $\pi_{bA\phi} > 1$. Now we show that for $\theta_A < \theta_B$, the persuader has no incentives to induce $A\phi$. It is sufficient to show that $\pi_{bA\phi} > \pi_{bAB}$. Indeed:

$$\begin{aligned}\pi_{bA\phi} > \pi_{bAB} &\iff \frac{\rho(a,A) - Q}{\rho(a,A) - \rho(b,\phi)} > 1 - Q\frac{1}{\rho(a,A)} \\ &\iff \frac{\rho(a,A) - Q}{\rho(a,A) - \rho(b,\phi)} > \frac{\rho(a,A) - Q}{\rho(a,A)} \\ &\iff \rho(a,A) > \rho(a,A) - \rho(b,\phi)\end{aligned}$$

which is always satisfied. Therefore,

$$\mathbb{E}[W_A(\pi_{AB}, AB)] > \mathbb{E}[W_A(\pi_{A\phi}, A\phi)].$$

This shows that, for $\theta_A < \theta_B$, the pair (π_{AB}, AB) uniquely satisfies equilibrium condition i) and ii). □

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